All right. Good morning, all. So we take another big step forward today and get onto a new plane of understanding, if you will. In the last week and a half, our focus was on the storage element or storage elements called inductors and capacitors. And capacitors stored change and inductors essentially stored energy in the field, the magnetic flux. And the state variable for an inductor was the current while that for a capacitor was the capacitor voltage. We also looked at circuits containing a single storage element, we looked at RC circuits and we also looked at circuits containing a single inductor. And this was a single inductor with a resistor and a current source or a voltage source and so on. What we are going to do today is do what are called "second-order systems".

So they are on the next plane now.

And with this second-order of systems, they are characterized by circuits containing two independent storage elements.

They could be an inductor and a capacitor or two independent capacitors. And you will see towards the end what I mean by two independent capacitors.

If I have two capacitors in parallel, they can be represented as a single equivalent capacitor so that doesn't count. It has to be two independent energy storage elements and resistors and voltage sources and so on. And what we end up getting is what is called "second-order dynamics".

And much as first order circuits were represented using first order differential equations, this kind you end up getting second-order differential equations.

Before we go into this, I would like to start motivating this and give you one example of why this is important to study. There are many, many examples but I will give you one.

What I would like to do is draw your attention to our good old inverter driving a second inverter.

The same circuit that we used to motivate RC studies, one inverter driving another. So let me draw the circuit.

Here is one inverter. This is, let's say, 5 volts and this is, let's say, 2 kilo ohms.

And I connect the output of this inverter to a second inverter. And what we saw in the last few lectures was that in this specific example there was a parasitic capacitor or a capacitor associated with the gate of this MOSFET. And that could be modeled by sticking a capacitor CGS between the gate of the MOSFET and ground. And we saw that the waveforms here, if I had some kind of step here.

Let's say, for example, a step that went from high to low. Then out here I would have a transition that instead of going up rapidly like this would transition a little bit more slowly.

And this transition was characterized by an RC time constant. And this is what gave rise to a delay in the eventual output. So that is what we saw previously, single energy storage element.

Today what we are going to do is we are going to look at the same circuit, the exact same circuit, and have some fun with it. What we are going to say is look, this thing is pretty slow, so what I would like to do is -- why don't we go ahead and put that up.

What we are going to see is that the yellow waveform is the waveform at the input here. And the green waveform here is the waveform at this intermediate node.

And notice that this waveform here is characterized by the slowly rising characteristics that are typical of an RC circuit. There are some other weirdnesses and so on going on here like a little bump and stuff like that. You can ignore all of that for now. It happens because of certain other very subtle circuit effects that you won't be dealing with, called Miller effects and so on that you won't be dealing with in 6.002.

So focus then on this part here.

It is pretty slow. And because of that slow rising, I get a very slow transition and I get some delay in my inverter. So you say ah-ha, we learned about this in 6.002, I can make it go faster.

How can you make the circuit go faster?

What could you do? This is rising very slowly.

How can you make it go faster? Anybody?

You have multiple choices, actually.

What are your choices here? Pardon.

Decrease the time constant. And how would you decrease the time constant? The capacitance is connected to this MOSFET gate here. I didn't want it in the first place but it is there, I cannot help it, so I can decrease the resistance.

Good. Let me go ahead and do that.

What I will do is I am going to knock this sucker out and stick in a new resistance that is say 50 ohms, a much smaller resistance. That should speed things up, right? That should make things go much faster because this is a smaller time constant because R is smaller, correct? OK, let's go do it.

And let's see if we get what we expect.

I have a little switch here. And using that switch, I am going to switch in this little resistance.

Whoa, what on earth is happening out there?

This is so much fun. What I did is I switched in a small resister here to decrease the time constant, but it looks like I got a whole bunch of crapola that I did not bargain for. This is certainly very fast, it goes up really fast, but I am not sure where it is going, though. Let's stare at that a little while longer. Let me expand the time scale for you. Look at this.

Instead of a nice little smooth thing going up.

I get something that looks like this.

It looks something like a sinusoid.

It looks sinusoidal, but then it is a sinusoid that kind of gives up and kind of gets tired and kind of goes away. Right?

It kind of dies out. So nothing that you have learned so far has prepared you for this.

And, trust me, when I first did some circuit designs myself a long, long time ago I got nailed by that. I looked at my circuit, and what ended up happening was I was noticing these sharp lines at all my transitions. When I looked at my scope, I expected to see nice little square waves but I saw these little nasty spikes sitting out there.

And then when I stared at it more carefully, those spikes were really sinusoids that seemed to kind of get tired and kind of go away. So those are nasty, those are real and they happen all the time.

And what we will do today is try to get into that and understand why that is the case. We will understand how to design that away. And that is a real problem, by the way. And the reason that is a real problem is the following. Look at this.

Look down here. Because this intermediate voltage is meandering all over the countryside here, at this particular point the intermediate voltage dips quite low. And because it dips quite low look at the output. The output has a bump here.

And it is quite possible for this output bump to now go into the forbidden region. Or worse.

If this swing here was higher, this could have actually gone onto a one, so I would have gotten a false one pulse here.

Instead of having a nice one to zero transition, I would have gotten a one to zero, oh, back to one, oh, back to zero and then back down to zero.

So this is nasty stuff, really, really nasty stuff.

What we will do is understand why that is the case today and see if we can explain it. What is going on here?

What is really going on here is take a look at this circuit here. I will take a look at this path here. So this is your VS voltage source. Path kind of goes like this and around. It turns out that this circuit is a loop here. And when there is current flow, going down to basic physics you remember that I also enclose some amount. So there is a current flowing in a loop. And because of that there is an effective inductance here. And, in fact, any current flowing through a wire above a ground plane, for that matter, can be characterized by the inductance. So I can model that by sticking a little inductor here. So my real circuit is not exactly a resistor and a capacitor, but my real circuit is an inductor as well that comes into play because of this wire. Every wire, when there is a current flow, has an inductance associated with it. And because of that the real circuit is resistor, inductor and capacitor.

So I end up with two storage elements now, and the dynamics of that are very different from that with a single storage element. That is just a bit of motivation for why our study of inductors is important.

And I can draw a quick circuit here.

If you look at the circuit, start from ground, the voltage VS and there is a resistor here.

And then I have an inductor and then I have a capacitor.

So it is a voltage source, resistor, inductor and capacitor. For this whole week we will be looking at circuits like this. Today what I would like to do is start very simple, start with the simplest possible form of this so that you can begin building up your insight and then go into more complicated cases.

Today what I will do is simply begin with a case where I don't have a resistor here and simply study a voltage source, an inductor and a capacitor and understand what the voltage looks like out here. So we look at the dynamics of a little system like this. Before we go on, I want to caution you about something.

It is just happenstance that I have introduced for you capacitors based on the parasitic capacitance here and inductance based on parasitic inductance.

I would hate to leave you with the impression that inductors and capacitors are "bad". Because when you think of a parasitic, you know, parasites.

These are parasitic. You didn't expect them there, didn't expect this here and we got the weird behavior.

So parasitics have a bad connotation to them.

I do not want to leave you with a bad taste in your mouth about capacitors and inductors that these are just bad things.

We just have to deal with them and deal with second-order differential equations and all that stuff because they're just bad stuff and we just have to deal with them.

I don't want you to end up going through life hating capacitors and inductors. Just because of my choice of examples, it just happened to be introducing them as capacitors.

I want to point out that these are fundamental lumped elements in their own right. They are very, incredibly important and useful circuits where we designed capacitors and inductors because we want to have them in there.

There are many circuits that we will look at where we really want the inductor in there. We will design an inductor by wrapping wire around in a coil and get bigger inductances and so. Just remember that this can be parasitic in some cases, but in many cases it's good, inductors are good, so just stick with that thought. These are mostly good so don't go around hating them. All right.

Let's go on and analyze a basic circuit like this.

And what I would like to cover in the next hour are the foundations of something like that.

I will take you through the foundations so you understand how it works. And, as always, what I am going to end up with is build up the foundations, help you understand why we got where we were and then help you build intuition. And then show you a really, really simple intuitive way of doing things in terms of how experts do it. And the real cool thing about EECS is that the way experts do things, things are really, really very simple in the end. But you need to build up some intuition to get there. So our circuit looks like this in terms of my two storage elements.

I have a voltage vI, inductor L, capacitor C and I am going to look at the voltage across the capacitor and my current through the capacitor. So v(t) is the voltage across the capacitor and my current is the current through this loop here, which is the same as the current through the capacitor or the current through the inductor.

And we are going to proceed in exactly the same manner as we did for first order differential equations, write the equations down and just boom, boom, boom, boom, go down the same sets of steps but just get to some place different. We are going to start by writing a node equation for this node here.

That's the only node for which I have an unknown voltage.

The node here is vI, so I need to find this, there's just one unknown node voltage.

And I am going to need some element laws.

For the capacitor I know the iV relation is given by the i for the capacitor is Cdv/dt. And just to show the capacitor I am just calling it dvc/dt. Similarly, for an inductor, L, the voltage across the inductor is given by Ldi/dt.

So this is the vI relation for the capacitor, the vI relation for an inductor.

It also suits us to write this in an integral form.

So if I integrate both sides of this equation and I bring L down to this side, I end up getting something like this, 1/L minus infinity to t, VLdt, and that is simply iL.

I am just simply replacing this with an integral form.

So this is a VI relationship for the inductor and this is for the capacitor. So let me now go ahead and apply the node method for my circuit here.

Here, for the node method, I have to equate the currents coming into the node or sum the currents coming into the node and equate that to zero. And while I do that I simply replace the currents by the corresponding voltages using the element laws. So what do I get?

I get the current going in here to the inductor is equal to the current going through the capacitor.

What is the current going the capacitor?

In terms of its v relationship it is Cdv/dt.

And the current going to the inductor is given by this relation here, which is simply 1/L minus infinity to t. The voltage across the capacitor is simply (vI-v)dt. I have just written down the node quotation for this node here.

Now I will just apply a bit of math and simplify it and get the resulting equation. What I can do is simply differentiate with respect to t here.

And get this to be Cd^2v/dt^2, the second derivative of v.

And here what I end up getting is 1/L(vI-v).

So I just differentiated the whole thing by d/dt here.

And then I just move L up here. I bring d^2v/dt^2 out here.

And then I get a minus v here, and that will be equal to, oh, I'm sorry. Let me leave this here.

Bring the minus v to this side so it becomes a plus and leave vI on this side. So I end up getting LCd^2v/dt^2. I bring L up here.

And then I take v to the other side.

Plus v and leave vI here so I get vI.

That is second order differential equation that governs the characteristics of the voltage, v.

So much as the voltage across the capacitor was a state variable in our RC circuits or the current through the inductor was a state variable in our RL circuits, out here both the current through the inductor and the voltage across the capacitor are my two state variables. And so here I have a second-order equation in my voltage, v.

Again, going through the foundations here, I am now going to go through a bunch of math.

Up to here it was circuit analysis, and now I am just going to do math. For the next three or four blackboards just math. You can solve this second-order differential equation any which way you want.

But just to keep things as simple as possible, in 6.002 I solve all the differential equations, it turns out we are fortunate enough we can do that, using the exact same method again and again and again, the same thing can be applied. And the method that we use to solve it is the method of homogenous and particular solutions. So the first step we are going to find the particular solution, vP.

Second step we find the homogenous solution, vH. And the third step we are going to find the total solution as the sum of, v is simply the particular plus the homogenous solution and then solve for constants based on the initial conditions and the applied voltage. So let's write down initial conditions. Let's assume, for simplicity, that my initial conditions are simply the voltage across the capacitor is zero to begin and the current through my inductor is also zero as I begin life.

Now, this is what is called "zero state".

v and i are both zero, and so the response of my circuit for some input is going to be called ZSR.

You've probably heard this term in one of your recitations.

So zero state response simply says I start with my circuit at rest and looks at how it behaves for some given input.

That is a little term you may end up using.

My input next. I am going to use the following input. vI of t is going to be a step, is going to look like this. My input is at t=0 v is going from zero to some voltage VI and then stay at that voltage.

It is going to be a step. Kaboom.

And you can see why I am going with this set of variables, because I want make this situation as close as possible to the funny behavior we observed there.

Remember we had a step, and because of the step we had some behavior at that node? So I will try to bring you as close to that. In tomorrow's lecture, I am going to close the loop around that and derive for you exactly the behavior we saw on the scope.

And to get there I am going to be try to be as close as possible to the constants and other parameters in the demo.

So VI is a step and zero state. Just in terms of notation, this kind of a step input occurs pretty frequently.

And we just have a special notation for it.

We simply call it VI is the final value here.

And we call it u(t). So VIu(t), u(t) simply represents a step at time t=0, steps from zero volts to VI.

That is just a little more notation that will come in handy at some point. More math now.

Three steps, particular solution, homogenous solution, total solution/constants.

This is almost like a mantra here, like a chorus.

Homogenous solution we compute using a four-step method.

And four-step method for homogenous solutions, it turns out that it happens to be that way for all the equations we will see in our course.

The first step would be assume a solution of the form Ae^st.

Exactly as with RCs. If you close your eyes and do exactly what you did for RCs you will get to where you want to be. You assume a solution of the form Ae^st. Substitute that into your homogenous equation. Obtain the characteristic equation. Solve for the roots.

And then write down your homogenous solution.

Same sort of steps again and again and again until you get bored to tears. Particular solution.

For the particular solution, I simply need to find a solution, any solution, if not the most general one but any solution that satisfies the particular equation which satisfies that equation. LCd^2vP/dt^2+vP=VI.

My input is a step and I am going to look for the solution for time t greater than zero. Notice that for time t less than or equal to zero, v is going to be zero.

So I am looking for a solution greater than t=0.

Here, if I substitute vP=VI, that is a particular solution.

Because if I substitute VI here this goes to zero and then I get VI=VI, so this works. I promised you this was going to be simple. You cannot get any simpler than that. I have done my first step.

I found the particular solution.

And VI is a good enough particular solution so I will use it, I will take it. As my second step I am going to find vH or the solution to the homogenous equation.

And the homogenous equation is simply that equation with drive set to zero. What I get here is LCd^2vH/dt^2+vH=0. That is my homogenous equation.

I simply set the drive to be zero.

And to find the solution here, I go through my four-step method. Again, in 6.002 following the kind of Occam's principle, we just show you the absolute minimum necessary to get to where you want.

The absolute minimum necessary is it turns out that we can solve all our differential equations that we use here by using the methods of homogenous and particular solutions.

And every homogenous solution can be solved by a four-step method. That is about as minimal as it can get. So no extraneous stuff there.

The four-step method, four steps.

The first step is assume a solution of the form vH=Ae^st.

What I have noticed is that students starting out are usually scared of differential equations.

I know I was when I was a student.

And the trick with differential equations is that it is all a matter of psych. Just because you see some squigglies and squagglies and a bunch of math and so on you say oh, that must be hard. But differential equations are actually the simplest thing there is because in a large majority of cases the way you solve them is you assume you know the answer, someone tells you the answer.

And then all you are left to do is shove the answer into the equation and find out the constants that makes it the answer. Just a matter of psych.

Psych yourselves that this stuff is easy, because I am telling you what the solution is.

All you have to do is substitute and verify.

If you think about differential equations that way or a large majority of them, it really is very simple if you can just get past the squigglies here.

Just get past the squigglies and then just simply stick in some simple stuff and it works. I mean it just cannot get any easier. I cannot think of any other field where the way you find a solution is assume you know the solution and stick it in. It has never made any sense to me but that is how it is. So we assume the solution to the form Ae^st, you stick it in there, and you have to find out the A and s that make it so.

It cannot get any simpler than that.

Let's stick the sucker in here and see what we can get.

Substitute Ae^st here I get LCA, and second derivative, so it's s^2 e^st. And Ae^st on this one here.

And that equals zero. And then let me just solve for whatever I can find. Assuming I don't take the trivial case A=0, I cancel these guys out.

And what I am left with is simply LCs^2+1=0.

In other words, what I end up getting is B, s^2=-1/LC. My first step was, I am giving you solutions, stick them in there, assume a solution of this form. Second step is get the characteristic equation. And the way you get the characteristic equation is that you simply stick this guy in there. And what you end up getting is some equation in s^2. Do you remember what you got for first order circuits? What s was?

What is s? For first order circuits, what did you get as a characteristic equation?

s+1/RC=0. The same thing.

Just remember to blindly apply the steps.

It will lead you to the answer. This is called the "characteristic equation". This is incredibly important.

You will see in about a couple weeks from now that once you write the characteristic equation down for a circuit, it tells you all there is to know about the circuit.

And often times you can stop solving right here.

To experienced circuit designers this tells me everything there is to know. This is really key.

That's why it's called a characteristic equation.

I believe in problem number three of the homework that will be coming out this week, that is exactly what you are going to do. I am going to give you a circuit, ask you to get to the characteristic equation quickly and then from there intuit the solution.

Write the characteristic equation and then just intuit solution, it's that simple. So, step A, assume a solution of the form, step B, write the characteristic equation down. And let me just simplify that a little bit. I go ahead and find my roots.

And my roots here, remember that j is the square root of minus one. And so what I end up getting is, my two roots here are, plus j square root of 1/LC and minus j square root of 1/LC. Two roots.

And just as a shorthand notation, much like I had a shorthand notation for RC, what was my shorthand notation for RC? Tau.

Just as tau was big in first order, we have a corresponding thing that is big in second order and that is omega nought.

Omega nought is simply square root 1/LC.

Just as tau was RC, omega nought is a shorthand here. And so s is simply plus or minus j omega nought. Notice that in this equation here, if you take the square root of LC there that has units of time, so one divided by that has units of frequency.

Notice that this guy is a frequency in radians.

I end up getting my roots of the homogenous equation, and that is my third step. And as my fourth step, I simply write down the homogenous solution as substituting s with its roots and writing the most general possible form of the solution, and that would be A1e^(j omega nought t)+A2e^(-j omega nought t).

Done. Some constant times this solution plus some other constant times, the other solution. Plus zero omega nought.

Remember it comes from here, Ae^st.

I assume the solution of this form, so my solution in this most general case would be s being j omega nought in one case, minus j omega nought in the other case, and I sum the two to get the most general solution.

So blasting ahead. I now have my homogenous solution. And as my third step of solution to differential equations I write down the total solution, v=vP+vH, particular plus the homogenous solutions. And v=VI, was my particular solution, +A1e^(j omega nought t)+A2e^(-j omega nought t) is my complete solution. The final step, write down the total solution and find the constants from the initial conditions. To find the constants from the initial conditions, let's start with, the voltage is zero to begin with.

This equation governs the characteristics of v, so I need to find the initial conditions.

First of all, I know that know that v(0)=0.

From there I substitute t=0. And so this goes to one, this goes to one, and I end up getting 0=VI+A1+A2. That is my first expression.

And then I am also given that i(0)=0.

And so I can get that as well. How do I get i?

This is v. I know that i=Cdv/dt, so I can get i by simply multiplying by C and differentiating this with respect to t.

I get C, this guy vanishes so I get d/dt of this.

So it is CA1(j omega nought) e^(j omega nought t)+CA2(-j omega nought)e^(-j omega nought t).

From here I am given that that is zero, and so therefore this guy becomes a one, this guy becomes a one, j omega nought, j omega nought cancel out.

What I end up getting is A1=A2. From the second initial condition I get A1=A2. From these two, if I substitute here for A2, I get VI + 2A1 = 0, or A1=-VI/2. That is also equal to A2.

Therefore, my total solution now can be written in terms of the actual values of the constants I have obtained.

I get VI-VI/2. So A1 and A2 are equal.

I just pull them outside. I pull VI-2 outside and I stick these two guys in parenthesis in.

Again, I promised you no more circuits from here on until the very last board or something like that.

It is all math, so not much else happening there. More math.

If you would like, I could skip all the way to the end and show you the answer. But I just love to write equations on the board so let me just go through that.

I am going to simplify this a little further here.

And we should remember this form by the Euler relation, ejx=cos x+j sin x. And by the same token, (e^jx + e^-jx)/2=cos x. You all should know this from the Euler relation. So were are using this guy here, ej^x + e^-jx=2cos x. And so this one is 2 cosine of omega nought t, 2 and 2 cancel out, and what I am left with is v(t)=VI-VI cos( omega nought t).

And the current is Cdv/dt, which is simply CVI sin( omega nought t). Just remember that omega nought is the square root of 1/LC. We are done.

In fact, I did not give that answer the importance that was due so let me just draw.

There. That is better.

Enough math. In a nutshell, what did we do. We wrote the node method, it's a very simple circuit, to write down the equation governing that circuit. And then we grunged through a bunch of math. Not a whole lot here.

It is pretty simple. And ended up with a relation that says the voltage across the capacitor for a step input, assuming zero state, is a constant VI-VI cos omega t. Notice that even though I have a step input, the circuit dynamics are such that I get a cosine in there. You can begin to see where these cosines are coming from now.

They come in here. And if you recall the example I showed you earlier of the inverter circuit, remember there was a cosine that decayed, that was sort of losing energy and kind of dying out?

So you can see where the cosines are coming from.

And just to draw you a little sketch here.

Let me draw v and i for you and let me plot omega t, pi/2, pi and so on. Let me plot VI.

When time t=0, VI=0, cosine omega t is one, and so VI-VI=0. That is simply a cosine that starts out at zero here, and at pi I get cosine omega t is minus one, so I get plus VI on the other side. So I end up at +2VI.

At this point the voltage is here.

And notice that this guy looks like this.

It is a cosine that is translated up so that its mean value is not zero but VI. It is just a translation up of a cosine. Similarly, in this case for the current it is a sinusoidal characteristic.

And it looks something like this where the peak is given by CVI, oh, I messed up.

When I differentiated this is missed the omega nought out there.

What I would like to do now -- This is the form of the output for a step input. What I would like to do next is show you a demo. But before I show you a demo, I always found it strange that I have a step input and then I have two little elements, how can I get a sine coming out of the output? I would like to get some intuition as to why things behave the way they are.

I could go and pray to find out, but let me just give you some very basic insight as to why this behaves the way it does. Let me draw the circuit for you here. And this is my inductor L and capacitance C. Remember this is v.

Let me just walk you through what is happening there and get you to understand this. Now, you have seen sines occur before. If you go and write down the equation of motion of a pendulum, you know, you have a pendulum, you move it to one side, let go. It is also governed by sinusoidal characteristics. And you will find that the equation governing its motion is very much of the same form, and you get the sinusoid where you have energy that is sloshing back and forth between maximum potential energy to maximum kinetic energy and zero potential energy back to maximum potential energy, zero kinetic.

So it is energy sloshing back and forth.

The same way here. Capacitors and inductors store energy. Let's walk through and see what happens. I start off with both of them having the stage zero, zero current, zero voltage. I apply a step here.

Boom, the step comes instanteously to VI.

I notice that the capacitor voltage cannot change instantly unless there is an infinite pulse of a sort, so this guy cannot change instantly.

And so its voltage starts off being zero.

So the entire voltage here, KVL must be true no matter what. They are absolutely fundamental principles from Maxwell's equations.

KVL must hold, which means that the entire voltage VI must appear across the inductor.

I put a big voltage across the inductor and its current begins to build up. There you go.

A voltage across the inductor, its current begins to build up.

As its current begins to build up that current must flow through the capacitor, too.

And as current flows through a capacitor it is depositing charge into the capacitor. As the capacitor begins to get charge deposited on it, its voltage begins to rise.

Let's see what happens here. Its voltage keeps rising.

At some point, the voltage across the capacitor is equal to VI. But then VI equals this VI here. So when the two become VI, the inductor has zero volts across it.

So there is no longer a potential difference that is increasing the current in that direction.

At that point, at pi divided by 2, I have some current going into the inductor so there is no longer a pressure that is forcing more current through the inductor because this voltage reaches VI.

But remember capacitors like to sit around holding voltages.

Just remember that demo. That rinky-dink capacitor sat there stubbornly holding its voltage.

And it had a huge spark towards the end.

It just sat there holding its voltage.

In the same manner, inductors love to sit around holding a current. They will do whatever they can to keep the current going through them.

It has got the current going through.

And few forces on earth can change that.

And so therefore, even though the capacitor voltage is VI and the voltage drop across the inductor is zero, it still keeps supplying a current.

It has got the current. It's got inertia.

It keeps going. It is like a runaway train.

You may not be pushing the train from the back, but once it is running it has got kinetic energy and is going to run no matter what for a least some more time, even if you take away the force on the train.

So I have taken away the force on the punching more current through, but it has kinetic energy.

It has current flowing through it so it continues to supply a current. Because it continues to supply the current the capacitor voltage keeps increasing.

This is a subtle insight which is absolutely spectacular that with zero volts across it, it still keeps pumping that current. Capacitor voltage has gone up.

And guess what? The voltage on this side is higher now but this guy is still pumping a current.

Man, I have been born to do this, you know, I shall pump a current. However, because the voltage has now gone up here gradually the current begins to diminish.

So the capacitor is concerned. You pump a current into me, my voltage goes up. At some point, like a runaway train, it comes to a halt.

The current through the capacitor drains and now goes to zero and the capacitor voltage reaches 2VI.

So this is at 2VI now and this is at VI.

Now the situation is not in equilibrium.

At this point there is zero current through it, but guess what? I have a VI pumping in this direction now. I have the same VI punching in this direction. So guess what?

Its current must now build up in this direction and its current begins to build up in that direction.

That begins to discharge the capacitor and the capacitor then goes on to a negative, or the current goes down to a maximum negative current, and this process continues.

What you are seeing here is energy.

It is sloshing back and forth between the two, and that is kind of a key. I will just quickly put up a demo that you can watch as you are walking out.

With a step input, notice the green is the voltage across the capacitor and the orange is the current through the capacitor.