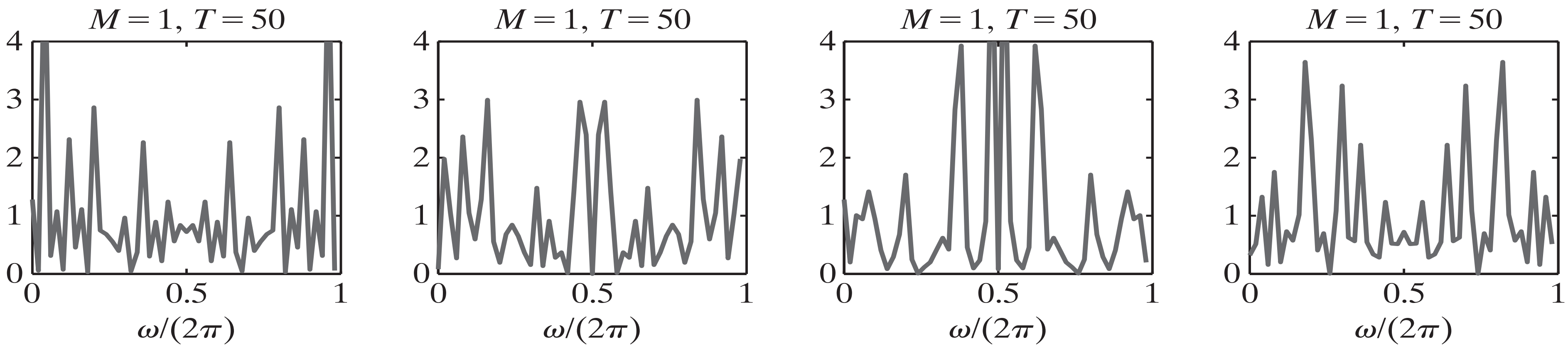
Einstein-Wiener-Khinchin theorem, PSD applications, modeling filters

6.011, Spring 2018

Lec 19

Periodograms
(e.g., a unit-intensity "white" process)



$$\text{CT case:} \quad X_T(j\omega) \leftrightarrow x(t) \ \text{ windowed to } \ [-T, T]$$

$$\text{Periodogram} = \frac{|X_T(j\omega)|^2}{2T}$$

$$\text{DT case:} \quad X_N(e^{j\Omega}) \leftrightarrow x[n] \ \text{ windowed to } \ [-N, N]$$

$$\text{Periodogram} = \frac{|X_N(e^{j\Omega})|^2}{2N+1}$$

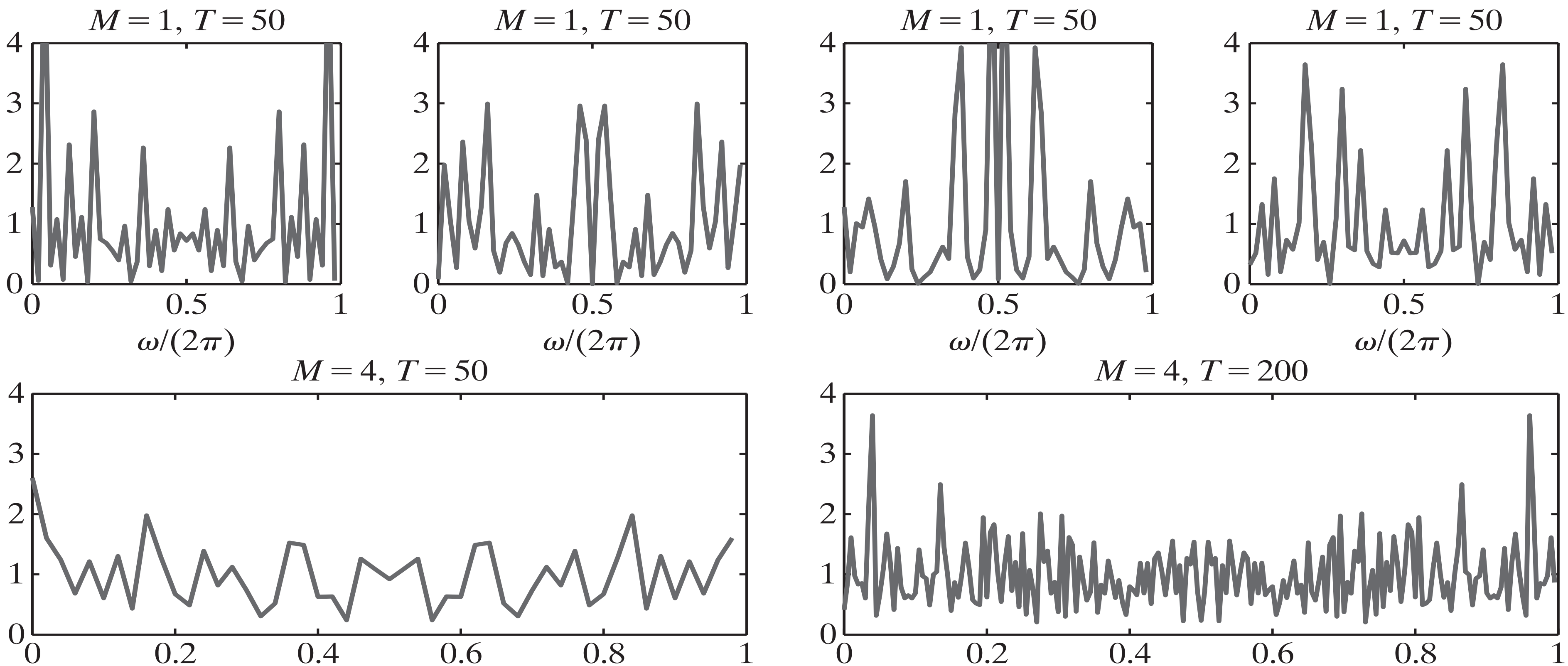
Einstein-Wiener-Khinchin theorem

$$\frac{1}{2T} E\left[|X_T(j\omega)|^2\right] = S_{xx}(j\omega) \star \frac{\sin^2(\omega T)}{\pi\omega^2 T}$$

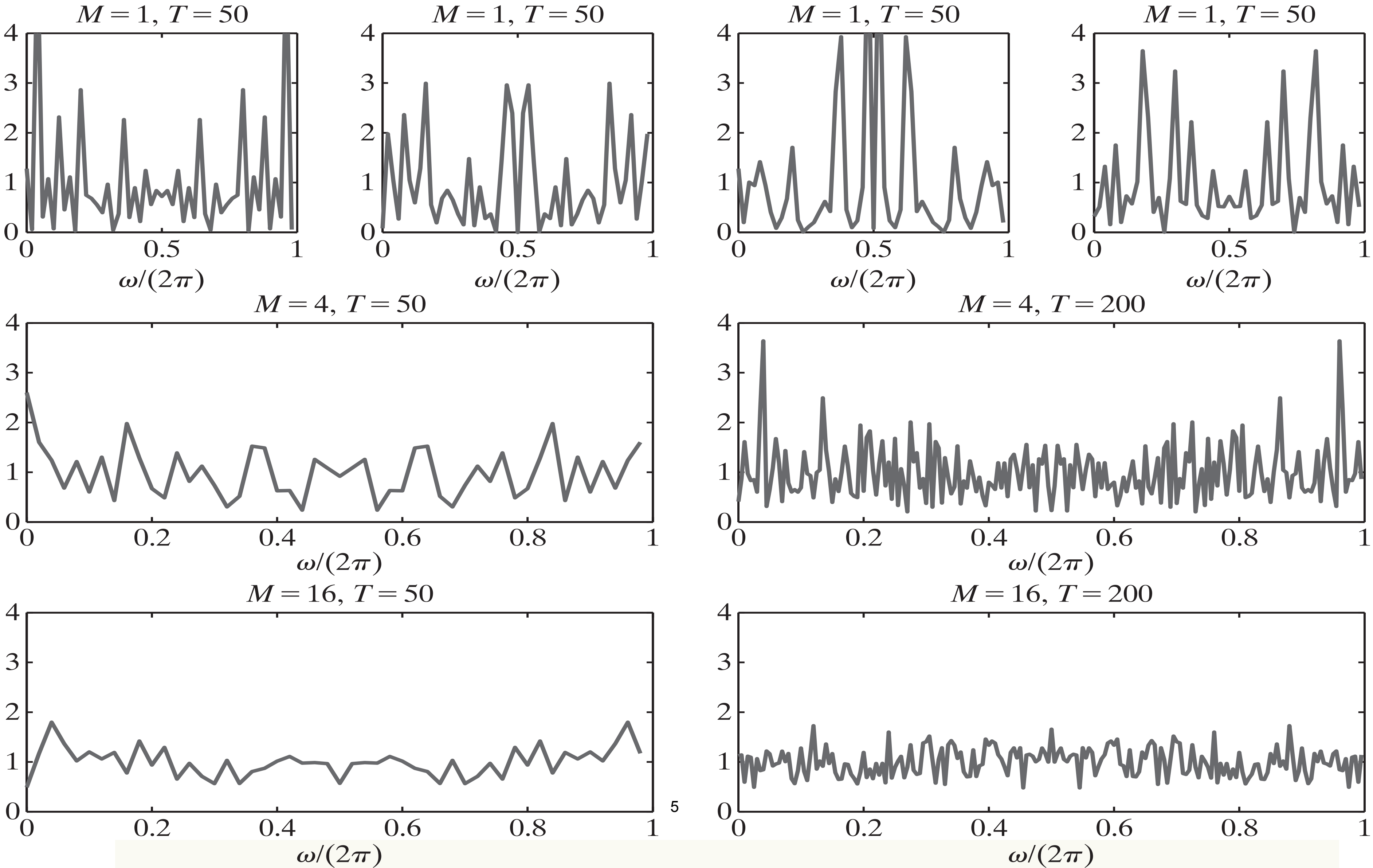
$$\text{Since} \quad \lim_{T\to\infty} \frac{\sin^2(\omega T)}{\pi\omega^2 T} = \delta(\omega) \ ,$$

$$\lim_{T\to\infty} \frac{1}{2T} E\left[|X_T(j\omega)|^2\right] = S_{xx}(j\omega)$$

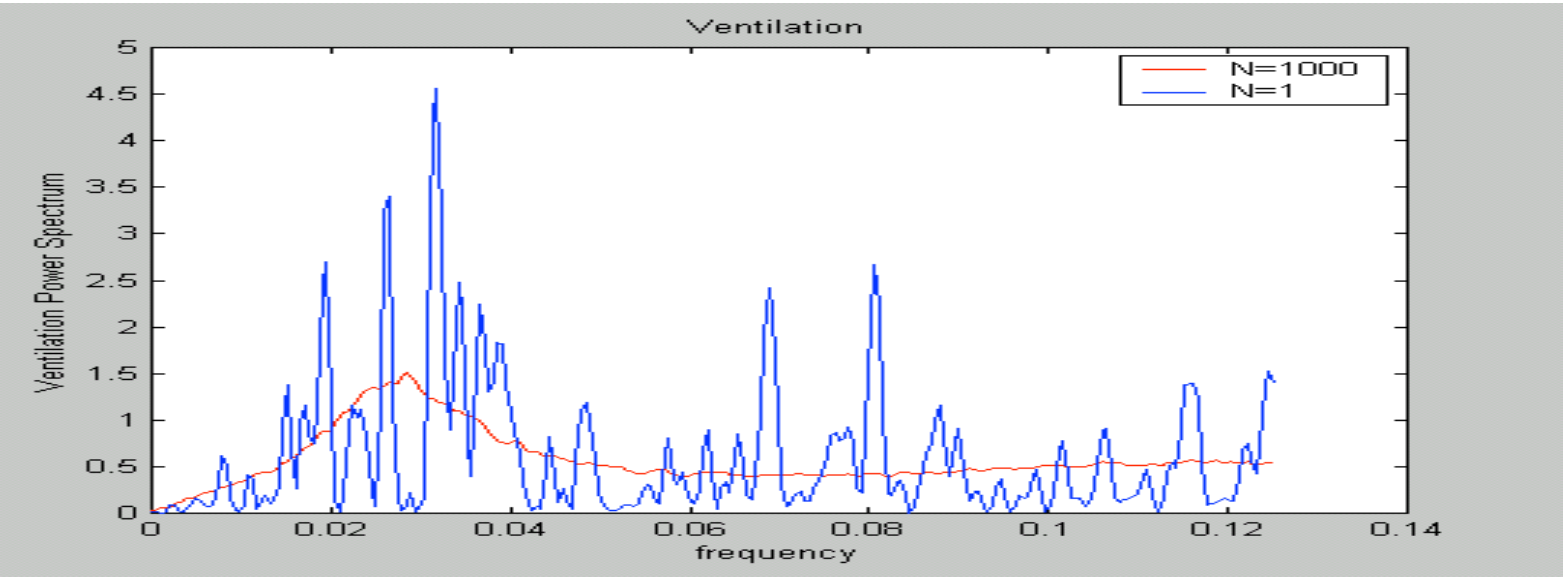
Periodogram averaging (illustrating the Einstein-Wiener-Khinchin theorem)

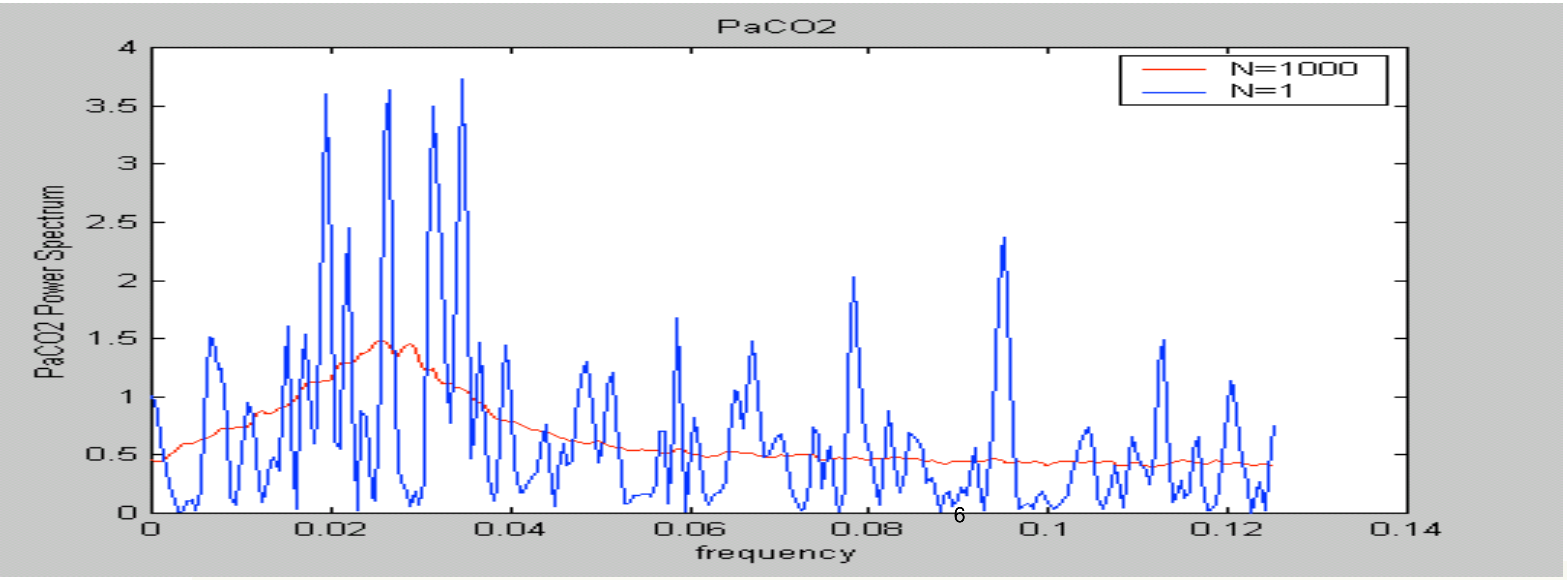


Periodogram averaging (illustrating the Einstein-Wiener-Khinchin theorem)



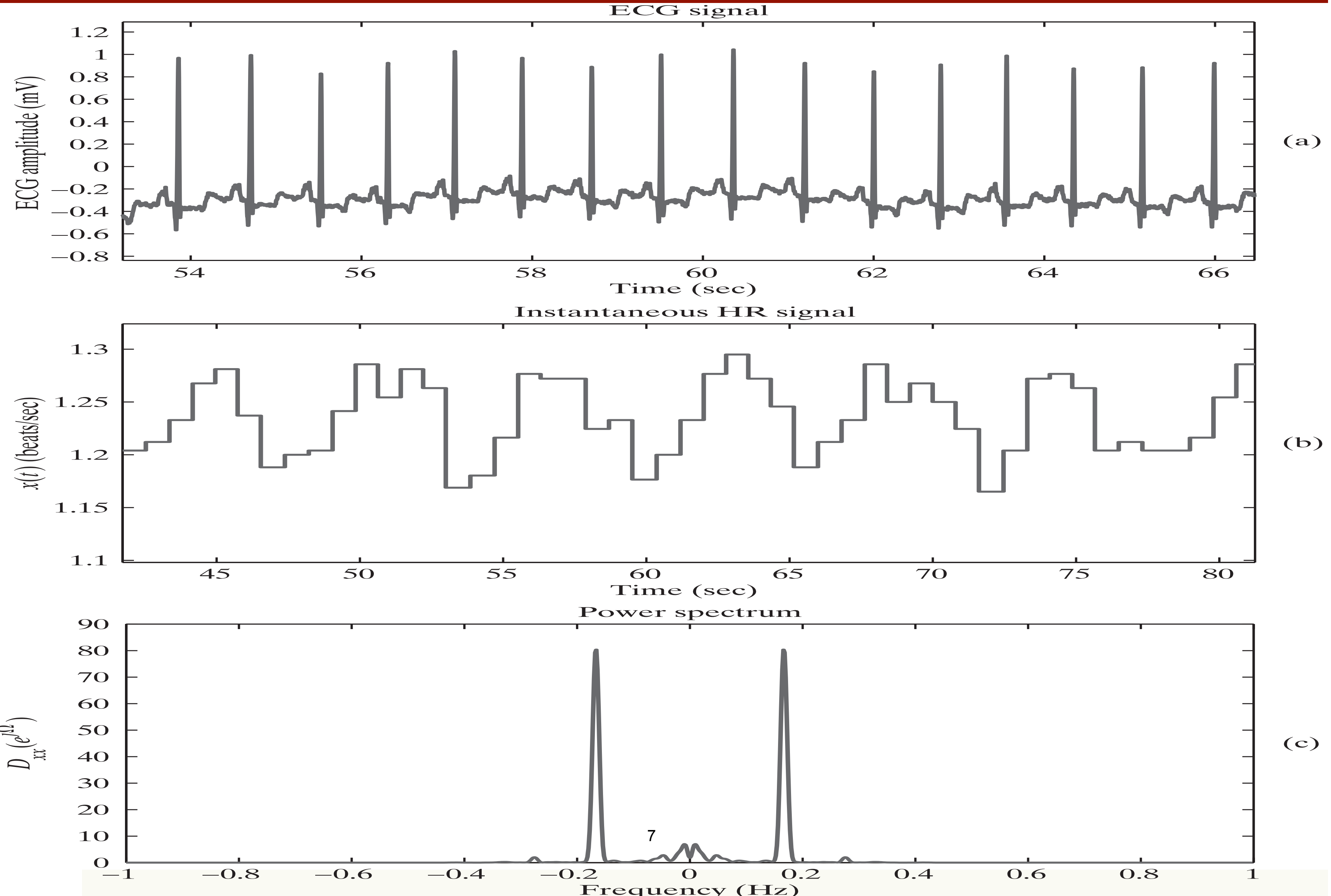
Respiratory model





cf. Khoo's textbook for N=1

Heart rate variability



Modeling filters

e.g., generate sample functions of WSS $y[\cdot]$ that has specified μ_y

and $\quad C_{yy}[m] = \sigma_y^2(\rho\delta[m-1] + \delta[m] + \rho\delta[m+1])$

$$\downarrow$$

Try $h[n] = a\delta[n] + b\delta[n-1]$ driven by unit-intensity white noise $x[\cdot]$,

$$H(z) = a + bz^{-1}\ , \quad |H(e^{j\Omega})|^2 = D_{yy}(e^{j\Omega})$$

More generally, $H(z)A(z)$ for allpass $A(z)\,,\ \ A(z)A(z^{-1}) = 1$

Need to add mean μ_y to the output

MIT OpenCourseWare
https://ocw.mit.edu

6.011 Signals, Systems and Inference
Spring 2018

For information about citing these materials or our Terms of Use, visit: https://ocw.mit.edu/terms.