Magnetostatics (Free Space With Currents & Conductors)

Suggested Reading - Shen and Kong – Ch. 13

André-Marie Ampère,
1775-1836

Outline

Review of Last Time: Gauss's Law
Ampere's Law
Applications of Ampere's Law
Magnetostatic Boundary Conditions
Stored Energy

Electric Fields

$$\oint_S \epsilon_o \overline{E} \cdot d\overline{A} = \int_V \rho dV$$

$$= Q_{enclosed}$$

$$\oint_C \overline{E} \cdot d\bar{l} = -\frac{d}{dt}\left(\int_S \overline{B} \cdot d\overline{A}\right)$$

Magnetic Fields

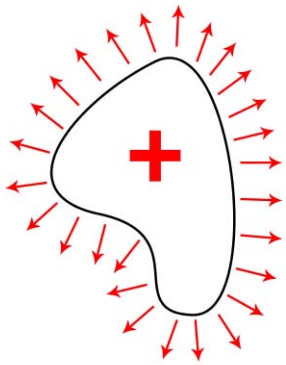
$$\oint_S \overline{B} \cdot d\overline{A} = 0$$

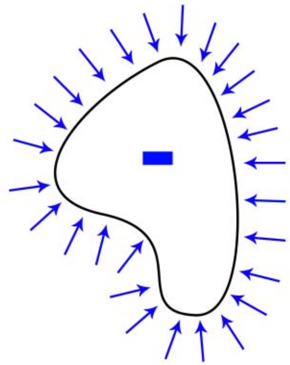
$$\oint_C \overline{H} \cdot d\bar{l}$$

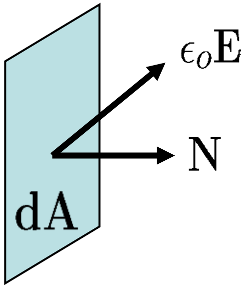
$$= \int_S \overline{J} \cdot d\overline{A} + \frac{d}{dt} \int_S \epsilon E dA$$

1^{st} Observation: Coulomb's Law

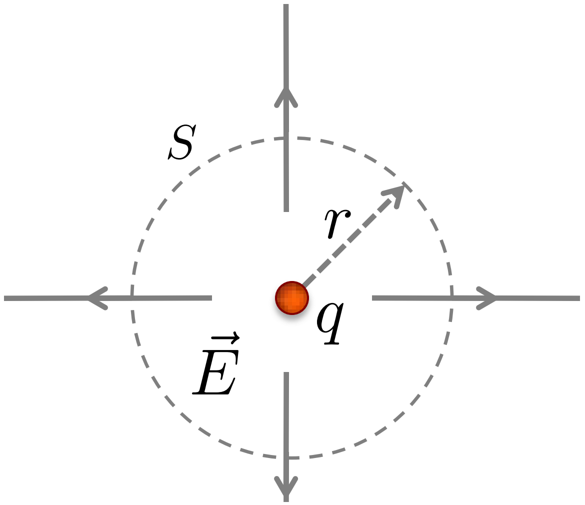
Fields fall-off as $1/r^2$ from point charge…







Gauss's Law: $\int_S \epsilon_o \mathbf{E} \cdot \mathbf{dA} = \int_{\mathbf{V}} \rho \mathbf{dV} = Q_{enclosed}$



$$\oiint_S \epsilon_o \vec{E} \cdot dS = q$$

$$E_r = \frac{q}{4\pi\epsilon_o r^2}$$

Gauss's Law encompasses all observations related to Coulomb's Law…

2^{nd} Observation: Force and Potential Energy

From 8.01: $f = -\frac{dV}{dx}$ $\qquad$ $f = -\nabla V$

From 8.02: $f = qE$ $\qquad$ $V = q\phi$

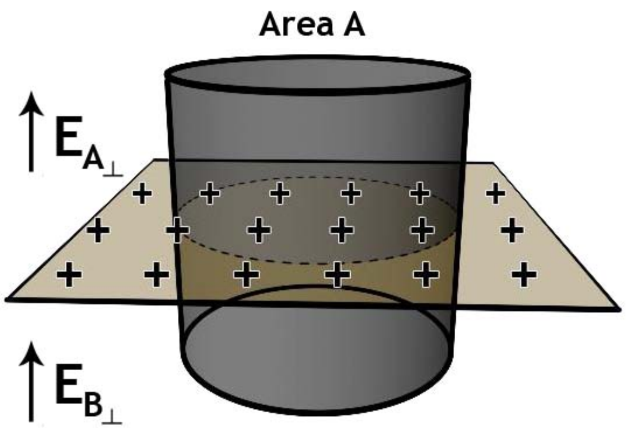
$E = -\nabla \phi$ $\qquad$ Where is this in Maxwell's Equations?

Integral form: $\Phi_a - \Phi_b = \int_a^b \vec{E} \cdot d\vec{C}$

For closed loops (where a=b): $\oint \vec{E} \cdot d\vec{C} = 0$

Faraday's Law (static) accounts for the E-field being a conservative force…

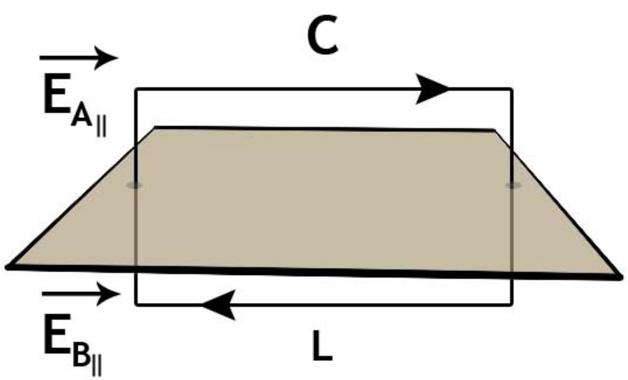
Boundary Conditions from Maxwell's Laws



$$\lim_{\delta \to 0} Gauss \Rightarrow (\epsilon_0 E_{A\perp} - \epsilon_0 E_{B\perp})A = \rho_s A$$

$$\hat{n} \cdot (\epsilon_0 E_A - \epsilon_0 E_B) = \rho_s$$

Normal $\vec{E}$ is discontinuous at a surface charge.

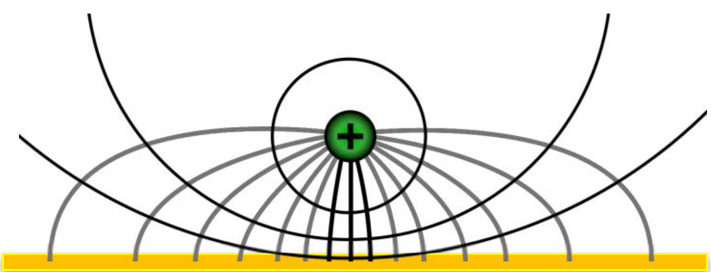


$$\lim_{\delta \to 0} Faraday \Rightarrow (E_{A\parallel} - E_{B\parallel})L = 0$$

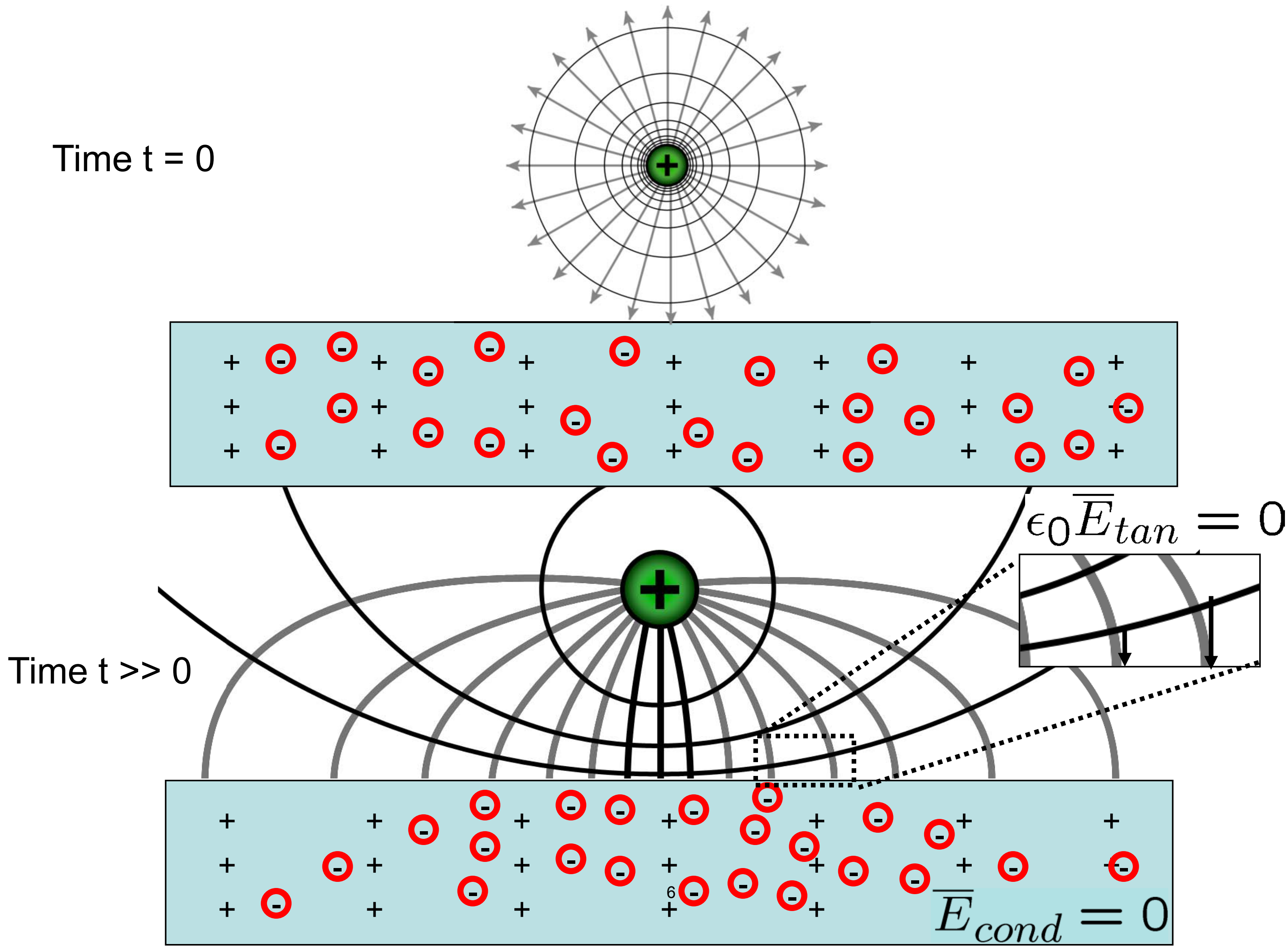
$$\hat{n} \times (E_A - E_B) = 0$$

Tangential $\vec{E}$ is continuous at a surface.

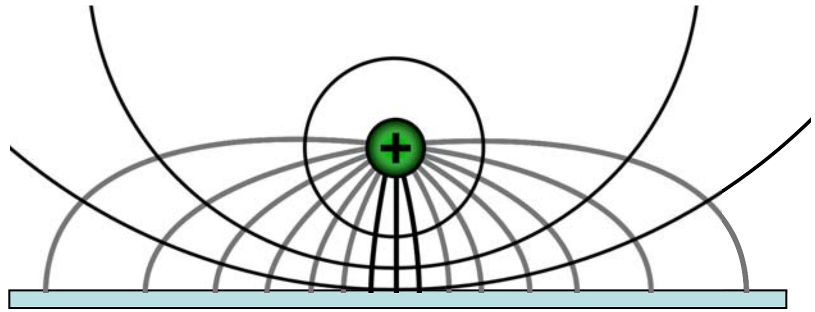
A static field terminates perpendicularly on a conductor

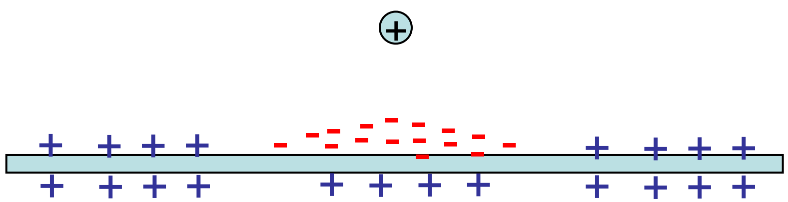


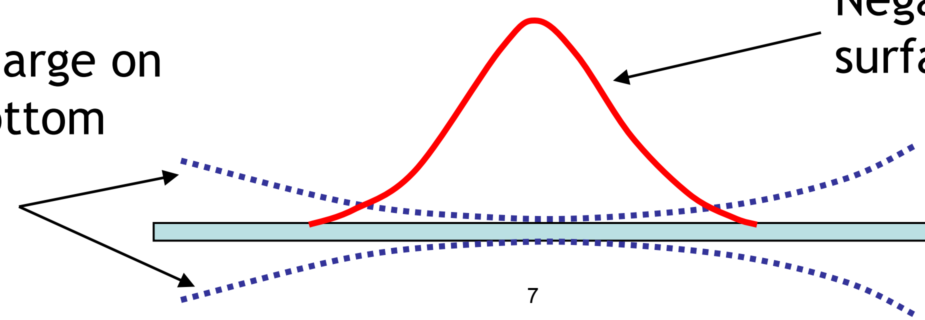
Point Charges Near Perfect Conductors



Point Charges Near Perfect Conductors







Hans Christian Ørsted

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In 1820, which Ørsted described as the happiest year of his life, Ørsted considered a lecture for his students focusing on electricity and magnetism that would involve a new electric battery. During a classroom demonstration, Ørsted saw that a compass needle deflected from magnetic north when the electric current from the battery was switched on or off. This deflection interestred Ørsted convincing him that magnetic fields might radiate from all sides of a live wire just as light and heat do. However, the initial reaction was so slight that Ørsted put off further research for three months until he began more intensive investigations. Shortly afterwards, Ørsted's findings were published, proving that an electric current produces a magnetic field as it flows through a wire. This discovery revealed the fundamental connection between electricity and magnetism, which most scientists thought to be completely unrelated phenomena.

His findings resulted in intensive research throughout the scientific community in electrodynamics. The findings influenced French physicist André-Marie Ampère's developments of a single mathematical form to represent the magnetic forces between current-carrying conductors. Ørsted's discovery also represented a major step toward a unified concept of energy.

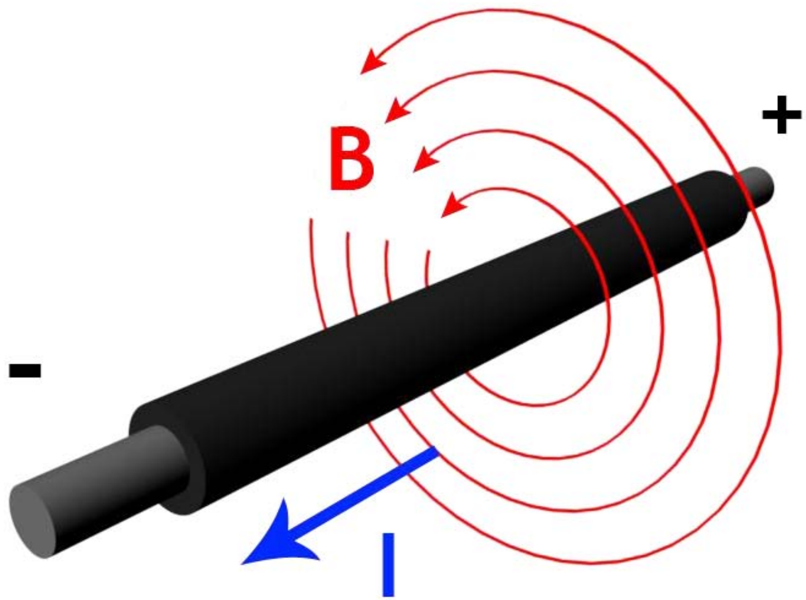
http://www.bookrags.com/biography/hans-christian-orsted-wop/

http://en.wikipedia.org/wiki/Hans_Christian_Oersted

3rd Observation: Magnetic Fields from Wires

Ampere observe that:

1) the H-field is rotationally symmetric around wire
2) the H-field falls off as 1/r
3) the H-field is proportional to the current in the wire



Andre-Marie Ampere, Memoir on the Mathematical Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience (1826)

Ampere's Law for Magnetostatics

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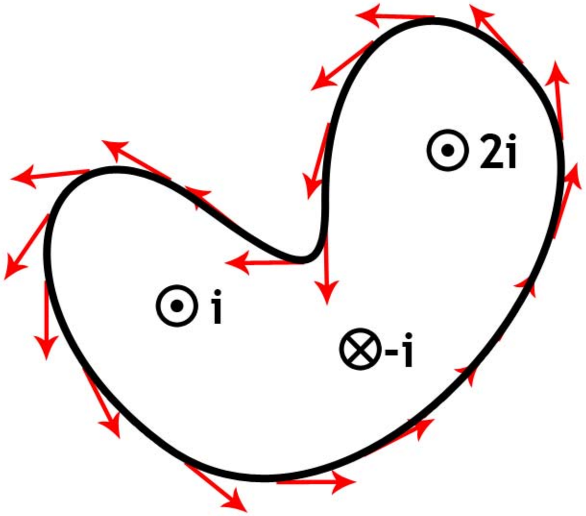
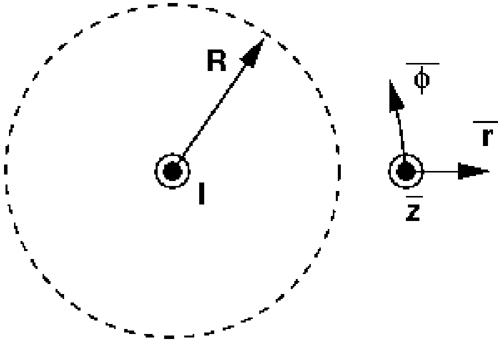


$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A}$$

$$= I_{enclosed}$$

Andre-Marie Ampere, Memoir on the Mathematical Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience (1826)

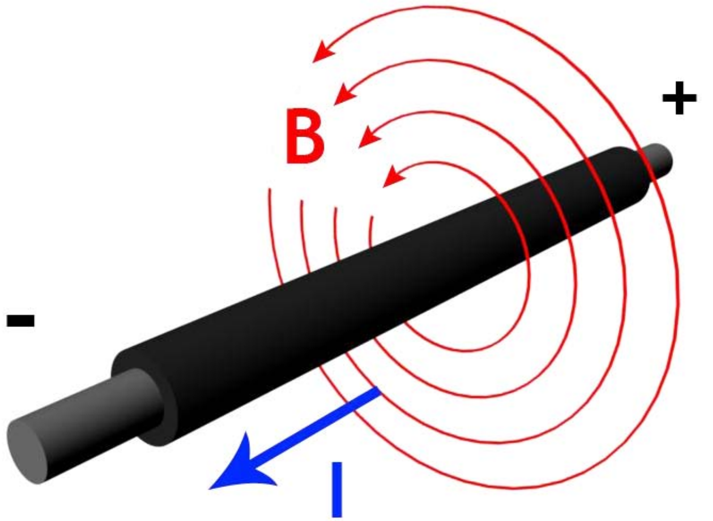
Magnetic Field Around a Very Long Wire Carrying Current in the z-Direction



$$\int_C \overline{H} \cdot d\overline{l} = \int_S \overline{J} \cdot d\overline{A}$$

$$H_\phi 2\pi r = I$$

$$\overline{H} = \frac{I}{2\pi r}\widehat{\phi}$$



Ampere observe that:

1) the H-field is rotationally symmetric around wire
2) the H-field falls off as 1/r
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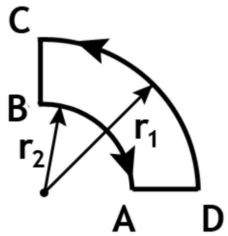
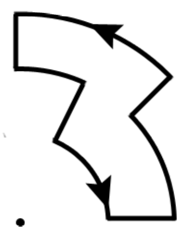
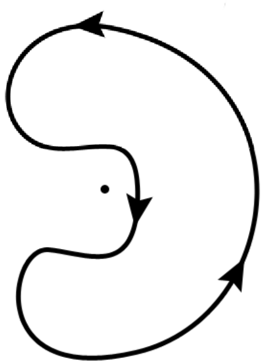
Ampere's Law Examples



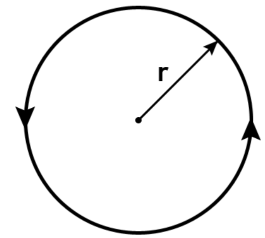
(a) Path lying in plane perpendicular to wire



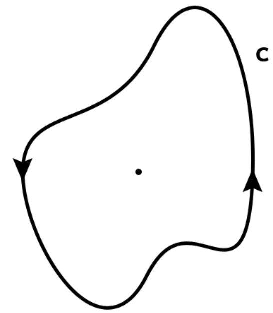
(b) Path constructed of Radial segments and arcs



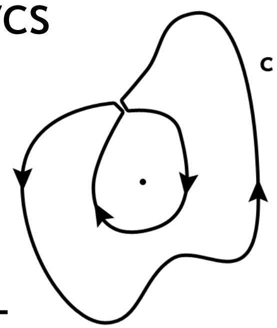
(c) Path which does not Enclose the wire



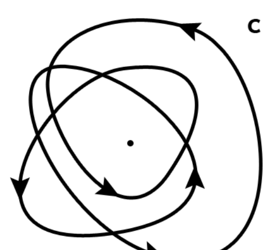
(d) Circular path enclosing wire



(e) Crooked path enclosing wire



(f) Circular and crooked path NOT enclosing wire

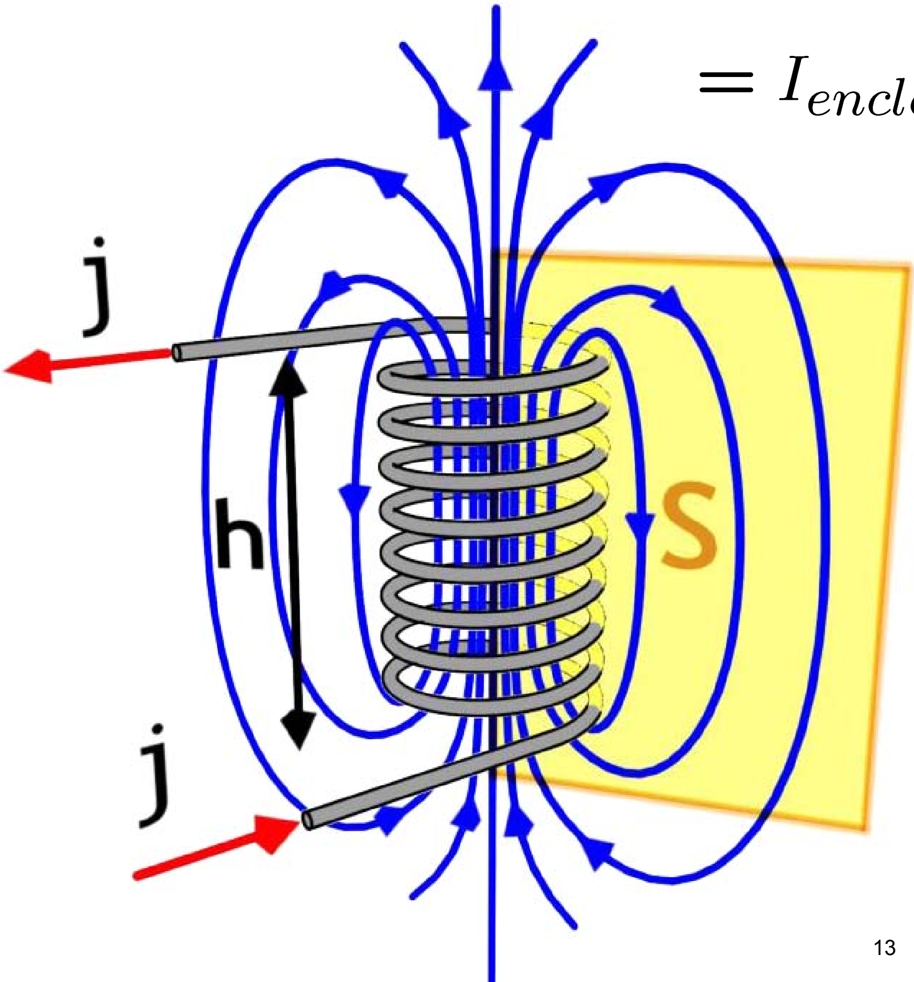


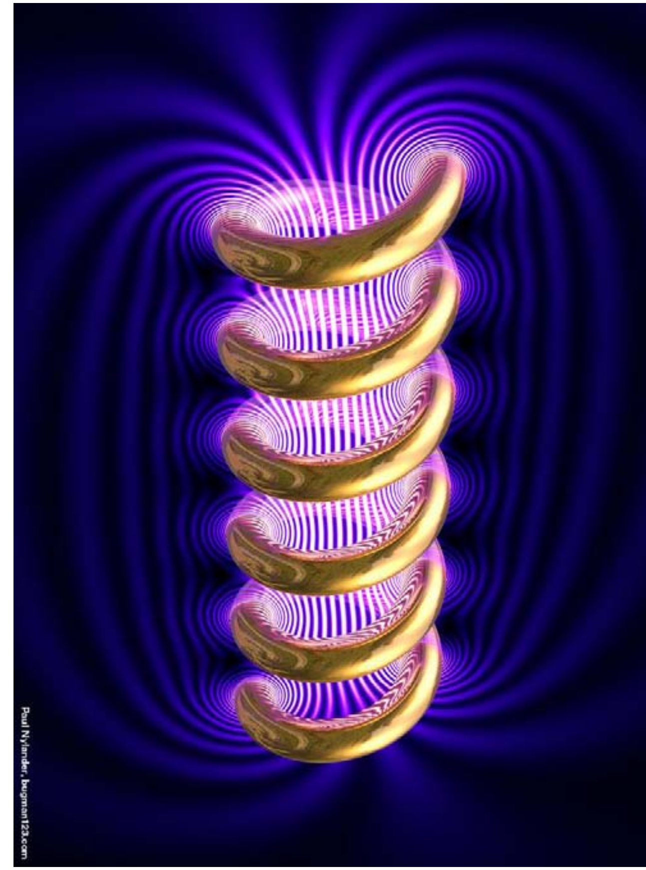
(g) Loop of N turns enclosing wire

Fields from a Solenoid

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A}$$

$$= I_{enclosed}$$



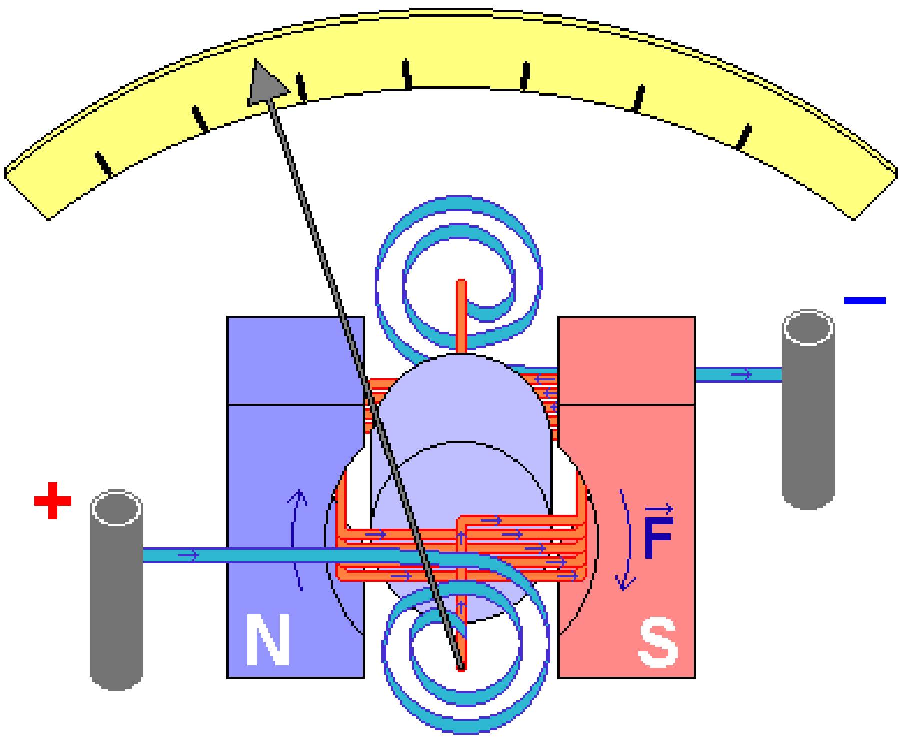


Courtesy of Paul Nylander. Used with permission.

$$H_{inside} \approx \boxed{}$$

A galvanometer is a type of an electric current meter. It is an analog electromechanical transducer that produces a rotary deflection of some type of pointer in response to electric current flowing through its coil.

Ampere invented the galvanometer.
Schweigger used a coil (1821).
Nobili improved on it in 1825 with
two opposite magnets, one of which is in the coil.



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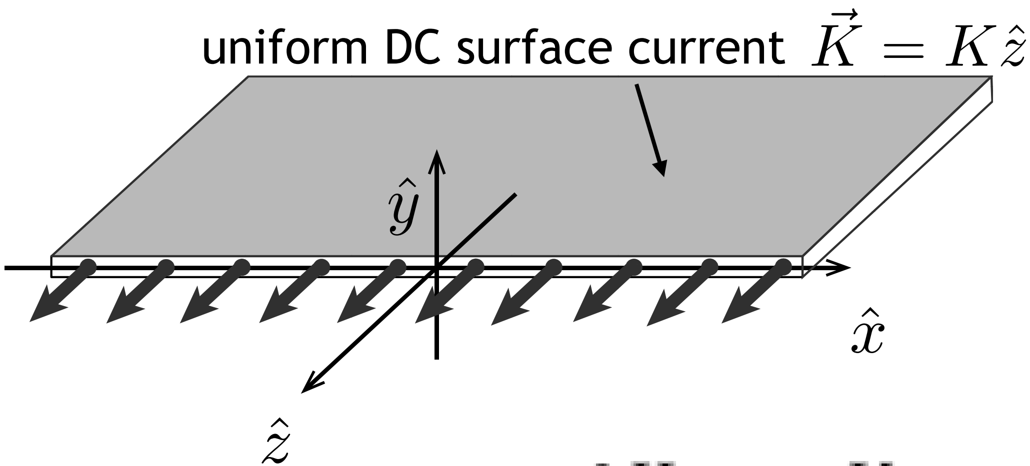
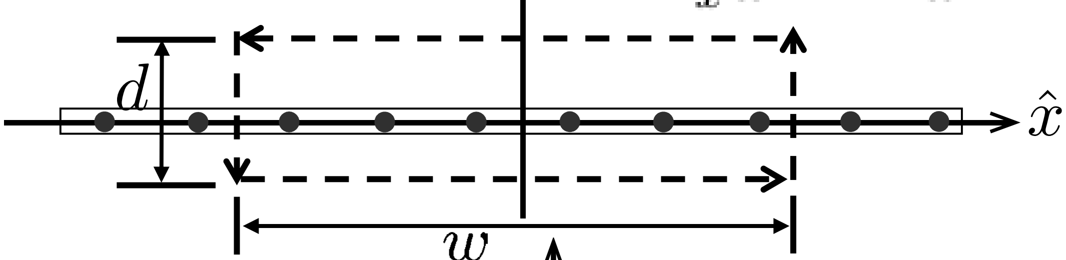
Magnetic Field Above/Below a Sheet of Current

... flowing in the $\hat{z}$ direction with current density K

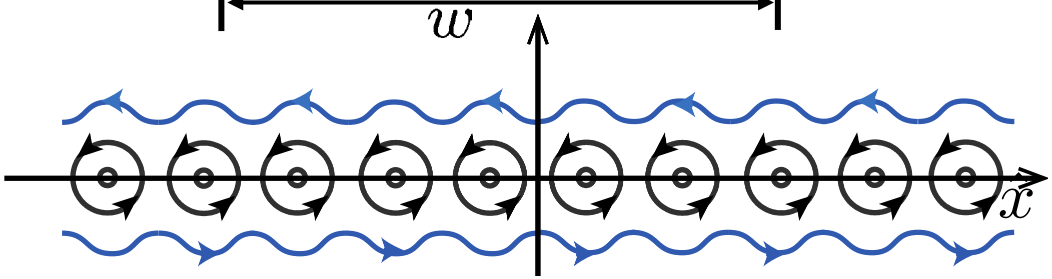


As seen "end on", the current sheet may be thought of as a combination of parallel wires, each of which produces its own H field. These H fields combine, so that the total H field above and below the current sheet is directed in $-\hat{x}$ and $\hat{x}$ direction, respectively.

$$2H_x w = Kw$$



$$\vec{H} \cdot \vec{dl} \begin{cases} H_x\, dx & on\ AB \\ 0 & on\ BC \\ (-H_x)(-dx) & on\ CD \\ 0 & on\ DA \end{cases}$$



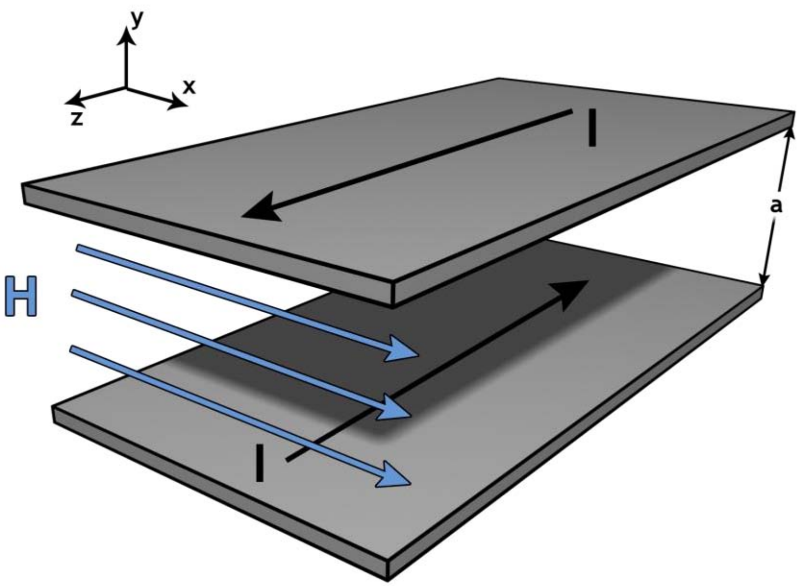
In between the wires the fields cancel

$$\vec{H} = \begin{cases} \dfrac{-\hat{x}K}{2} & y > 0 \\ \dfrac{\hat{x}K}{2} & y < 0 \end{cases}$$

What happens if we place near by each other ...

Two Parallel-Plate Conductors

... with currents flowing in opposite directions



$$\overline{H_A} = \begin{cases} \frac{-\hat{x}K}{2} & y > a \\ \frac{\hat{x}K}{2} & y < a \end{cases}$$

$$\overline{H_B} = \begin{cases} \frac{\hat{x}K}{2} & y > 0 \\ \frac{-\hat{x}K}{2} & y < 0 \end{cases}$$

Solve by using SUPERPOSITION

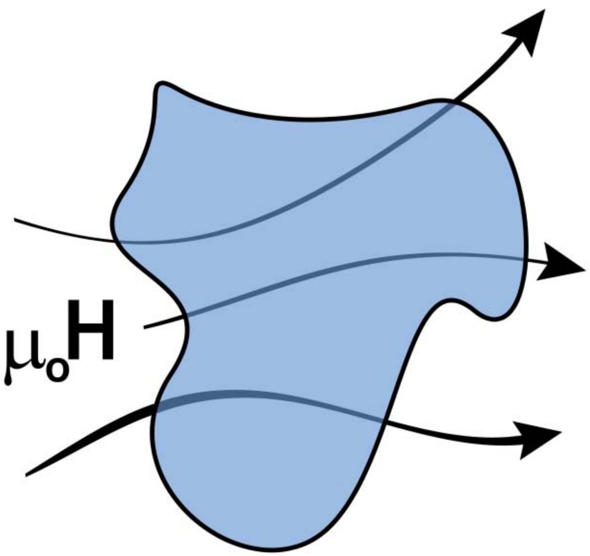
$$\overline{H} = \overline{H_A} + \overline{H_B} = \begin{cases} 0 & y > a \\ \hat{x}K = \frac{\hat{x}I}{w} & a > y > 0 \\ 0 & y < 0 \end{cases}$$

All the magnetic field is confined between the two current plates !

4^{th} Observation: No Magnetic Monopoles

$$\nabla \cdot \mu_o \vec{H} = 0 \qquad \text{and} \qquad \oint\!\!\!\oint_S \mu_o \vec{H} \cdot d\vec{S} = 0$$

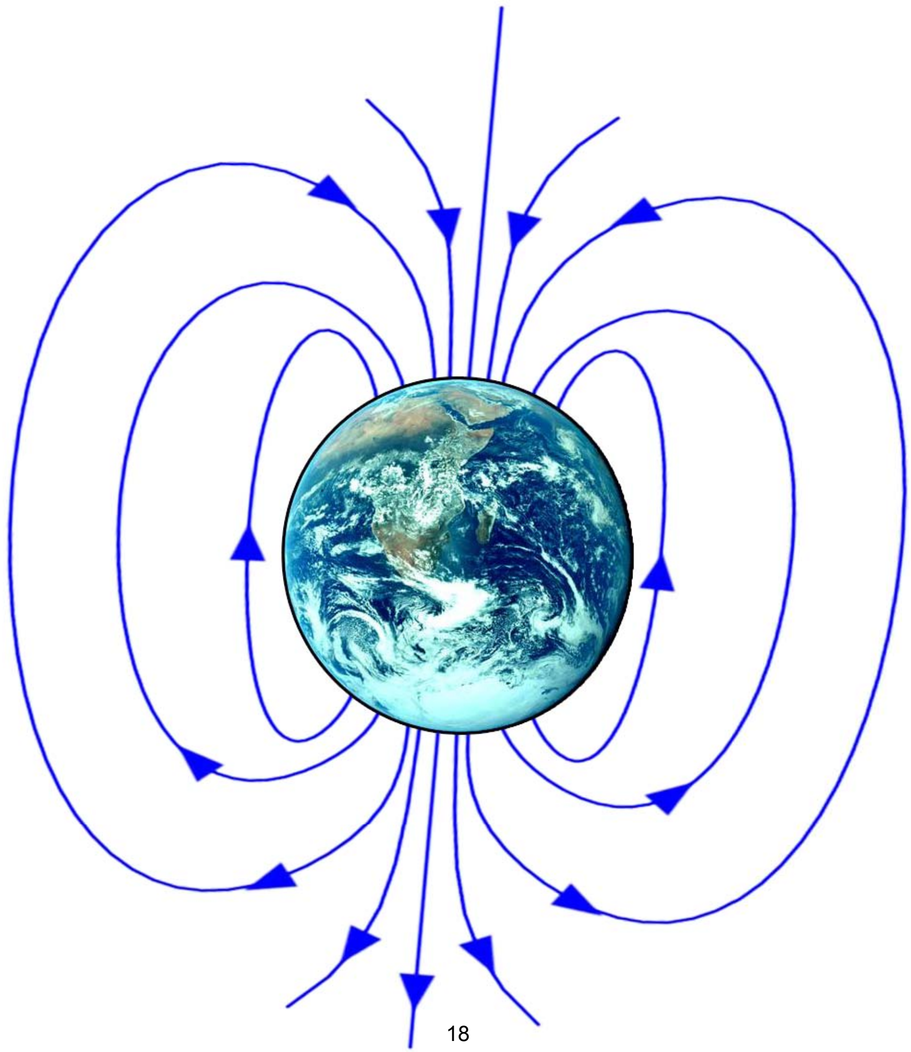
Gauss' Law for Magnetic Fields



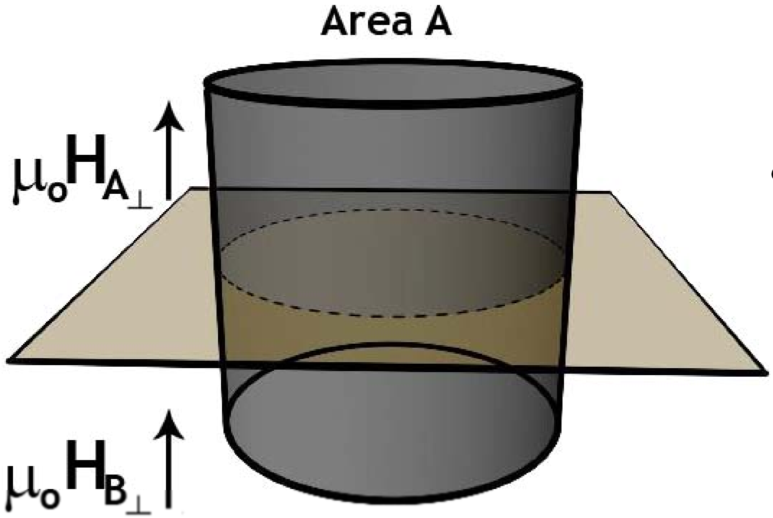
No net magnetic flux enters of exits a closed surface.
What goes in must come out.

Lines of magnetic flux ($\mu_o \vec{H}$) never terminate.
Rather, they are solenoidal and close on themselves in loops.

Earths Magnetic Field



Magnetostatic Boundary Conditions

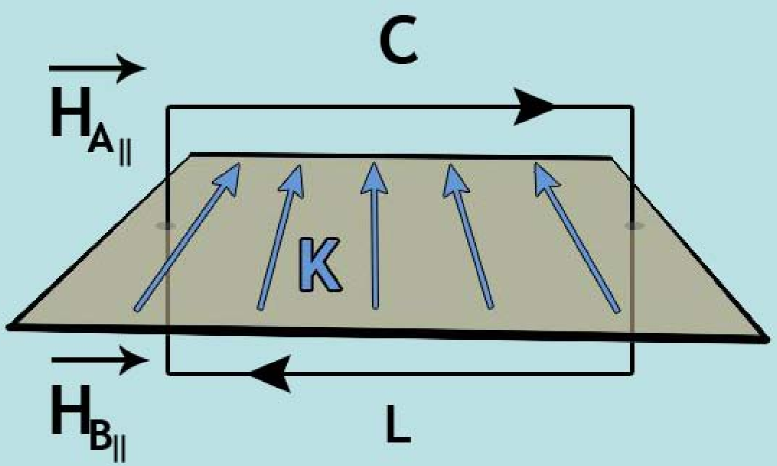


$$\lim_{\delta \to 0} \; Gauss \Rightarrow (\mu_0 H_{A\perp} - \mu_0 H_{B\perp})A = 0$$

$$\hat{n} \cdot (\mu_0 H_A - \mu_0 H_B) = 0$$

GAUSS'LAW:

Normal $\mu_o \vec{H}$ is continuous at a surface.



$$\lim_{\delta \to 0} \; Ampere \Rightarrow (H_{A\|} - H_{B\|})L = K\,L$$

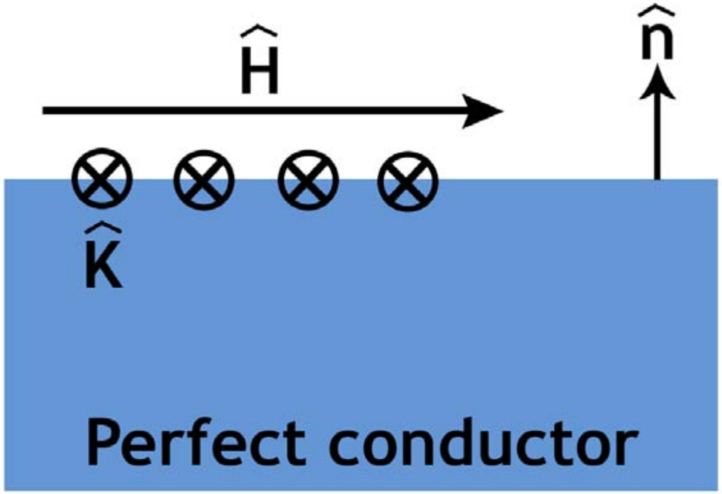
$$\hat{n} \times (H_A - H_B) = \vec{K}$$

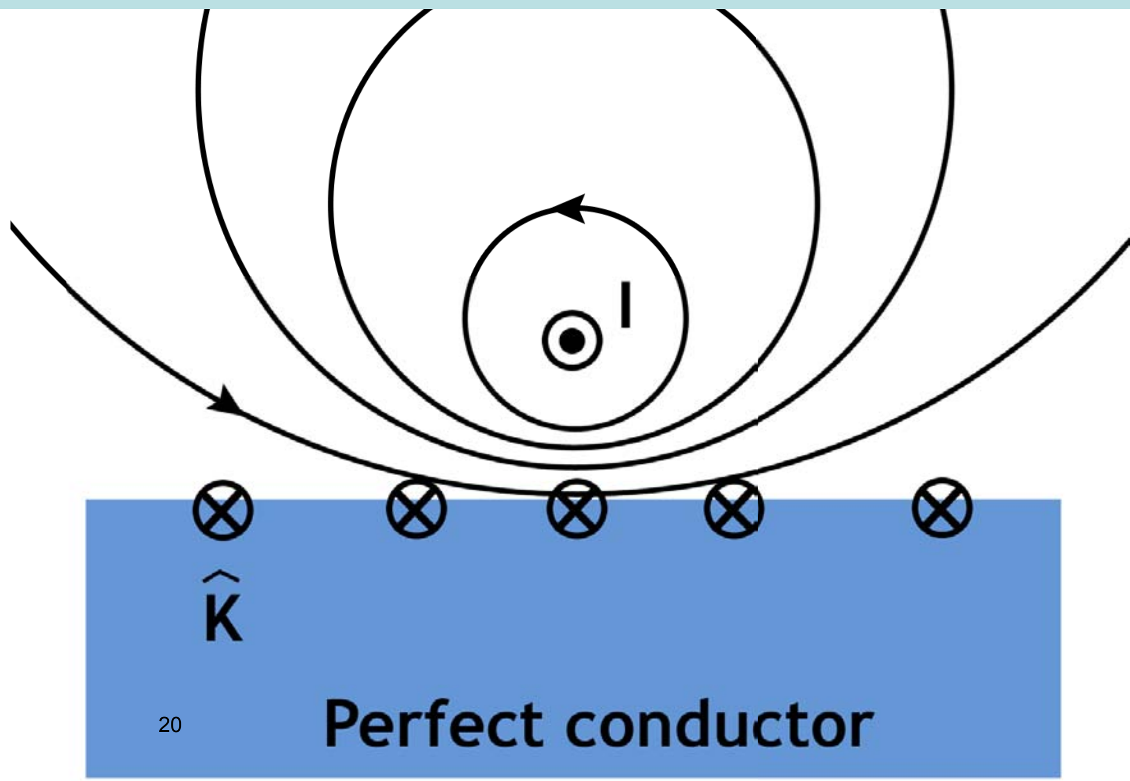
AMPERE'S LAW

Tangential $\vec{H}$ is discontinuous at a surface current $\vec{K}$.

Magnetic Fields at Perfect Conductors

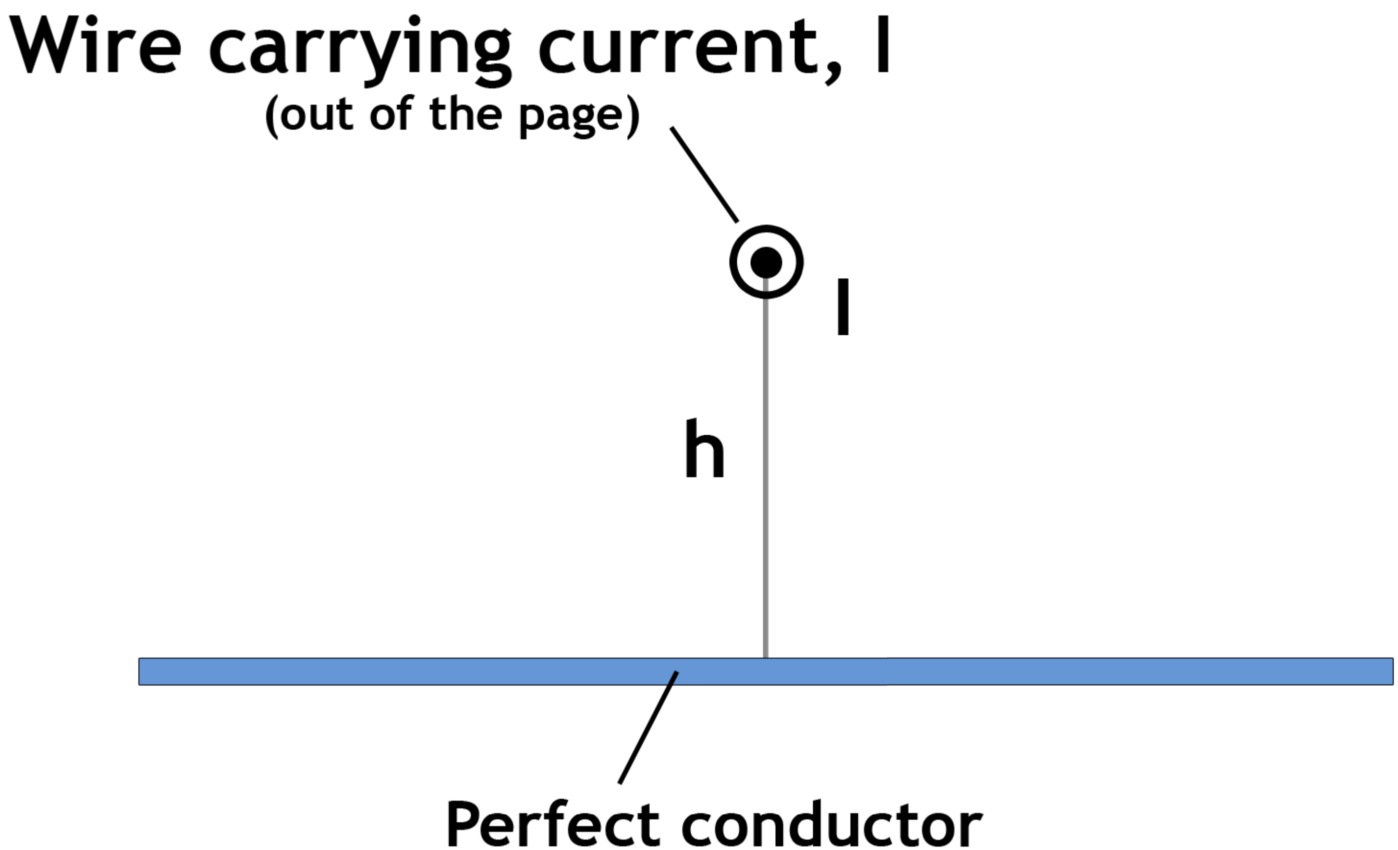
Perfect conductors exclude magnetic fields. Since normal $\mu_o \vec{H}$ is continuous across a surface, there can be no normal $\mu_o \vec{H}$ at the surface of a perfect conductor. Thus, only tangential magnetic fields can be present at the surface. They are terminated with surface currents.





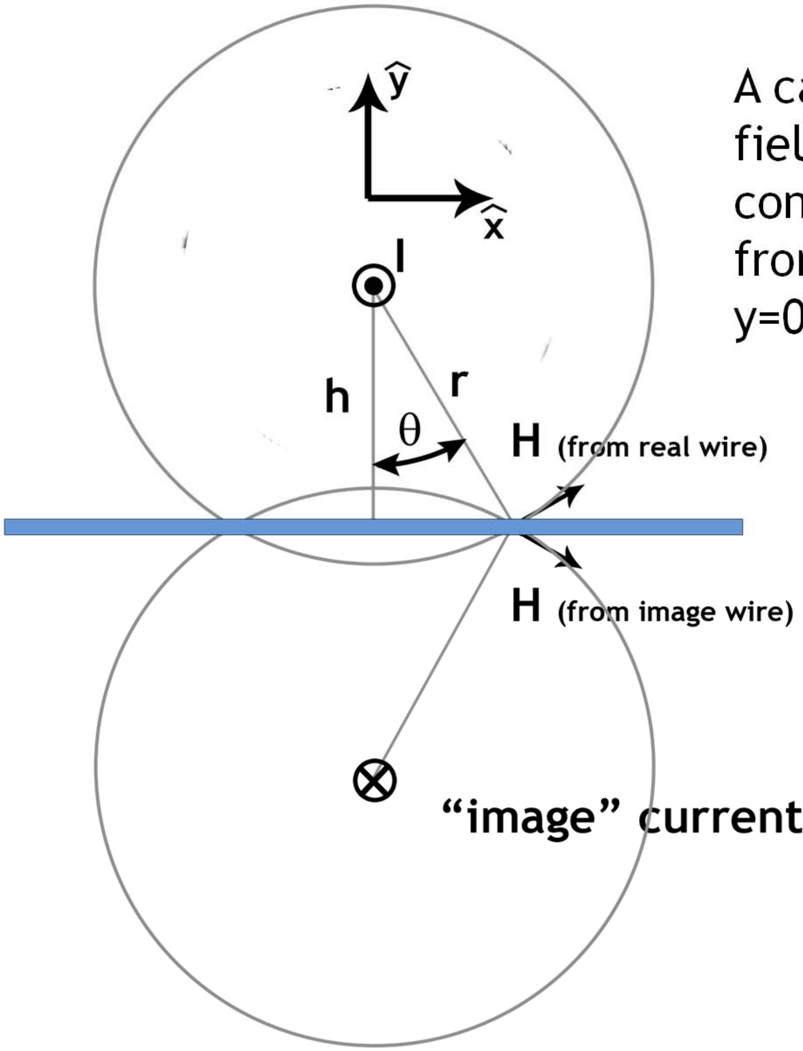
Boundary Condition Example: Magnetic Field at a 'perfect conductor'

There can be no fields (E or H) inside such perfect conductors, so any H field just at the surface must be parallel to the surface.



Solution uses the 'Method of Images':

A negative 'image' of the real current is situated below the surface, the same distance as the actual current, ensuring that the magnetic field at the surface is tangential.



A calculation of the x-directed (horizontal field at the surface of the 'perfect' conductor employs superposition of fields from the two (real and image) sources. At y=0:

$$H_x = \frac{I}{2\pi r}\cos\theta + \frac{I}{2\pi r}\cos\theta$$

$$H_y = \frac{I}{2\pi r}\sin\theta - \frac{I}{2\pi r}\sin\theta = 0$$

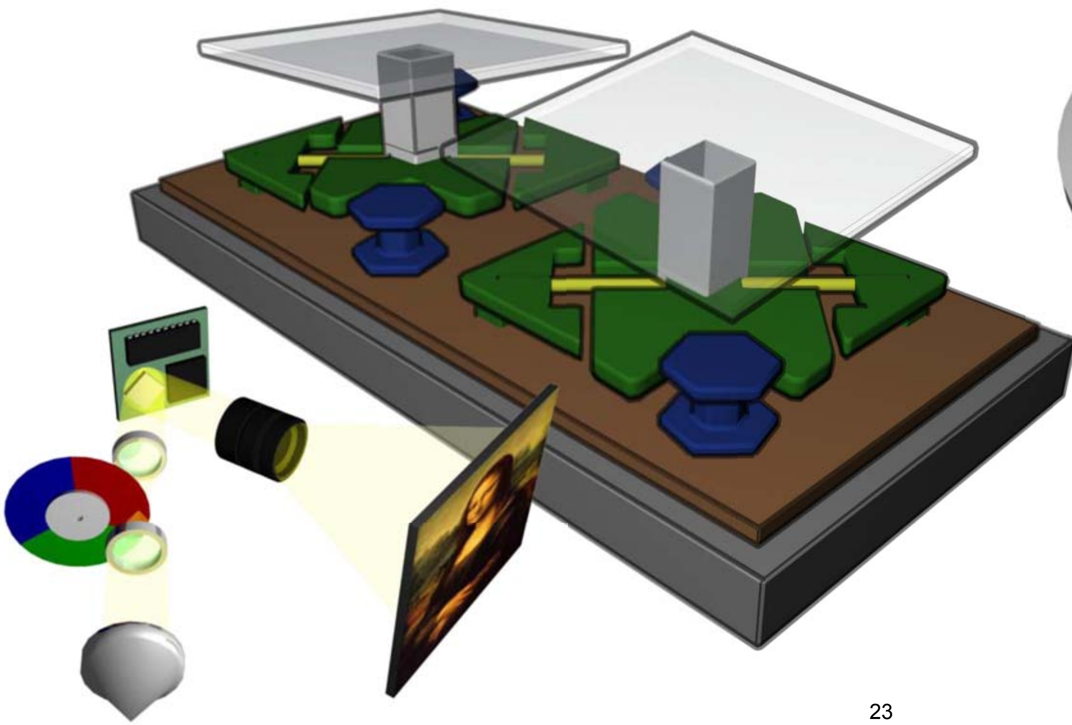
$$r = \sqrt{h^2 + x^2} \qquad \cos\theta = \frac{h}{\sqrt{h^2 + x^2}}$$

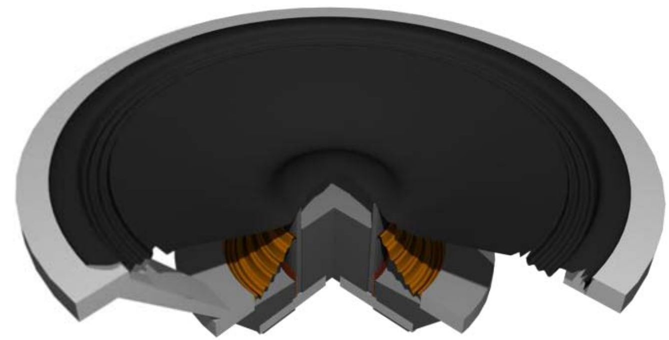
$$H_x = \frac{I}{\pi}\frac{h}{h^2 + x^2}$$

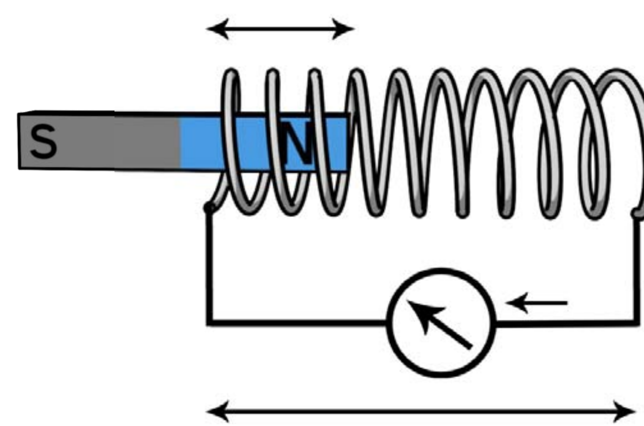
Actuators

Now that we know how to calculate
charges & E-fields, currents and H-fields
we are ready to calculate the forces that make things move

$$\overline{f} = q(\overline{E} + \overline{v} \times \mu_o \overline{\mathrm{H}})$$







KEY TAKEAWAYS

- Maxwell's Equations (in Free Space with Electric Charges present):

	DIFFERENTIAL FORM	**INTEGRAL FORM**
E-Gauss:	$\nabla \cdot \epsilon_o \vec{E} = \rho$	$\oiint_S \epsilon_o \vec{E} \cdot dS = \iiint_V \rho dV$
Faraday:	$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\mu_o \vec{H}$	$\oint_C \vec{E} \cdot d\vec{C} = \iint_S -\frac{\partial}{\partial t}\mu_o \vec{H} \cdot d\vec{S}$
H-Gauss:	$\nabla \cdot \mu_o \vec{H} = 0$	$\oiint_S \mu_o \vec{H} \cdot d\vec{S} = 0$
Ampere:	$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\epsilon_o \vec{E}$	$\oint_C \vec{H} \cdot d\vec{C} = \iint_S (\vec{J} + \frac{\partial}{\partial t}\mu_o \vec{H}) \cdot d\vec{S}$

- Boundary conditions for **E**-field:
  . Normal E-field – discontinuous
  . Tangential E-field - continuous

- Boundary conditions for **H**-field:
  . Normal H-field – continuous
  . Tangential H-field - discontinuous