

# Introduction to Political Economy 14.770

## Problem Set 2

Due date: October 6, 2017.

### Question 1:

This question will walk you through a model of *probabilistic voting* with the possibility of group-specific transfers.

A society is a two party democracy with population normalized to 1, with political parties  $R$  and  $D$  competing to maximize their vote share. Parties compete by proposing a tax rate  $\tau \in [0, 1]$  with proceeds distributed lump-sum to each member of society. Taxing income introduces distortions, so the tax revenue is  $(\tau - v(\tau))\bar{y}$ , where  $\bar{y}$  is average income in society and  $v(0) = v'(0) = 0$  and  $v'(1) = \infty$ , and the government budget constraint is

$$T \leq (\tau - v(\tau))\bar{y}$$

The society is stratified into  $n$  groups. The size of each group varies, but members of the same group have the same income, denoted by  $y_j$ , with differing political ideologies. Let the political leaning towards party  $R$  of individual  $i$  in group  $j$  be  $\sigma_j^i$  and the size of group  $j$  be  $\alpha_j$ , with  $\sum_{j=1}^n \alpha_j = 1$  and naturally  $\sum_{j=1}^n \alpha_j y_j = \bar{y}$ . Assume that  $\sigma_j^i$  is drawn from a distribution  $F_j(x)$  symmetric around 0.

Assume that individuals of the society all share a common utility function

$$U_j^i(c_i, \sigma_j^i) = c_i + [\sigma_j^i + \delta]I_R$$

where  $I_R$  is an indicator for party  $R$  coming to power, and  $\delta$  is a random popularity measure for party  $R$ , drawn from distribution  $G(\cdot)$ .

1. First, ignore the ideological leanings of each group and the relative popularity measure (i.e.,  $\sigma_j^i = \delta = 0$ ). Find the equilibrium in the party competition game and the tax rate announced by the two parties. Does a pure strategy equilibrium always exist?

2. Now characterize the equilibrium with the ideological leanings (still with  $\delta = 0$ ). Does a pure strategy equilibrium always exist?
3. Now assume the parties can offer both a lump-sum redistribution to all members of each group, denoted by  $\omega_j \geq 0$ , so the government budget constraint becomes

$$\sum_{j=1}^n \alpha_j \omega_j + T \leq (\tau - v(\tau)) \bar{y}$$

Show that there is no pure strategy equilibrium for the game when  $\delta$  is known in advance to be 0. Determine conditions for an equilibrium to exist when  $\delta$  is random with distribution  $G$ , and characterize such an equilibrium. Will the two parties necessarily offer the same policy platform?

4. Now fully characterize the equilibrium in this probabilistic voting model assuming that  $\sigma_j^i$  is uniform over  $[-\phi_j^{-1}, \phi_j^{-1}]$  for all  $j$  and  $\delta$  is uniform over  $[-\psi^{-1}, \psi^{-1}]$ .

**Question 2:**

This question will walk you through a simplified version of Feddersen and Pesendorfer (1996)'s "Swing Voter's Curse" – a model we covered in Lecture 3.

Consider the following environment: There are two states of nature,  $\theta \in \{0, 1\}$ . The ex-ante probability of state 0 is  $\alpha := Pr\{\theta = 0\}$ , with  $\alpha < \frac{1}{2}$ .

There are  $N + 1$  voters, where  $N$  is even. The voters vote over two policies,  $x \in \{0, 1\}$ . The implemented policy is chosen via simple majority rule: the alternative that receives  $\frac{N}{2} + 1$  votes wins.

A voter may have one of the three types,  $t \in \{0, 1, i\}$ . A voter with type  $t = 0$  always votes for  $x = 0$ , and similarly, a voter with type  $t = 1$  always votes for  $x = 1$ . A voter with type  $i$  (an *independent voter*) has preferences given by:

$$U_i(x, \theta) = -\mathbf{1}(x \neq \theta)$$

where  $x$  is the chosen policy and  $\theta$  is the state of the world.

Each voter's type is drawn randomly and independently, according to the following distribution: each voter has probability  $\frac{2}{3}$  of being  $t = 0$ ,

probability  $\frac{\gamma}{2}$  of being  $t = 1$ , and probability  $1 - \gamma$  of being  $t = i$ . Here,  $\gamma \in [0, 1]$  parametrizes the expected share of *partisans* in the population. Conditional on being  $t = i$ , a voter is *informed* (i.e. learns the true value of  $\theta$ ) with probability  $q \in [0, 1]$  and *uninformed* with the complementary probability. We will consider the Bayesian Nash Equilibrium of the game induced by this setup.

**Warm-Up.** Consider the case where  $N = 0$ , i.e. there is only one voter. Observe that in the unique Bayesian Nash Equilibrium, an uninformed independent voter votes for  $x = 1$  with probability one. To do this, you first need to observe

$$Pr\{\theta = 0|t = i\} = \alpha < \frac{1}{2}$$

We will now argue that this behavior by uninformed independent voters does not always arise when  $N$  is larger, i.e. the voter no longer votes in isolation.

1. Now, consider the case  $N = 2$  (i.e., there are only three voters). Derive a condition to ensure that there can not be a Bayesian Nash Equilibrium (BNE) where all uninformed independent voters vote for  $x = 1$  with probability one. To do this:
  - Assume the contrary: suppose there is a BNE in which all uninformed independent voters vote for  $x = 1$  with probability one. Derive  $\sigma_{\theta,x}$ , the probability that a voter votes for  $x \in \{0, 1\}$  in state  $\theta \in \{0, 1\}$ , for each value of  $\theta$  and  $x$ .
  - When is an uninformed independent voter pivotal? Using the quantities derived above, calculate the posterior probability that  $\theta = 0$  conditional on the event that an uninformed independent voter is pivotal.
  - Based on the posterior derived above, what is a condition for an uninformed independent voter to deviate and vote for  $x = 0$  instead? (Note that there is no abstention here).

How does this condition depend on  $\alpha$ ,  $\gamma$  and  $q$ ?

2. Generalize the same condition to  $N \geq 2$ . Then show that there exists  $\bar{N}(\alpha, \gamma, q)$  such that for  $N > \bar{N}(\alpha, \gamma, q)$ , there can not be a Bayesian

Nash Equilibrium where all uninformed independent voters vote for  $x = 1$  with probability one.

How does  $\bar{N}(\alpha, \gamma, q)$  change with  $\alpha$ ,  $\gamma$  and  $q$ ?

**Question 3:**

This question will walk you through the Grossman and Helpman (1994) model of *lobbying* – a model we covered in Lectures 6 and 7.

A policymaker chooses the level of a policy vector,  $\mathbf{x}$ , which affects the welfare of several interest groups and the general public. Each group  $i$  offers a non-negative payment schedule  $C_i$  to influence policy. The schedule  $C_i$  is a contract stipulating that if the policymaker sets the policy at  $\mathbf{x}$ , then group  $i$  will pay the policymaker  $C_i(\mathbf{x})$ . The utility of the policymaker is  $G(\mathbf{x}) = a \sum_{i=0}^n W_i(\mathbf{x}) + \sum_{i=1}^n C_i(\mathbf{x})$ , where  $W_i(\mathbf{x})$  is the welfare of group  $i$ , and this formulation implicitly assumes that there are  $n$  groups that are organized and group  $i = 0$  is unorganized and represents all other citizens. The utility of each group  $i$  is  $U_i(\mathbf{x}, c_i) = W_i(\mathbf{x}) - C_i(\mathbf{x})$ . Assume  $W_0, W_1, \dots, W_n$  are strictly concave, twice continuously differentiable functions.

The order of play is as follows: First, all groups simultaneously choose their payment schedules. Next, the policymaker observes the schedules and chooses  $\mathbf{x}$ . An equilibrium is defined as a subgame-perfect Nash equilibrium.

1. Show that if contribution schedules are continuously differentiable, then each group  $i > 0$  making a positive payment in equilibrium will offer a payment schedule that must satisfy  $\partial C_i(\mathbf{x}^*) / \partial x_j = \partial W_i(\mathbf{x}^*) / \partial x_j$ , for each component  $x_j$  of  $\mathbf{x}$ . Interpret this condition. What happens if we do not make this continuous differentiability assumption? Is this assumption plausible?
2. Show that the equilibrium policy maximizes a weighted sum of aggregate welfare and the sum of the groups' welfares. What are the weights?
3. Suppose  $\mathbf{x} = (x_1, x_2)$ , and suppose  $W_0$  can be written as  $W_0(\mathbf{x}) = \beta W_0^1(x_1) + (1 - \beta)W_0^2(x_2)$ . Also, suppose there are two lobby groups, one that cares only about  $x_1$  and one that cares only about  $x_2$ . Suppose the first policy dimension becomes relatively more salient to the public, in the sense that  $\beta$  increases. What happens to  $x_1$  and  $x_2$ , and to the equilibrium contributions made by each group?

**Question 4:**

Consider the following regressions. In each case, explain the reasoning and criticize it. Feel free to elaborate as much as you like, in particular, giving suggestions of how you would improve on the empirical strategy.

A) A researcher wants to find out whether greater ethnic fragmentation leads to worse political decisions. For this reason, she runs a regression of the fraction of local government revenues in U.S. cities spent for education on an index of ethnic diversity in the city.

B) A researcher wants to find out whether common (British) law leads to better political outcomes. For this reason, he runs a regression of an index of corruption on a dummy for having common law rather than French civil law or German legal code.

C) Another researcher wants to answer the same question, and he runs a regression of an index for corruption on a dummy for having common law, and instruments this using a dummy for having been a British colony.

D) A researcher wants to investigate the relationship between democracy and inequality, so he runs a regression of various measures of democracy on measures of inequality.

E) A researcher wants to investigate whether political instability in a country's neighbors has a negative effect on economic performance. So he runs a regression of log income on a variety of controls, an index of political instability in the country, and the average of the index of political instability among the country's neighbors.

F) A researcher wants to investigate the relationship between inequality and growth, so he runs a regression of growth on initial inequality using cross-sectional data. He also runs a panel regression of growth in a five-year period on inequality during the five-year period, as well as country fixed effects and time effects.

G) A researcher wishes to show that Downsian policy convergence fails, so runs the regression of economic policy and various economic outcomes on the identity of the party that is elected at the local level.

MIT OpenCourseWare  
<https://ocw.mit.edu>

14.770 Introduction to Political Economy  
Fall 2017

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.