Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications

Problem Set #10 — Issued: 11/22/05
Fall Term 2005 — Due: 11/30/05

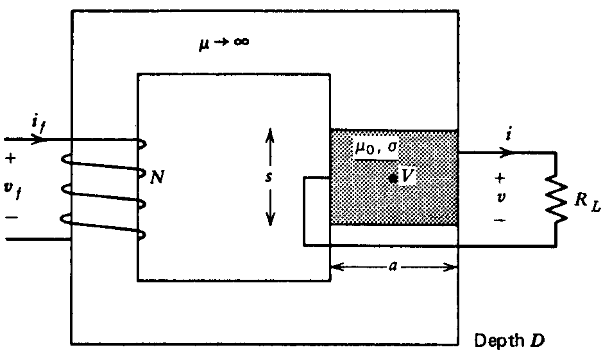
Suggested Reading Assignment: Lecture Notes 18

Problem 10.1

A magnetohydrodynamic (MHD) machine of height s and width a is placed within a magnetic circuit. The fluid moving at velocity V out of the paper obeys Ohm's law

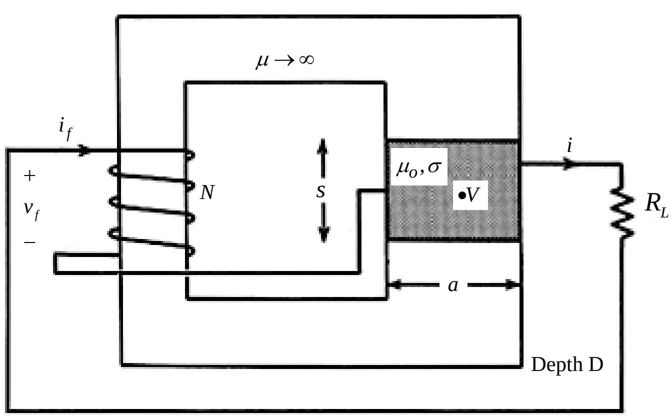
$$\bar{J} = \sigma\left(\bar{E} + \bar{V} \times \bar{B}\right)$$

and has magnetic permeability μ_o.



(a) A constant dc current $i_f = I_0$ is applied to the N turn coil. What is the magnetic field $\bar{B}$ in the MHD machine?

(b) What is the voltage v across the load resistor R_L?

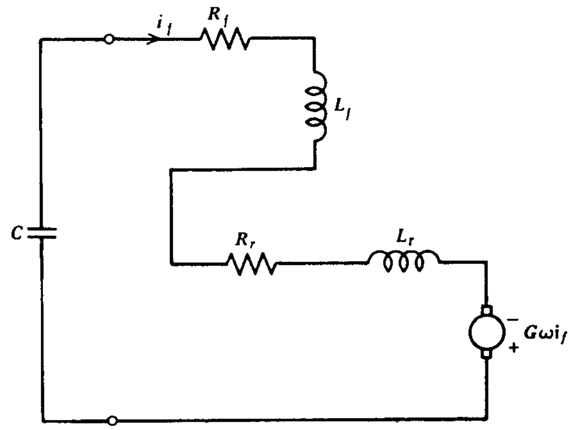


(c) The MHD machine and load resistor R_L are now connected in series with the N turn coil that has a resistance R_f as shown in the figure to the left.
No current is applied but at $t = 0$ noise causes a small current so that $i_f(t=0) = i(t=0) = I_0$. Neglecting magnetic saturation of the iron core of the magnetic current, what is $i_f(t)$ for $t \geq 0$?

(d) Under what conditions is $i_f(t)$ unbounded as $t \to \infty$? Such a system is called DC self-excited.

Problem 10.2

The field winding of a homopolar generator is connected in series with the rotor terminals through a capacitor C as shown in the equivalent circuit below. The rotor is turned at constant speed ω.



(a) For what minimum value of rotor speed is the system self-excited?

(b) For the self-excited condition of (a) what range of values of C will result in dc self-excitation or in ac self- excitation?

(c) What is the frequency for ac self-excitation?

Problem 6.21 in *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, 1987. Used with permission.

Problem 10.3

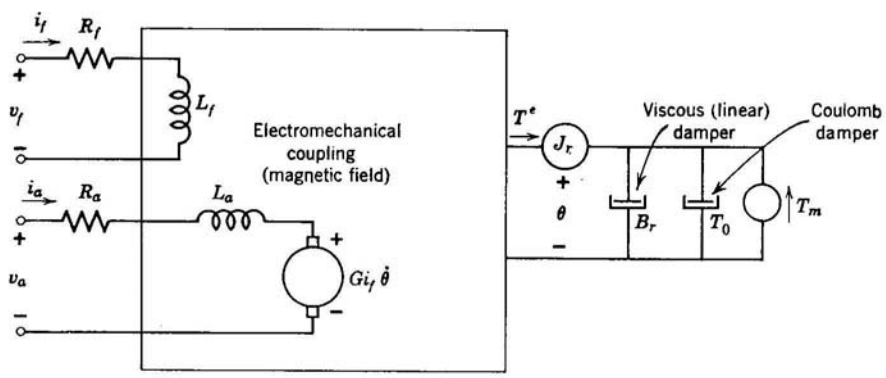


Figure 6.4.5 in *Electromechanical Dynamics*, by Herbert H. Woodson and James R. Melcher, 1968.

The electromechanical equivalent circuit of a commutator machine is shown above. The field and armature windings can be connected in parallel (shunt excitation) or in series (series excitation) as shown below:

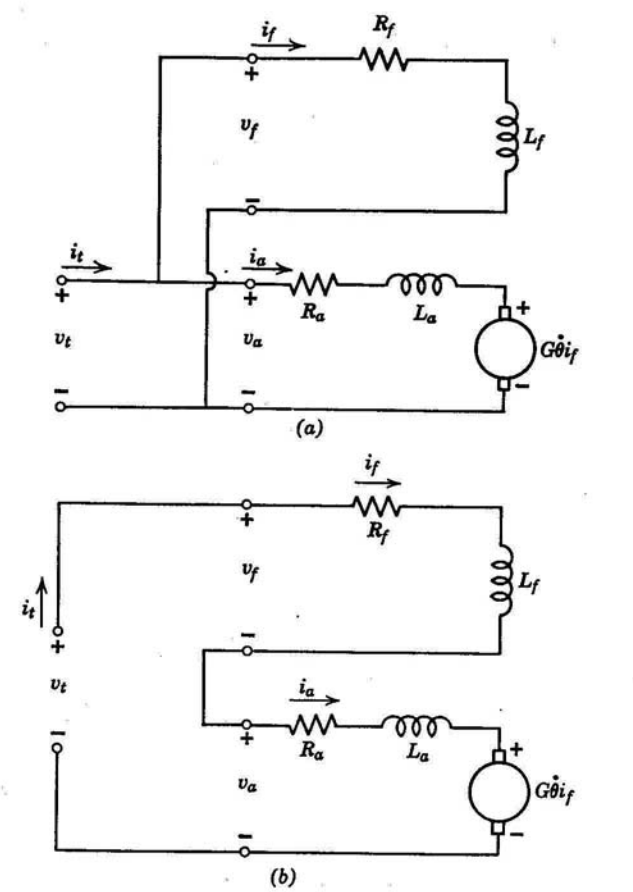


Figure 6.4.9 in *Electromechanical Dynamics*, by Herbert H. Woodson and James R. Melcher, 1968.

The electromagnetic torque on the armature (rotor) with moment of inertia J_r is $T^e = Gi_f i_a$.

a) A DC voltage source V_0 is connected across the v_t terminals for shunt and series excitations. In the DC steady state, find the terminal current i_t and torque $T^e = Gi_f i_a$ as a function of armature angular speed $\omega = \dot{\theta}$ for shunt and series excitations.

b) The voltage source of part (a) that was connected across the v_t terminals is now removed and is replaced by a load resistor R_L for shunt and series configurations. Assume that all circuit variables vary as e^{st} and find natural frequency s for each configuration. For what rotor speed $\left(\omega = \dot{\theta}\right)$, direction and magnitude, will the generator be self-excited for each configuration?
Hint: The shunt excitation algebra can be involved without some forethought. To simplify the math manipulations for this linear model it is convenient to immediately write the currents i_f and i_a as

$$i_f = I_f e^{st}$$

$$i_a = I_a e^{st}$$

Solve for solutions for s and find unstable roots.