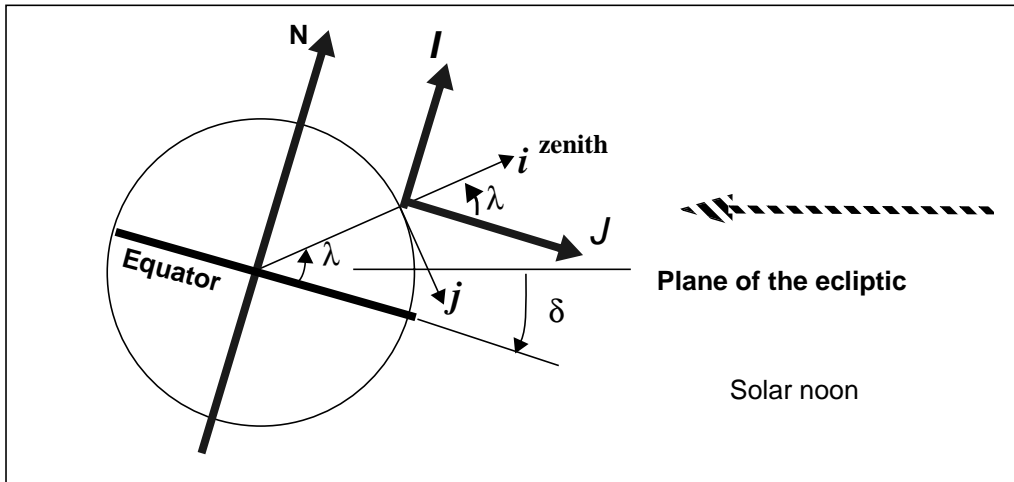


Photovoltaic Solar Energy Systems - The Solar Resource

Solar Geometry

We derive the machinery for calculating the flux onto an arbitrarily oriented, photovoltaic array from the daily and seasonal motion of the earth about its axis and about the sun. For this we introduce two sets of measures of the sun's position relative to the earth. One set gives the sun's position relative to a coordinate frame fixed to the earth with one axis directed overhead, another orthogonal axis directed toward the horizon - the \mathbf{i}, \mathbf{j} coordinate frame in the figure below. The other set is fixed at the same point on the earth but it has one axis directed to the polar, north star, another orthogonal axis directed parallel to the equator - the \mathbf{I}, \mathbf{J} coordinate frame shown in the figure below.



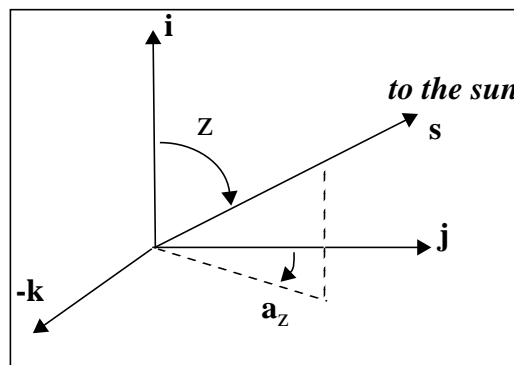
We have:

$$\begin{aligned} \mathbf{i} &= (\sin \lambda) \mathbf{I} + (\cos \lambda) \mathbf{J} \\ \mathbf{j} &= (-\cos \lambda) \mathbf{I} + (\sin \lambda) \mathbf{J} \\ \mathbf{k} &= \mathbf{K} \end{aligned}$$

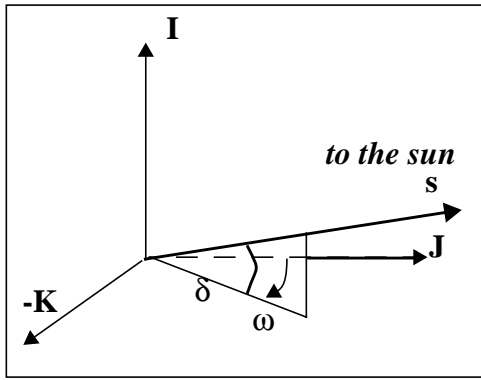
where λ is the latitude of the site.

The \mathbf{i}, \mathbf{j} coordinate reference frame with the \mathbf{i} axis pointing directly overhead, and the \mathbf{j} axis pointing toward the horizon in the direction of the sun's position at solar noon (when it reaches its highest point in the sky) is shown at the right.

Z is the zenith angle, and a_z the azimuth angle. A unit vector pointing toward the sun is then given by:



$$\mathbf{s} = (\cos z) \mathbf{i} + (\sin z \cos a_z) \mathbf{j} - (\sin z \sin a_z) \mathbf{k}$$



The second coordinate system fixed so that one axis continually points to the north star and another in the direction of the sun's position at solar noon, lying in the plane of the equator follows.

$$\mathbf{s} = (\sin \delta)\mathbf{I} + (\cos \delta \cos \omega)\mathbf{J} - (\cos \delta \sin \omega)\mathbf{K}$$

Equating these two expressions for \mathbf{s} , we obtain, using the first set of equations relating \mathbf{i} to \mathbf{I} , \mathbf{j} to \mathbf{J} , \mathbf{k} to \mathbf{K} :

$$\begin{aligned} \cos z &= (\sin \delta)(\sin \lambda) + (\cos \delta)(\cos \lambda) \cos \omega \\ (\sin z)(\cos a_z) &= (\cos \delta)(\cos \omega) \sin \lambda - (\cos \lambda) \sin \delta \\ (\sin z) \sin a_z &= \cos \delta (\sin \omega) \end{aligned}$$

If one specifies the day of the year, this fixes the *declination* δ , according to

$$\delta = 23.45 \cdot \sin\left(360 \cdot \frac{284 + n}{365}\right)$$

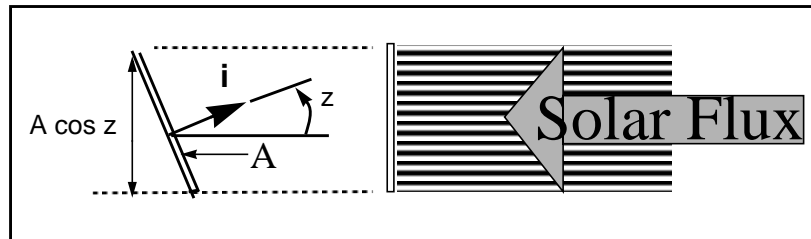
Given also the latitude λ at which the local coordinate frame is fixed to the surface of the earth, then the first of these defines how the zenith angular distance varies from sunrise to sunset — as ω , the *hour* angle, varies over the course of the day. The length of the day is defined by this relationship since, at sunrise and sunset, the zenith distance is 90° or $\pi/2$, i.e. the sun is at the horizon.

Setting $\cos z = \cos 90 = 0$, this gives:

$$\omega_{\text{sunrise/sunset}} = \omega_{\text{ss}} = \arccos[-\tan \delta \tan \lambda]$$

The remaining equations give the value of the azimuth angle, a_z as a function of the hour angle.

But the first equation for $\cos z$ gives us something more, namely, the instantaneous solar flux onto the horizontal plane. If we think of the sun's radiation, or solar insolation, as a fluid streaming toward earth from the sun, then the flow through or across a plane area whose normal is oriented at some angle to the direction of the flow is proportional to the cosine of that angle.



An alternate interpretation is to think of the flux onto the array as the flow normal to the area A which again is given by $A \cos z$.

Now the expression for $\cos z$ is of the form

$$\cos z = C_1 + C_2 \cos \omega$$

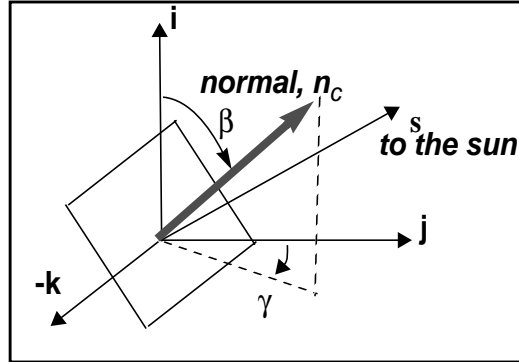
which plots as a cosine shifted up or down depending upon the sign of C_1 which depends, in turn, on the sign of the declination. Note that C_2 is always positive.

Flux onto a tilted array.

We can do better than placing our solar array on the horizontal, at our latitude at any rate. To find the flux on a tilted array we consider the orientation of the array relative to our local coordinate frame:

In this, β is the array tilt, and γ is, well lets just call it γ . The unit normal to the tilted array can be written:

$$\mathbf{n}_c = (\cos \beta) \mathbf{i} + (\sin \beta)(\cos \gamma) \mathbf{j} - (\sin \beta)(\sin \gamma) \mathbf{k}$$



If the array is south facing, usually the case for a fixed array (no tracking), this simplifies to:

$$\mathbf{n}_c = (\cos \beta) \mathbf{i} + (\sin \beta) \mathbf{j}$$

Now the flux onto the array is proportional to the cosine of the angle between the unit vector to the sun, \mathbf{s} , and the unit normal to the array, \mathbf{n}_c . But this is given by the scalar product of these two vectors. This becomes, using all of the machinery presented above:

$$\cos(\mathbf{n}_c, \mathbf{s}) = \sin \delta \sin(\lambda - \beta) + \cos \delta \cos(\lambda - \beta) \cos \omega$$

Which again, for prescribed declination, latitude and array inclination is a shifted cosine.

Monthly average daily radiation and other awkward expressions:

Say you are in outer space, in earth orbit, and your solar array is horizontal — that is, lying on the surface of the sphere whose radius is equal to the radius of your orbit. There is nothing between you and the sun, not even a wisp of aether to attenuate the solar radiation. The flux up there, what we call the extra-terrestrial flux is

$$\phi_{ext} = 1.350 \text{ kw/m}^2$$

Over the course of a day in space, the total solar radiation flux impinging upon your horizontal array will be given by

$$2\phi_{ext} \int_0^{\omega_{ss}} \left(\cos z \cdot \frac{24}{2\pi} \right) d\omega \equiv H_{ext}$$

With $\cos z$ given as before, this integrates to:

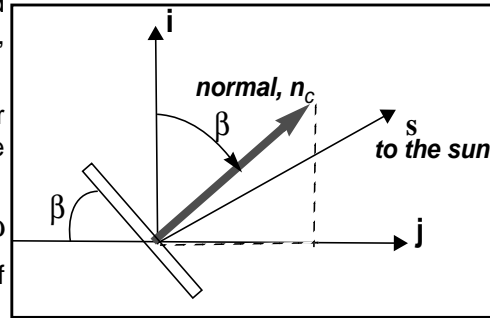
$$H_{ext} = \frac{24}{\pi} \phi_{ext} \cos \delta \cos \lambda [\sin \omega_{ss} - \omega_{ss} \cos \omega_{ss}]$$

Say you are still outside the earth's atmosphere and your solar array is tilted up an angle β to the horizontal, but is south facing i.e., $\gamma=0$.

Then the cosine of the angle between a unit vector perpendicular to the array and a unit vector towards the sun is as above:

$$\cos(\mathbf{n}_c, \mathbf{s}) = \sin \delta \sin(\lambda - \beta) + \cos \delta \cos(\lambda - \beta) \cos \omega$$

which, again, is a function of the hour angle, or time of day.



Integrating this over the course of a day, from sunrise to sunset gives

$$H_{ext}(\beta) = \frac{24}{\pi} \phi_{ext} \cos \delta \cos(\lambda - \beta) [\sin \omega'_{ss} - \omega'_{ss} \cos \omega'_{ss}]$$

where here we assume sun set on the panel before the horizontal. That is setting $\cos(\mathbf{n}_c, \mathbf{s}) = \cos 90 = 0$, gives:

$$\omega'_{ss} = \arccos[-\tan \delta \cdot \tan(\lambda - \beta)]$$

Some Clearness Indexes

The "clearness index" K_T is a number derived from experience, from measured, total daily solar insolation values for a particular location. It is defined as the ratio of monthly average, daily total horizontal insolation on the earth's surface \bar{H} to the monthly average, daily total horizontal insolation outside the atmosphere, in outer space, \bar{H}_{ext} .

$$K_T = \bar{H} / \bar{H}_{ext}$$

The table shows values for clearness index at various locations.

Month	Boston $\lambda=42.1$	Madison $\lambda=42.1$	Albuquerque $\lambda=35$	Miami $\lambda=35$
Jan	0.41	0.44	0.64	0.51
Feb	0.45	0.49	0.67	0.54
Mar	0.45	0.50	0.68	0.56
Apr	0.44	0.47	0.71	0.57
May	0.48	0.51	0.73	0.53
June	0.52	0.53	0.74	0.48
July	0.53	0.55	0.70	0.50
Aug	0.49	0.55	0.70	0.49
Sept	0.49	0.52	0.71	0.49
Oct	0.47	0.50	0.71	0.52
Nov	0.42	0.39	0.67	0.52
Dec	0.42	0.38	0.63	0.52

The Contribution of Diffuse Radiation at Tilt.

With the clearness index for a site, and our calculation of the extraterrestrial radiation/energy impinging onto the horizontal in space, we can estimate the monthly average, daily total solar insolation on a *horizontal* array at the earth's surface at the site. The "horizontal" clearness index is a

number that is computed from the collection of field data at each site. As such it includes the effects of vagaries of the weather, diffuse and reflected, as well as direct, radiation on the total daily flux onto the horizontal. If we set the array at a tilt angle β up from the horizontal (the beta above), we must take account of the diffuse (and reflected) radiation in another way. In other words, we need to construct a site-specific K_t for the tilted surface.

The method we present relates the monthly-average daily total radiation incident on an inclined surface to the monthly-average daily radiation on a horizontal plane at the earth's surface by a factor R . The method is a first approximation. It was originally developed by Liu and Jordan^{1,2}, evaluated by Klein³, then subsequently improved and refined by Klein and Theilacker⁴. We present the method in its first approximation form.

$$\bar{H}_\beta = R \cdot \bar{H}$$

where

$$R = \left[1 - \left(\frac{\bar{H}_d}{\bar{H}} \right) \right] \cdot R_b + \left(\frac{\bar{H}_d}{\bar{H}} \right) \cdot \left(\frac{1 + \cos \beta}{2} \right) + \rho \cdot \left(\frac{1 - \cos \beta}{2} \right)$$

The first of these states that the monthly-average, daily total radiation incident on a *surface inclined at an angle* β is given by the product of two factors: the monthly-average, daily total radiation incident on a *horizontal surface at the earth's surface* and a coefficient R defined by the next equation. So far nothing ventured, nothing much gained.

The factor R is a bit more interesting. In this, $\left(\frac{\bar{H}_d}{\bar{H}} \right)$ is the ratio of the monthly-average, daily diffuse radiation to the monthly average, monthly-average, daily total radiation incident on a horizontal surface - to be determined below - and R_b is the ratio of the monthly-average *beam radiation on the tilted surface* to that on a horizontal surface. The first ratio is constructed from experimental field data, the second derives from our earlier geometric analysis. The parameter ρ is a measure of the site's surface reflectivity and is best obtained through measurement.

But first, consider the case when β , the tilt angle is zero, i.e., the surface is horizontal. In this case, our expression for R comes to $R = R_b$ which is as it should be. So what is going on here is, in the first term of R , we "back out" the contribution of the diffuse from the total incident on *the horizontal* - so that the first term represents just the direct beam contribution to the horizontal - then add it back in as a diffuse contribution in the second term.

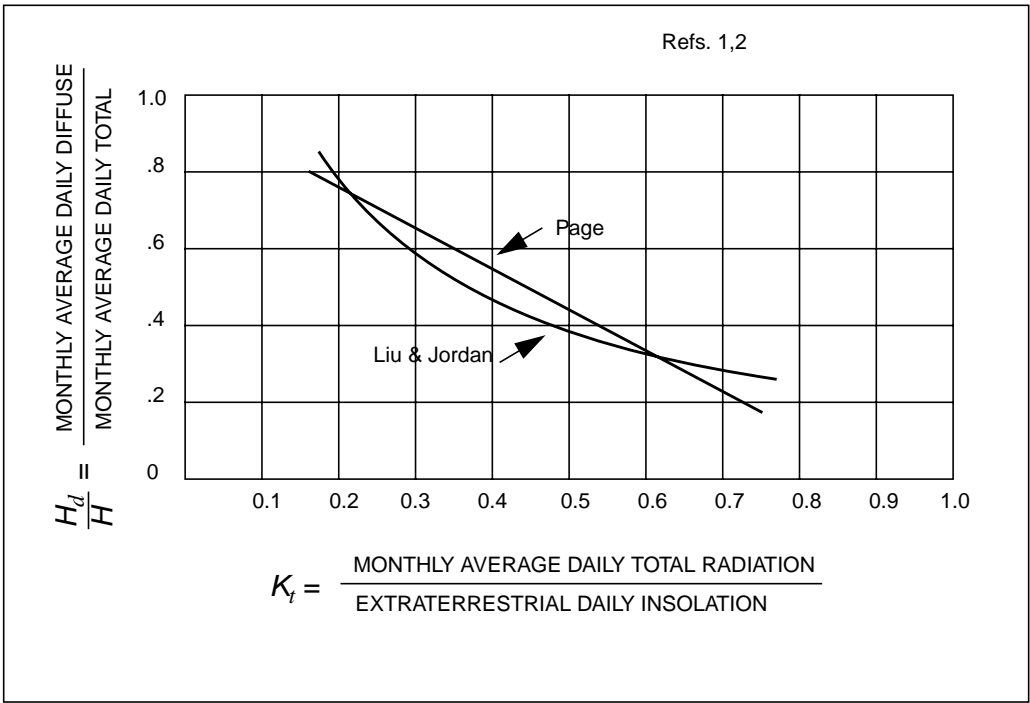
R_b is then "estimated" to be the ratio of total, daily, direct extraterrestrial radiation on the tilted surface, what we labeled $H_{ext}(\beta)$, to that on a horizontal surface, H_{ext} , so we have:

$$R_b = \frac{H_{ext}(\beta)}{H_{ext}} = \frac{\cos \delta \cos(\lambda - \beta) [\sin \omega'_{ss} - \omega_{ss} \cos \omega_{ss}']}{\cos \delta \cos \lambda [\sin \omega_{ss} - \omega_{ss} \cos \omega_{ss}]}$$

In this ω'_{ss} is the sunset hour on the tilted array while ω_{ss} , without the prime, is the sunset hour on the horizontal.

We still must reckon with the diffuse contribution. As noted, for this we rely upon experimental data. Two curve fits (at least) have been proposed. We show them both in graphical and algebraic form below:

1. Liu, B.Y.H & Jordan, R.C., "The interrelationship and characteristic distribution of direct, diffuse, and total solar radiation", *Solar Energy* 4(3) 1960.
2. Liu, B.Y.H & Jordan, R.C., "Daily insolation on surfaces tilted toward the equator", *Trans. ASHRAE* 526-541, 1962.
3. Klein, S.A. "Calculation of Monthly Average Insolation on Tilted Surfaces", *Solar Energy*, vol. 19, 325-329, 1977.
4. Klein, S.A. & Theilacker, J.C., "An Algorithm for Calculating Monthly-Average Radiation on Inclined Surfaces", *Jour. of Solar Energy Engineering*, vol. 103, 29-33, 1981.



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