Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis (Fall 2010)

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- 1. Show $\rho(aX + b, Y) = \rho(X, Y)$.
- 2. Romeo and Juliet have a date at a given time, and each, independently, will be late by amounts of time, X and Y, respectively, that are exponentially distributed with parameter λ .
 - (a) Find the PDF of Z = X Y by first finding the CDF and then differentiating.
 - (b) Find the PDF of Z by using the total probability theorem.
- 3. Problem 4.16, page 248 in text.

Let X and Y be independent standard normal random variables. The pair (X, Y) can be described in polar coordinates in terms of random variables $R \ge 0$ and $\Theta \in [0, 2\pi]$, so that

$$X = R\cos\Theta, \quad Y = R\sin\Theta.$$

Show that R and Θ are independent (i.e. show $f_{R,\Theta}(r,\theta) = f_R(r)f_{\Theta}(\theta)$).

- (a) Find $f_R(r)$.
- (b) Find $f_{\Theta}(\theta)$.
- (c) Find $f_{R,\Theta}(r,\theta)$.
- 4. Problem 4.20, page 250 in text. Schwarz inequality. Show that for any random variables X and Y, we have

$$(\mathbf{E}[XY])^2 \le \mathbf{E}[X^2]\mathbf{E}[Y^2].$$

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