$\qquad$

# Department of Electrical Engineering and Computer Science Massachusetts Institute of Technology 

# 6.012 Electronic Devices and Circuits 

Exam No. 1<br>Wednesday, March 15, 2006<br>7:30 to 9:30 pm

## Notes:

1. Unless otherwise indicated, you should assume room temperature and that $\mathrm{kT} / \mathrm{q}$ is 0.025 V . You should also approximate [(kT/q) ln 10] as 0.06 V .
2. Closed book; one sheet (2 pages) of notes permitted.
3. All of your answers and any relevant work must appear on these pages. Any additional paper you hand in will not be graded.
4. Make reasonable approximations and assumptions. State and justify any such assumptions and approximations you do make.
5. Be careful to include the correct units with your answers when appropriate.
6. Be certain that you have all ten (10) pages of this exam booklet and the two (2) page formula sheet, and make certain that you write your name at the top of this page in the space provided.
7. An effort has been made to make the various parts of these problems independent of each other so if you have difficulty with one item go on, and come back later.

PROBLEM 1 $\qquad$
$\qquad$ (out of a possible 33)
PROBLEM 2
$\qquad$ (out of a possible 33)

TOTAL
$\qquad$
Problem 1-(34 points)
A silicon resistor is shown schematically below. The silicon is doped uniformly with a boron (B) concentration of $2 \times 10^{16} \mathrm{~cm}^{-3}$, and an arsenic (As) concentration of 1 x $10^{16} \mathrm{~cm}^{-3}$. (Recall that boron is in Column III of the periodic table and As is in Column V.) There are ohmic contacts at each end of the silicon resistor. Assume the mobilities are $\mu_{\mathrm{e}}=1,600 \mathrm{~cm}^{2} / \mathrm{V}$-s and $\mu_{\mathrm{h}}=600 \mathrm{~cm}^{2} / \mathrm{V}$-s, and the minority carrier lifetimes are $\tau_{\mathrm{e}}=$ $\tau_{\mathrm{h}}=10^{-7} \mathrm{~s}$.

a) [18 pts] Assuming thermal equilibrium $\left(\mathrm{v}_{\mathrm{AB}}=0\right)$ at room temperature, find the following.
i) The total acceptor concentration, $\mathrm{N}_{\mathrm{a}}$ :

$$
\mathrm{N}_{\mathrm{a}}=
$$

$\qquad$ $\mathrm{cm}^{-3}$
ii) The net acceptor concentration, $\mathrm{N}_{\mathrm{A}}=\left(\mathrm{N}_{\mathrm{a}}-\mathrm{N}_{\mathrm{d}}\right)$ :
$\qquad$ $\mathrm{cm}^{-3}$
iii) The hole concentration, $\mathrm{p}_{\mathrm{o}}$ :

$$
\mathrm{p}_{\mathrm{o}}=\ldots \mathrm{cm}^{-3}
$$

## Problem 1 continued

iv) The electron concentration, $\mathrm{n}_{\mathrm{o}}$ :

$$
\mathrm{n}_{\mathrm{o}}=
$$

v) The electrical conductivity, $\sigma_{0}$ :

$$
\sigma_{\mathrm{o}}=\ldots \mathrm{S} / \mathrm{cm}
$$

vi) The electrostatic potential relative to intrinsic silicon, $\phi_{p}$ :

$$
\phi_{\mathrm{o}}=
$$ V

b) [ 9 pts$]$ Let the applied voltage, $\mathrm{v}_{\mathrm{AB}}$, be 2 Volts. Find the following:
i) The end-to-end resistance of the resistor.

$$
\mathrm{R}=
$$

ii) The electric field, E , in the silicon.

$$
\mathrm{E}=
$$

$\mathrm{V} / \mathrm{cm}$
iii) The total drift current density, $\mathrm{J}^{\mathrm{dr}}$ :
$\mathrm{J}^{\mathrm{dr}}=$ $\qquad$ $\mathrm{A} / \mathrm{cm}^{2}$
c) [7 pts] Assume the silicon is illuminated by a steady state light which generates electron-hole pairs uniformly throughout the bulk, with a rate $g_{L}=10^{23} \mathrm{~cm}^{-3} \mathrm{~s}^{-1}$. Find the following under illumination assuming steady state.
i) The minority carrier diffusion length, $\mathrm{L}_{\text {min }}$ :

$$
\mathrm{L}_{\text {min }}=
$$

$\qquad$ cm
ii) The excess minority carrier population and the conductivity, $\sigma$, in the middle portion of the resistor many minority diffusion lengths from the ohmic contacts:

$$
\begin{aligned}
\mathrm{n}^{\prime} & =\ldots \mathrm{cm}^{-3} \\
\sigma & =\quad \mathrm{S} / \mathrm{cm}
\end{aligned}
$$

## End of Problem 1

## Problem 2 (33 points)

The p-type silicon sample illustrated below is made from silicon having a net acceptor concentration of $5 \times 10^{17} \mathrm{~cm}^{-3}$; minority carrier diffusion length, $\mathrm{L}_{\text {min }}$ of $100 \mu \mathrm{~m}$; electron mobility, $\mu_{\mathrm{e}^{\prime}}$ of $1600 \mathrm{~cm}^{2} / \mathrm{V}-\mathrm{s}$; and hole mobility, $\mu_{\mathrm{h}^{\prime}}$ of $600 \mathrm{~cm}^{2} / \mathrm{V}-\mathrm{s}$. The sample is $10 \mu \mathrm{~m}$ long $\left(10 \mu \mathrm{~m}=10^{-3} \mathrm{~cm}\right)$. All of the surfaces are reflecting boundaries except for the ohmic contact on the right end. The sample is illuminated with a constant light that generates $10^{19}$ hole-electron pairs / $\mathrm{cm}^{3}-\mathrm{s}$ uniformly throughout the bulk.

a) [8 pts] What is the boundary condition on the excess minority carrier concentration or its derivative at each end of the sample?
i) Boundary condition at $x=0$ :
ii) Boundary condition at $\mathrm{x}=\mathrm{L}(=10 \mu \mathrm{~m})$ :
b) [5pts] What is the minority carrier lifetime, $\tau_{\text {min }}$ in this sample?

$$
\tau_{\min }=
$$

c) [8 pts] The figures you need to answer this question are on the next page.
i) From the six sketches on the next page, select the one which best illustrates the magnitude of the minority carrier current in this sample when it is uniformly illuminated as described above. Explain your answer.

Sketch $\qquad$ because

## Problem 2 continued







ii) From the six sketches above, select the one which best illustrates the profile of the excess minority carrier concentration in this sample when it is illuminated as described above. Explain your answer.

Sketch $\qquad$ because
d) [6 pts] Assuming infinite minority carrier lifetime (and thus no recombination in the bulk), what is the minority carrier current density, $\mathrm{J}_{\text {emin }}$, flowing into, or out of, the ohmic contact, when the sample is illuminated as described?

$$
\mathrm{J}_{\min }\left(\mathrm{x}=\mathrm{L}^{-}\right)=
$$

$\qquad$ $\mathrm{A} / \mathrm{cm}^{2}$
e) [6 pts] We know that the minority carrier lifetime is not infinite, and there is some recombination in the bulk.
i) What is the rate per unit area at which hole-electron pairs are recombining in the entire bulk of the sample away from the ohmic contact? First give a formula, and then a numerical value.

Algebraic expression $=$ $\qquad$
Recombination =
$\qquad$ pairs $/ \mathrm{cm}^{2}-\mathrm{s}$
ii) Compare your answer above to your answer in Part d. How would you expect the excess carrier fluxes into the ohmic contact to compare with the rate of holeelectron recombination occurring in the bulk? Explain your answer.

Much greater than $\qquad$ Much less than $\qquad$ Similar to because:
$\qquad$

## Problem 3-(33 points)

You are given the following asymmetrical silicon ( Si ) $\mathrm{n}^{+}$- p junction diode structure with $\mathrm{w}_{\mathrm{n}}=1 \mu \mathrm{~m}, \mathrm{w}_{\mathrm{p}}=4 \mu \mathrm{~m}$, and $\tau_{\mathrm{e}}=\tau_{\mathrm{h}}=\infty$. The n-region of this diode is heavily doped, $\mathrm{N}_{\mathrm{D}}=10^{19} \mathrm{~cm}^{-3}$. The p-region is composed of two sub-regions with doping $\mathrm{N}_{\mathrm{A} 1}$ and $\mathrm{N}_{\mathrm{A} 2}$ as shown. The width of the first sub-region, $\mathrm{w}_{1}$, is $2 \mu \mathrm{~m}$. Assume the mobilities are $\mu_{\mathrm{e}}=$ $1,600 \mathrm{~cm}^{2} / \mathrm{V}$-s and $\mu_{\mathrm{h}}=600 \mathrm{~cm}^{2} /$ V-s.

a) [15 pts] First consider a diode in which the two p-type sub-regions have the same doping level with $\mathrm{N}_{\mathrm{A} 1}=\mathrm{N}_{\mathrm{A} 2}=10^{15} \mathrm{~cm}^{-3}$. In Parts i) and ii) assume there is zero applied bias, $\mathrm{v}_{\mathrm{AB}}=0 \mathrm{~V}$; in Part iii) assume forward bias, $\mathrm{v}_{\mathrm{AB}}>0$. Calculate the following quantities:
i) The built-in potential of the junction, $\phi_{b}$.

$$
\phi_{\mathrm{b}}=
$$

$\qquad$ V
ii) The width of the depletion region on the $p$-side of the junction, $x_{p}$.

$$
\mathrm{x}_{\mathrm{p}}=
$$

$\qquad$ $\mu \mathrm{m}$
iii) The ratio of the electron to hole current density crossing the junction.

$$
\mathrm{J}_{\mathrm{e}} / \mathrm{J}_{\mathrm{h}}=
$$

$\qquad$
b) [18 pts] Next consider a device in which the second region is doped much more heavily p-type with $\mathrm{N}_{\mathrm{A} 2}=10^{18} \mathrm{~cm}^{-3}$. The doping level in the first region is the same as before, i.e., $\mathrm{N}_{\mathrm{A} 1}=10^{15} \mathrm{~cm}^{-3}$. The width of this first region is $2 \mu \mathrm{~m}$.
i) Consider the situation with zero applied bias, i.e., $\mathrm{v}_{\mathrm{AB}}=0 \mathrm{~V}$. What is the built-in potential, $\phi_{b}$, of the junction?

$$
\phi_{\mathrm{b}}=\ldots \mathrm{V}
$$

ii) A bias, $v_{A B}=V_{1}$, is applied so that the width depletion region on the $p$-side of this diode, $\mathrm{x}_{\mathrm{p}}$, is $2 \mu \mathrm{~m}$, i.e. the first p-region is fully depleted. What is $\mathrm{V}_{1}$ ?

$$
\mathrm{V}_{1}=\mathrm{v}_{\mathrm{AB}}\left(\text { such that } \mathrm{x}_{\mathrm{p}}=\mathrm{w}_{1}\right)=
$$

$\qquad$
iii) With a bias applied as in Part b) ii) so that $x_{p}=w_{1}=2 \mu \mathrm{~m}$, what is the small signal linear equivalent junction capacitance per unit area, $\mathrm{C}_{\mathrm{ab}}\left(\mathrm{V}_{1}\right) \equiv \partial \mathrm{q}_{\mathrm{AB}} /\left.\partial \mathrm{v}_{\mathrm{AB}}\right|_{\mathrm{vAB}=\mathrm{V} 1}$ ? Hint: Think "depletion region width" rather than "complicated formula."
$\qquad$ $\mathrm{F} / \mathrm{cm}^{2}$
iv) The small signal linear equivalent capacitance of this junction in the reverse bias region, i.e. $\mathrm{V}_{\mathrm{AB}}<0 \mathrm{~V}$, looks like one of the sketches below as the bias is changed. Chose the correct one and explain why. (Note that correct choice with no explanation receives no points.) Hint: See hint for Part (iii).


Figure $\qquad$ because

End of Problem 3; End of Exam

## Extra pieces :

## Problem 1: Add to problem if use on future Problem Set

iii) On the axes provided below sketch and dimension the excess minority carrier population within 3 minority carrier diffusion lengths of the ohmic contact on the right end of the bar.


## Problem 3: ditto

c) [8 pts] Finally consider a diode in which the first p-type sub-region is very thin, $\mathrm{W}_{1}=10 \mathrm{~nm}\left(=10^{-2} \mu \mathrm{~m}\right)$, and is more heavily doped than the second p-type sub-region: $\mathrm{N}_{\mathrm{A} 1}=10^{16} \mathrm{~cm}^{-3}, \mathrm{~N}_{\mathrm{A} 2}=10^{15} \mathrm{~cm}^{-3}$. There is zero volts applied bias, $\mathrm{v}_{\mathrm{AB}}=0$. You may assume that $\mathrm{w}_{1} \ll \mathrm{x}_{\mathrm{p}}$ and treat the depletion charge in Sub-region 1 as a delta function at $\mathrm{x}=0^{+}$.
i) Calculate electrostatic potential step, $\phi_{b}$, crossing the junction.

$$
\phi_{\mathrm{b}}=\ldots \mathrm{V}
$$

ii) Calculate width of the depletion region on the $p$-side, $x_{p}$.

$$
x_{p}=
$$

$\qquad$ $\mu \mathrm{m}$

MIT OpenCourseWare
http://ocw.mit.edu

### 6.012 Microelectronic Devices and Circuits

Fall 2009

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.

