Tutorial 5
October 14/15, 2010

1. Let Q be a random variable which is uniformly distributed between 0 and 1. On any given day, a particular machine is functional with probability Q. Furthermore, given the value of Q, the status of the machine on different days is independent.

   (a) Find the probability that the machine is functional on a particular day.

   (b) We are told that the machine was functional on m out of the last n days. Find the conditional PDF of Q. You may use the identity

   $$\int_0^1 p^k(1-p)^{n-k}dp = \frac{k!(n-k)!}{(n+1)!}$$

2. Let X be a random variable with PDF f_X. Find the PDF of the random variable $Y = |X|$

   (a) when $f_X(x) = \begin{cases} 1/3, & \text{if } -2 < x \leq 1, \\ 0, & \text{otherwise;} \end{cases}$

   (b) when $f_X(x) = \begin{cases} 2e^{-2x}, \text{if } x > 0, \\ 0, \text{otherwise;} \end{cases}$

   (c) for general $f_X(x)$.

3. An ambulance travels back and forth, at a constant specific speed v, along a road of length ℓ. We may model the location of the ambulance at any moment in time to be uniformly distributed over the interval $(0, \ell)$. Also at any moment in time, an accident (not involving the ambulance itself) occurs at a point uniformly distributed on the road; that is, the accident's distance from one of the fixed ends of the road is also uniformly distributed over the interval $(0, \ell)$. Assume the location of the accident and the location of the ambulance are independent.

   Supposing the ambulance is capable of *immediate* U-turns, compute the CDF and PDF of the ambulance's travel time T to the location of the accident.

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