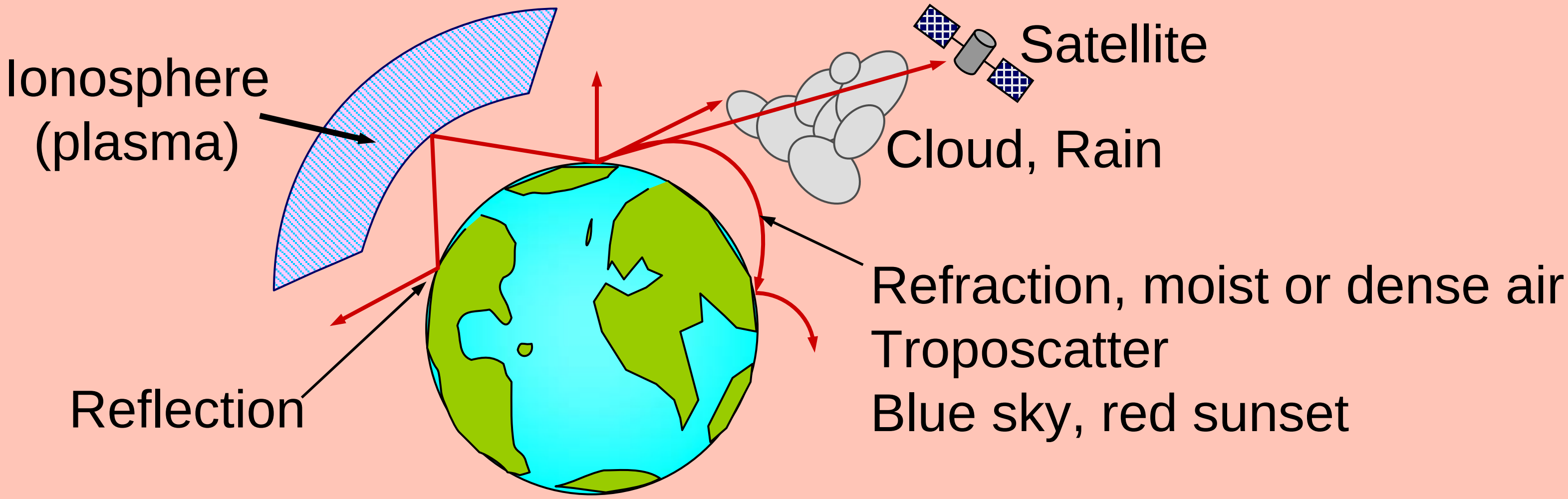
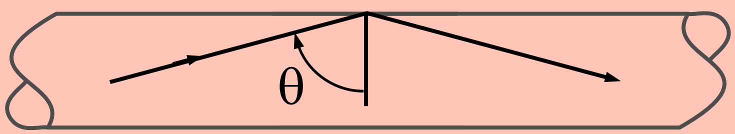
WAVES IN MEDIA

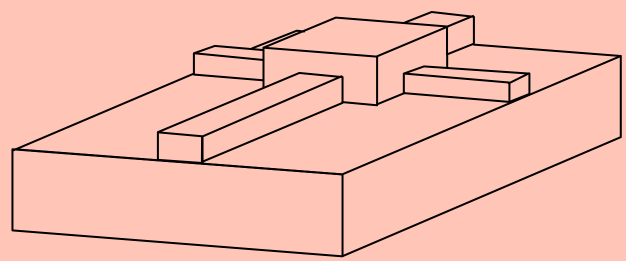
Radio Communications



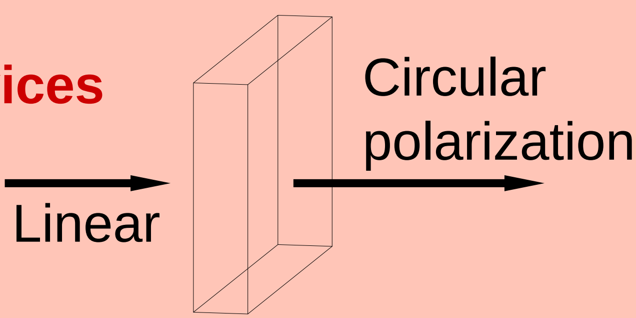
Optical Fibers



Optoelectronics on chips



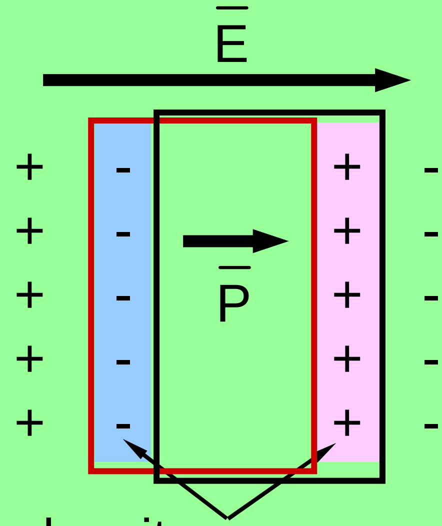
Devices



WAVES IN MEDIA – Constitutive Relations

Vacuum: $\bar{D} = \varepsilon_o \bar{E}$ $\quad \nabla \bullet \bar{D} = \rho_f$

ρ_f = free charge density



Dielectric Materials:

$$\bar{D} = \varepsilon \bar{E} = \varepsilon_o \bar{E} + \bar{P}$$

$$\nabla \bullet \varepsilon_o \bar{E} = \rho_f + \rho_p$$

polarization charge density ρ_p

$$\nabla \bullet \bar{P} = -\rho_p$$

$\bar{P}$ = "Polarization Vector"

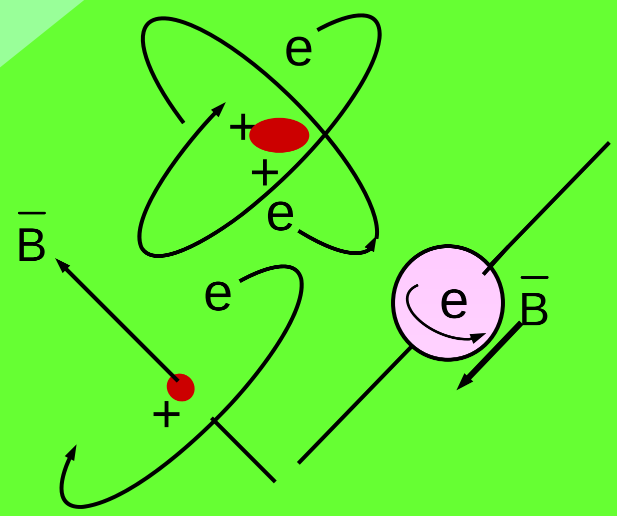
Magnetic Materials:

$$\nabla \bullet \bar{B} = 0$$

$\bar{B} = \mu_o \bar{H}$ in vacuum

$$\bar{B} = \mu \bar{H} = \mu_o \left(\bar{H} + \bar{M} \right)$$

$\bar{M}$ = "Magnetization Vector"



TYPES OF MEDIA

Properties are function of:	Designation:
Field direction	Anisotropic $\overline{D} = \overline{\overline{\varepsilon}}\,\overline{E}, \quad \overline{B} = \overline{\overline{\mu}}\,\overline{H}$
Position	Inhomogeneous
Time: $\neq f(t)$	Stationary
$\neq f(\text{history})$	Amnesic
Frequency	Dispersive
$\overline{E}$ or $\overline{H}$	Non-linear
Temperature	Temperature dependent
Pressure	Compressive

ANISOTROPIC DIELECTRICS

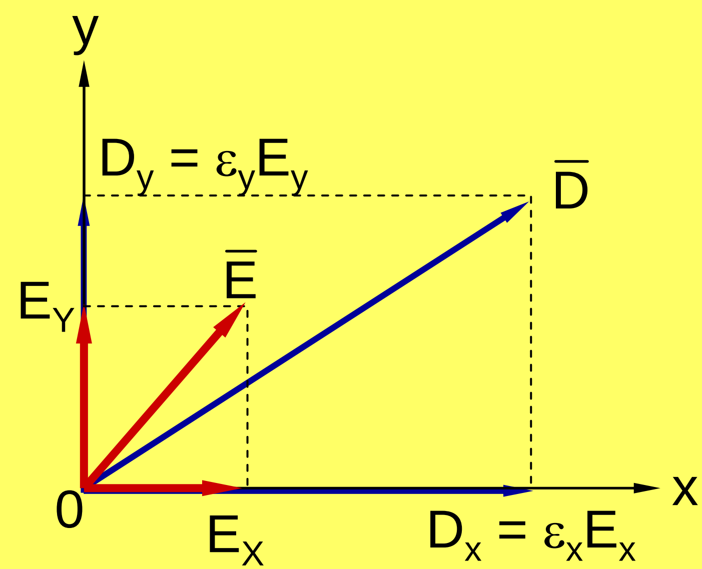
$$\underline{\overline{D}} = \underline{\overline{\overline{\varepsilon}}}\,\underline{\overline{E}}$$

$$\underline{D}_x = \underline{\varepsilon}_{xx}\underline{E}_x + \underline{\varepsilon}_{xy}\underline{E}_y + \underline{\varepsilon}_{xz}\underline{E}_z$$

$$\underline{D}_y = \underline{\varepsilon}_{yx}\underline{E}_x + \underline{\varepsilon}_{yy}\underline{E}_y + \underline{\varepsilon}_{yz}\underline{E}_z$$

$$\underline{D}_z = \underline{\varepsilon}_{zx}\underline{E}_x + \underline{\varepsilon}_{zy}\underline{E}_y + \underline{\varepsilon}_{zz}\underline{E}_z$$

Let $\underline{\overline{\overline{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$ ⇒



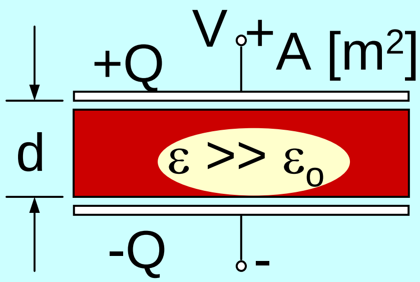
x,y,z are "principal axes"

Note: When $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$, $\overline{D} // \overline{E}$ iff $\overline{E} // \hat{x}, \hat{y}$, or $\hat{z}$

Real $\underline{\overline{\overline{\varepsilon}}}, \underline{\overline{\overline{\mu}}} \Rightarrow$ Lossless medium

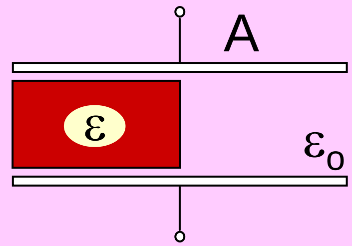
MAKING ANISOTROPIC MATERIALS

$C = Q/V$



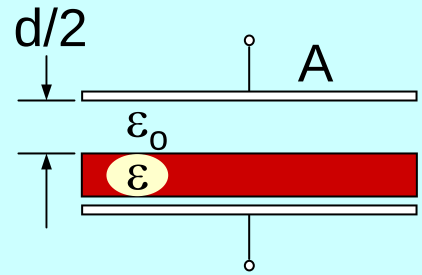
$$C = \frac{\varepsilon_{eff} A}{d}$$

$$\varepsilon_{eff} = \varepsilon$$



$$C \cong \frac{\varepsilon (A/2)}{d}$$

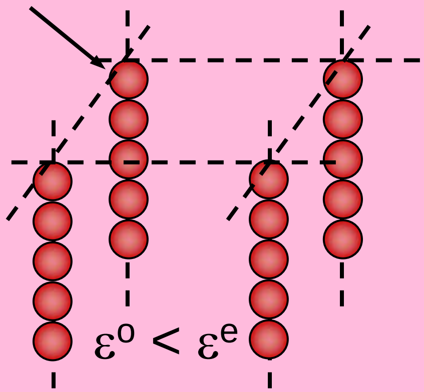
$$\varepsilon_{eff} \cong \varepsilon/2$$



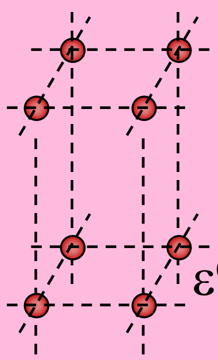
$$C \cong \frac{\varepsilon_o A}{(d/2)}$$

$$\varepsilon_{eff} \cong 2\varepsilon_o$$

Atom or molecule



$\varepsilon^o < \varepsilon^e$

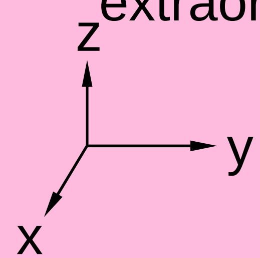


$\varepsilon^o > \varepsilon^e$

"Uniaxial Medium"

$\varepsilon_x = \varepsilon_y = \varepsilon^o$ "ordinary"

$\varepsilon_z = \varepsilon^e$ "extraordinary"



WAVES IN UNIAXIAL MEDIA

Derive wave equation:

$$\nabla \times \underline{\bar{E}} = -j\omega \underline{\bar{B}} \qquad \nabla \bullet \underline{\bar{D}} = \rho_f = 0$$

$$\nabla \times \underline{\bar{H}} = j\omega \underline{\bar{D}} \qquad \nabla \bullet \underline{\bar{B}} = 0$$

Assume:

$$\underline{\bar{D}} = \underline{\bar{\bar{\varepsilon}}}\,\underline{\bar{E}}$$

$$\sigma = 0,\ \underline{\bar{\bar{\mu}}} = \underline{\mu}_o$$

$$\underline{\bar{\bar{\varepsilon}}} = \begin{bmatrix} \varepsilon^e & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}$$

$$\nabla \times \left(\nabla \times \underline{\bar{E}}\right) = \nabla\left(\nabla \bullet \underline{\bar{E}}\right) - \nabla^2 \underline{\bar{E}} = -j\omega\mu \nabla \times \bar{H} = \omega^2 \mu \underline{\bar{\bar{\varepsilon}}}\,\underline{\bar{E}}$$

Can show $\nabla \bullet \underline{\bar{E}} = 0$ (can also test final solution)

Therefore:

$$\underbrace{\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right.}_{} \left. + \frac{\partial^2}{\partial z^2}\right]\left[\hat{x}\underline{E}_x + \hat{y}\underline{E}_y + \hat{z}\underline{E}_z\right] + \omega^2 \mu \underline{\bar{\bar{\varepsilon}}}\,\underline{\bar{E}} = 0$$

Assume = 0 (assume UPW in z direction)

Yields 3 decoupled equations (x,y,z components)

BIREFRINGENT MEDIA

Decoupled wave equations:

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2 \mu \varepsilon^e}_{\triangleq (k^e)^2}\right] \underline{E}_x = 0\ ,\quad k^e = \omega\sqrt{\mu\varepsilon^e} \qquad \text{(x-polarization equation)}$$

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2 \mu \varepsilon}_{\triangleq (k^o)^2}\right] \underline{E}_y = 0\ ,\quad k^o = \omega\sqrt{\mu\varepsilon} \qquad \text{(y-polarization equation)}$$

Solutions:

$$\underline{E}_x \propto e^{-jk^e z} = e^{-j(\omega / v^e) z} \quad \text{where} \begin{cases} v^e = 1/\sqrt{\mu\varepsilon^e} & \text{“extraordinary” velocity} \\ v^o = 1/\sqrt{\mu\varepsilon} & \text{“ordinary” velocity} \end{cases}$$

Thus x- and y-polarized waves propagate independently at different velocities

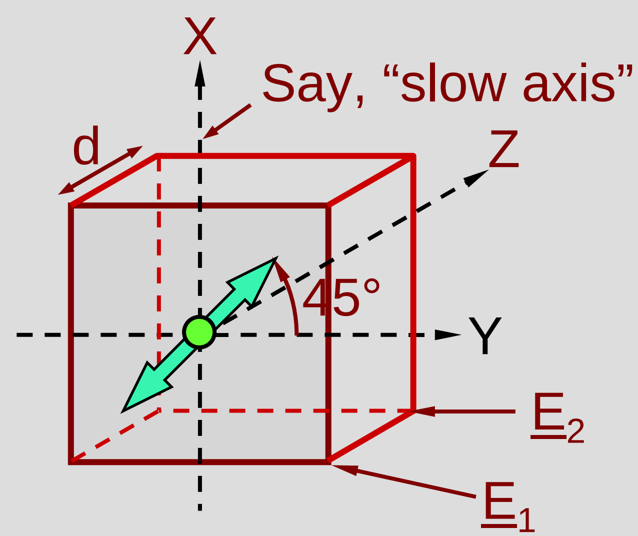
If $v^e < v^o$ then $v^e \rightarrow$ “slow-axis velocity”

BIREFRINGENT MEDIA

Example:

Input: $\underline{\overline{E}}_1 = E_o\left(\hat{x} + \hat{y}\right) \Rightarrow 45^o$ linear polarization

Output: $$\underline{\overline{E}}_2 = E_o \underbrace{\left(\hat{x}e^{-j\overbrace{\phi^e}^{-jk^e d}} + \hat{y}e^{-j\overbrace{\phi^o}^{-jk^o d}}\right)}_{\text{What pol.?}}$$



$$\Delta\phi \triangleq \phi^e - \phi^o = (k^e - k^o)d$$

$= \pi/2$ "Quarter-wave plate"

$= \pi$ "Half-wave plate"

Output: