## YOUR NAME

# Department of Electrical Engineering and Computer Science <br> Massachusetts Institute of Technology 

# 6.012 Electronic Devices and Circuits 

Exam No. 1<br>Wednesday, October 7, 2009

7:30 to 9:30 pm

## Notes:

1. An effort has been made to make the various parts of these problems independent of each other so if you have difficulty with one item go on, and come back later.
2. Some questions ask for an explanation of your answer. No credit will be given for answers lacking this explanation.
3. Unless otherwise indicated, you should assume room temperature and that $\mathrm{kT} / \mathrm{q}$ is 0.025 V . You should also approximate $[(\mathrm{kT} / \mathrm{q}) \ln 10]$ as 0.06 V .
4. Closed book; one sheet ( 2 pages) of notes permitted. Formula sheet provided.
5. All of your answers and any relevant work must appear on these pages. Any additional paper you hand in will not be graded.
6. Make reasonable approximations and assumptions. State and justify any such assumptions and approximations you do make.
7. Be careful to include the correct units with your answers when appropriate.
8. Be certain that you have all ten (10) pages of this exam booklet and the three (3) page formula sheet, and make certain that you write your name at the top of this page in the space provided.

PROBLEM 1
PROBLEM 2
PROBLEM 3
$\qquad$
$\qquad$
$\qquad$

TOTAL

## Problem 1-(34 points)

This problem contains 4 independent short problems that can be worked in any order.
a) [6 pts] You find a sample of silicon, but the accompanying data sheet has been partially destroyed so you only know that it is n-type, its resistivity is $100 \mathrm{Ohm}-\mathrm{cm}$, and its electron mobility is $1600 \mathrm{~cm}^{2} / V-\mathrm{s}$. You also remember that $\mathrm{n}_{\mathrm{i}}(\mathrm{Si})=10^{10} \mathrm{~cm}^{-3}$.
i) What is the approximate equilibrium electron concentration, $\mathrm{n}_{\mathrm{o}}$, in this sample?

$$
\mathrm{n}_{\mathrm{o}}=\ldots \mathrm{cm}^{-3}
$$

ii) What is the approximate equilibrium hole concentration, $\mathrm{p}_{\mathrm{o}}$, in this sample?

$$
\mathrm{p}_{\mathrm{o}}=\ldots \mathrm{cm}^{-3}
$$

b) [10 pts] The excess hole population illustrated on the right is established in an n-type silicon sample, $\mathrm{N}_{\mathrm{D}}=10^{17} \mathrm{~cm}^{-3}$. The minority carrier lifetime in this sample is $10^{-4} \mathrm{~s}$, the hole mobility is $640 \mathrm{~cm}^{2} / \mathrm{V}$-s, and the cross-section of the sample is $10^{-4} \mathrm{~cm}^{2}$.
i) What is the hole current, $\mathrm{i}_{\mathrm{h}}=\mathrm{A} \mathrm{J}_{\mathrm{h}}$ in this sample?


Note: $1 \mu \mathrm{~m}=10^{-4} \mathrm{~cm}$

$$
i_{h}=
$$

$\qquad$ Amps
ii) What is the total number of excess holes in this sample?

Excess holes = $\qquad$

## Problem 1 continued

iii) What is the total rate at which excess holes are recombining with excess electrons in this sample, and what is the corresponding hole current, $\mathrm{i}_{\mathrm{h}, \text { recomb }}$ ?

Total recombination occurring in sample $=$ $\qquad$ holes/s

Hole recombination current, $i_{\text {h,recomb }}=$ $\qquad$ Amps
c) [9 pts] Consider two $\mathrm{p}^{+}-\mathrm{n}$ diodes which are identical except for the fact that in Diode A the minority carrier lifetime is infinite making $L_{h} \gg w_{N}$, while in Diode $B$ the minority carrier lifetime is finite and small enough that $\mathrm{L}_{\mathrm{h}}<\mathrm{w}_{\mathrm{N}}$. Note: $\mathrm{w}_{\mathrm{N}}$ is the width of the $n$-type side, and $\mathrm{L}_{\mathrm{h}}$ is the minority carrier diffusion length.
i) Which of these two diodes has the larger saturation current, $I_{s}$ ? Explain your answer.

$$
\text { Diode A } \_ \text {Diode B } \_ \text {They are similar }
$$

Explanation:
ii) Which of these two diodes has the larger total number of excess holes in the n side quasi-neutral region when a forward biased $\mathrm{V}_{\mathrm{AB}}$ is applied?
Diode A $\qquad$ Diode B $\qquad$ They are similar $\qquad$
Explanation:
iii) Which of these two diodes has the wider space charge layer (depletion region) in thermal equilibrium?
Diode A $\qquad$ Diode B $\qquad$ They are similar $\qquad$
Explanation:

## Problem 1 continued

d) [9 pts] This question concerns the npn bipolar transistor shown below. There is negligible minority carrier recombination throughout except at the ohmic contacts.

i) On the axes provided, sketch and label the excess minority carrier profiles when both junctions are forward biased with $\mathrm{V}_{\mathrm{BE}}=\mathrm{V}_{\mathrm{BC}}=0.6 \mathrm{~V}$. Notice that this is not a bias in the forward active region.

ii) Calculate the forward current gain, $\beta_{\mathrm{f}}\left(\approx 1 / \delta_{\mathrm{E}}\right)$, for this device when it is biased in the forward active region, i.e. $\mathrm{v}_{\mathrm{BE}}>0$ and $\mathrm{v}_{\mathrm{CE}} \leq 0 . \mathrm{D}_{\mathrm{e}}=40 \mathrm{~cm}^{2} / \mathrm{s}, \mathrm{D}_{\mathrm{h}}=15 \mathrm{~cm}^{2} / \mathrm{s}$.

$$
\beta_{\mathrm{f}}=
$$

$\qquad$
iii) Redesign this transistor to increase the forward current gain, $\beta_{\mathrm{f}}$, to 100 by increasing the doping level of one of the three regions in this device. Indicate which region should be changed and to what the new doping level should be.

Region: $\qquad$
$\qquad$

Problem 2 - (32 points)
Consider the silicon diode pictured below. It is $4 \mu \mathrm{~m}$ long, with ohmic contacts at each end, and it is uniform p-type with $\mathrm{N}_{\mathrm{A}}=1 \times 10^{17} \mathrm{~cm}^{-3}$ for $1 \mu \mathrm{~m}$ on its far left end and uniform n-type with $\mathrm{N}_{\mathrm{D}}=1 \times 10^{17} \mathrm{~cm}^{-3}$ on its far right end. In between these two uniformly doped regions, the net concentration, $\mathrm{N}_{\mathrm{d}}(\mathrm{x})-\mathrm{N}_{\mathrm{a}}(\mathrm{x})$, slowly grades linearly over a distance of $2 \mu \mathrm{~m}$ from $-1 \times 10^{17} \mathrm{~cm}^{-3}$ on the left to $1 \times 10^{17} \mathrm{~cm}^{-3}$ on the right, as shown in the lower figure.

a) [6 pts] In thermal equilibrium, what is the electrostatic potential, $\phi(x)$, in the lefthand quasi-neutral region at $x=-1.5 \mu \mathrm{~m}$, and what is the electrostatic potential, $\phi(\mathrm{x})$, in the right-hand quasi-neutral region at $x=+1.5 \mu \mathrm{~m}$, and what is the built-in potential step, $\Delta \phi_{b}$, seen transiting from $x=-1.5 \mu \mathrm{~m}$ to $\mathrm{x}=+1.5 \mu \mathrm{~m}$ ? Use the 60 mV rule, and $\log 2=0.3$.

$$
\begin{aligned}
& \phi(-1.5 \mu \mathrm{~m})= \\
& \phi(1.5 \mu \mathrm{~m})= \\
& \mathrm{V} \\
& \Delta \phi_{\mathrm{b}}= \\
& \mathrm{V} \\
& \mathrm{~V}
\end{aligned}
$$

## Problem 2 continued

b) [4 pts] For the rest of this problem, a bias voltage, $\mathrm{V}_{\mathrm{AB}}$, is applied to this diode resulting in a total depletion region width of $1 \mu \mathrm{~m}$, and $\mathrm{x}_{\mathrm{N}}=\left|\mathrm{x}_{\mathrm{P}}\right|=0.5 \mu \mathrm{~m}$. On the axes provided below, plot and label the net charge density, $\rho(x)$, for $x$ in the range -2 $\mu \mathrm{m}<\mathrm{x}<2 \mu \mathrm{~m}$ with this bias voltage applied. Use the depletion approximation and assume that the regions outside the depletion region are quasi-neutral, i.e. $\rho(x) \approx 0$.

c) [4 pts] On the axes provided below, plot and label the electric field, $E(x)$, for $x$ in the range - $2 \mu \mathrm{~m}<\mathrm{x}<2 \mu \mathrm{~m}$ with the bias voltage $\mathrm{V}_{\mathrm{AB}}$ applied.

d) [4 pts] What is the change in potential, $\Delta \phi$, transiting the depletion region when the bias is the same as in Part b, i.e. what is $\phi(0.5 \mu \mathrm{~m})-\phi(-0.5 \mu \mathrm{~m})$ ?

$$
\Delta \phi_{\text {Depl.Reg. }}=\phi(0.5 \mu \mathrm{~m})-\phi(-0.5 \mu \mathrm{~m})=
$$

$\qquad$ Volts
e) [4 pts] What is the change in potential, $\Delta \phi$, in transiting the quasi-neutral n-type graded region between $x=0.5 \mu \mathrm{~m}$ and $x=1.0 \mu \mathrm{~m}$, i.e. what is $\phi(1.0 \mu \mathrm{~m})-\phi(0.5 \mu \mathrm{~m})$ ?

$$
\Delta \phi_{\mathrm{nQNR} .}=\phi(1.0 \mu \mathrm{~m})-\phi(0.5 \mu \mathrm{~m})=\square \text { Volts }
$$

f) [4 pts] On the axes provided below, plot and label the electrostatic potential, $\phi(\mathrm{x})$, for $x$ in the range $-2 \mu \mathrm{~m}<\mathrm{x}<2 \mu \mathrm{~m}$ with the same bias voltage applied as in Part c. Use the depletion approximation, and assume that the regions outside the depletion region are quasi-neutral. In your plot make $\phi(0)=0$ Volts.

$\mathrm{g})$ [2 pts] What is the applied bias voltage, $\mathrm{V}_{\mathrm{AB}}$ ?

$$
\mathrm{V}_{\mathrm{AB}}=\ldots \text { Volts }
$$

Problem 3 (34 points)
A p-type semiconductor sample with acceptor concentration $N_{A}$ and length L , illustrated below, has ohmic contacts at both its ends. A light source generates $M_{A}$ electron-hole pairs $/ \mathrm{cm}^{2}$-s in the plane at $x=X_{A}$, i.e. $g_{L}(x)=M_{A} \delta\left(X_{A}\right)$. Assume low-level injection and quasi-neutrality everywhere in the bar.


The general equation governing the excess minority carriers in a uniformly doped material is

$$
\frac{d^{2} n^{\prime}(x)}{d x^{2}}-\frac{n^{\prime}(x)}{L_{e}^{2}}=-\frac{1}{D_{e}} g_{L}(x)
$$

a) [4 pts] What boundary condition is imposed on the excess minority carriers $\mathrm{n}^{\prime}$ at $\mathrm{x}=$ 0 and $\mathrm{x}=\mathrm{L}$ ?

Boundary condition at $x=0$ : $\qquad$

Boundary condition at $\mathrm{x}=\mathrm{L}$ : $\qquad$
b) [4 pts] We now make the assumption that the minority carrier lifetime is very long, which simplifies the general equation to:

$$
\frac{d^{2} n^{\prime}(x)}{d x^{2}} \approx-\frac{1}{D_{e}} g_{L}(x)
$$

What quantitative restriction is placed on the minority carrier lifetime, $\tau_{e}$, for this assumption to be valid?

$$
\tau_{\mathrm{e}} \gg \approx \ll
$$

(circle one)
(fill in the blank)
c) [6 pts] Using the long-lifetime approximation in part (b), determine two constraints (i.e. boundary conditions) on the excess minority carriers at $x=X_{A}$, i.e. relating $\mathrm{n}^{\prime}\left(\mathrm{X}_{\mathrm{A}}{ }^{-}\right)$to $\mathrm{n}^{\prime}\left(\mathrm{X}_{\mathrm{A}}{ }^{+}\right)$.

Constraint on $n^{\prime}$ at $x=X_{A}$ : $\qquad$
(Hint: Finite currents imply finite $\mathrm{dn}^{\prime} / \mathrm{dx}$.)

Constraint on $\mathrm{dn}^{\prime} / \mathrm{dx}$ in the vicinity of $\mathrm{x}=\mathrm{X}_{\mathrm{A}}$ :
(Hint: What goes in must come out.)
d) [6 pts] On the axes provided sketch the excess minority carrier concentration, $\mathrm{n}^{\prime}(\mathrm{x})$, everywhere inside the semiconductor. Label your sketch with the relevant equations for $\mathrm{n}^{\prime}(\mathrm{x})$.

e) [6 pts] On the axes provided sketch the minority carrier diffusion current, $\mathrm{J}_{\mathrm{e}, \mathrm{diff}}(\mathrm{x})$, everywhere inside the semiconductor. Label your sketch with the relevant equations for $J_{e, \text { diff }}(x)$.


Problem 3 continues on the next page
f) [4 pts] In the space below, briefly explain (approx. 25 words or less) why the minority carrier diffusion is the dominant minority carrier current.
g) [4 pts] A second light source is added illuminating a single spot along the semiconductor at $x=X_{B}$, where $x_{B}>X_{A}$, and generating electron-hole pairs at a rate $M_{B}$, so that $g_{L}(x)$ is now

$$
g_{L}(x)=M_{A} \delta\left(X_{A}\right)+M_{B} \delta\left(X_{B}\right)
$$

Find $\mathrm{n}^{\prime}(\mathrm{x})$ and $\mathrm{J}_{\mathrm{e} \text {,diff }}(\mathrm{x})$ everywhere inside the semiconductor under this new illumination condition. If you could not do Parts $d$ and $e$, indicate how you would use the results of those parts to answer this question.

$$
\begin{aligned}
& \mathrm{n}^{\prime}(\mathrm{x})= \\
& \mathrm{J}_{\mathrm{e}, \mathrm{diff}}(\mathrm{x})= \\
&
\end{aligned}
$$

## End of Problem 3

## End of Exam One

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