MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 – Electromagnetics and Applications

Problem Set 2 (five problems)

Suggested Reading: Course notes (Staelin)
Sections 2.1-2.3.4, 2.4, 2.7.1-2.7.3, 3.2.1-3.2.2, Appdx. C.

Problem 2.1

Assume the "Whatever" vector $\bar{W}(x,y,z) = \hat{x}\sin y + \hat{y}y$.

(a) If an electric displacement vector $\bar{D} = \bar{W}$, what is the charge density $\rho(x,y,z)$ [C/m^3]?

(b) If the magnetic field $\bar{H} = \bar{W}$, what is the current density $\bar{J}(x,y,z)$ [A/m^2], assuming $\bar{H}$ is physically possible?

(c) Does the magnetic field $\bar{B} = \bar{W}$ satisfy all of Maxwell's equations? If not, which one is violated?

Problem 2.2

If the electric field $E(t) = R_e\{\underline{E}\, e^{j\omega t}\}$ where $\underline{E}$ is a *phasor*, then what is E(t) if:

(a) $\underline{E} = 1 - j$ (b) $\underline{E} = e^{j\pi/4} - 1$ (c) $\bar{\underline{E}} = j\hat{x} + (1 - j)\hat{y}$

(d) What is the complex vector $\bar{\underline{E}}$ if $\bar{E}(t) = \hat{x}\cos\omega t + \hat{y}\sin(\omega t + \pi/4)$?

[Hint: $E(t) = \cos\omega t$ for the case $\underline{E} = 1$]

Continued on next page

Problem 2.3

(a) What is the frequency f (Hz) of the wave having the magnetic field:

$$\bar{H} = \hat{x}\sin(10^7\pi t - 0.2z) + \hat{y}\cos(10^7\pi t - 0.2z - 2.5\pi)?$$

(b) What is its wavelength λ (meters)?

(c) What is the velocity of light c in this medium?

(d) Find the corresponding $\bar{E}(x,y,z,t)$ assuming $\mu = \mu_o$.

(e) What is the polarization of this wave? (e.g., "left circular"; polarization is usually characterized by the behavior of the electric field vector)

(f) What is the shortest non-zero time delay τ(sec) that could be added to the x component of the wave in order to achieve linear polarization? In this case, what is the direction θ of the linear polarization relative to the x axis? A sketch may help.

(g) Polarization can also be characterized by complex notation. What is the polarization of the z-propagating wave $\underline{\bar{E}} = (j-1)\hat{x} + (1-j)\hat{y}$? (e.g., "left circular".)

Problem 2.4

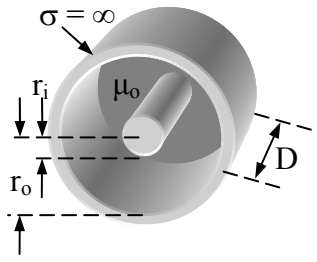
A 1-GHz uniform plane wave propagating in the +z direction in a medium μ, ε is characterized by $\underline{\bar{E}} = \hat{x}\,3$.

(a) What is the time average intensity [W/m^2] of this wave?

(b) What is the magnetic energy density $W_m(t)$ [J/m^3] at x = y = z = 0? What is the electric energy density $W_e(t)$ there?

Problem 2.5

Using the general expression for inductance L, find L for a coaxial inductor of length D and short circuited at one end, where the inner and outer radii of the two concentric conductors are r_i and r_o, respectively, as illustrated.



$$L = \frac{\Lambda}{i} = \frac{\mu N \iint_A \bar{H}\bullet d\bar{a}}{i}$$

6.013 Electromagnetics and Applications
Spring 2009