

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Problem Set 9
Topics: Bernoulli, Poisson
Due: May 3rd, 2006

1. Fred is giving out samples of dog food. He makes calls door to door, but he leaves a sample (one can) only on those calls for which the door is answered *and* a dog is in residence. On any call the probability of the door being answered is $3/4$, and the probability that any household has a dog is $2/3$. Assume that the events “Door answered” and “A dog lives here” are independent and also that the outcomes of all calls are independent.
 - (a) Determine the probability that Fred gives away his first sample on his third call.
 - (b) Given that he has given away exactly four samples on his first eight calls, determine the conditional probability that Fred will give away his fifth sample on his eleventh call.
 - (c) Determine the probability that he gives away his second sample on his fifth call.
 - (d) Given that he did not give away his second sample on his second call, determine the conditional probability that he will leave his second sample on his fifth call.
 - (e) We will say that Fred “needs a new supply” immediately *after* the call on which he gives away his last can. If he starts out with two cans, determine the probability that he completes at least five calls before he needs a new supply.
 - (f) If he starts out with exactly m cans, determine the expected value and variance of D_m , the number of homes with dogs which he passes up (because of no answer) before he needs a new supply.
 2. Each night, the probability of a robbery attempt at the local warehouse is $1/5$. A robbery attempt is successful with probability $3/4$, independent of other nights. After any particular successful robbery, the robber celebrates by taking off either the next 2 or 4 nights (with equal probability), during which time there will be no robbery attempts. After that, the robber returns to his original routine.
 - (a) Let K be the number of robbery attempts up to (and including) the first successful robbery. Find the PMF of K .
 - (b) Let D be the number of days until (and including) the second successful robbery, including the days of celebration after the first robbery. Find the PMF of D , or its transform (whichever you find more convenient).

During a successful robbery, the robber steals a random number of candy bars, which is 1, 2, or 3, with equal probabilities. This number is independent for each successful robbery and independent of everything else. No candy bars are stolen in unsuccessful robberies.

 - (c) Let S be the number of candy bars collected in two successful robberies. Find the PMF of S .
 - (d) Let Q be the number of candy bars collected in ten robbery attempts (whether successful or not). Find the PMF of Q , or its transform, whichever is easier. Find the expectation and the variance of Q .
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3. You are visiting the rainforest, but unfortunately your insect repellent has run out.
 - (a) As a result, at each second, a mosquito lands on your neck with probability 0.2.
 - i. What's the PMF for the time until the first mosquito lands on you?
 - ii. What's the expected time until the first mosquito lands on you?
 - iii. If you weren't bitten for the first ten seconds, what would be the expected time until the first mosquito lands on you (from time $t=10$) ?
 - (b) Instead, imagine the rainforest had only one mosquito, which arrived in the following way: the time of arrival is exponentially distributed with $\lambda = 0.2$.
 - i. What's the expected time until the first mosquito lands on you?
 - ii. If you weren't bitten for the first ten seconds, what would be the expected time until the first mosquito lands on you (from time $t=10$) ?
 4. A particular medical operation proves fatal in 1% of the cases. Find an approximation to the probability that there will be at least 2 fatalities in 200 operations.
 5. Arrivals of certain events at points in time are known to constitute a Poisson process, but it is not known which of two possible values of λ , the average arrival rate, describes the process. Our a priori estimate is that $\lambda = 2$ or $\lambda = 4$ with equal probability. We observe the process for t units of time and observe exactly k arrivals. Given this information, determine the conditional probability that $\lambda = 2$. Check to see whether or not your answer is reasonable for some simple limiting values for k and t .
 - G1[†]. A wombat in the San Diego zoo spends the day walking from a burrow to a food tray, eating, walking back to the burrow, resting and repeating the cycle. The amount of time to walk from the burrow to the tray (and also from the tray to the burrow) is 20 secs. The wombat, with probability $\frac{1}{3}$, will momentarily stand still (for a negligibly small time) during a walk to or from the tray, with all times being equally likely (and independently of what happened in the past). A photographer arrives at a random time and will take a picture at the first time the wombat will stand still. What is the expected value of the length of time the photographer has to wait to snap the wombat's picture ?
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