Recitation 5
February 28, 2006

Review/discussion:

The following five types of discrete random variables arise frequently in applications and in the remainder of the course. Their properties are tabulated on pages 116–117 of the text. Make sure you understand how these random variables arise and how to derive their means and variances.

- Discrete uniform over $[a, b]$ (or uniform over $\{a, a+1, \ldots, b\}$)

- Bernoulli with parameter p

- Binomial with parameters p and n

- Geometric with parameter p

- Poisson with parameter λ

Problems:

1. Problem 2.22, page 123 in the text. Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability p and the second with probability q. All tosses are assumed independent.

   (a) Find the PMF, the expected value, and the variance of the number of tosses.

   (b) What is the probability that the last toss of the first coin is a head?

2. Prove the following version of the Total Expectation Theorem:

$$\mathbf{E}[X] = \sum_{i=1}^{n} \mathbf{P}(A_i)\mathbf{E}[X \mid A_i]$$

   whenever $A_1, A_2, \ldots, A_n$ is a partition of the sample space.

3. Suppose a discrete random variable X can have only non-negative integer values. Show that

$$\mathbf{E}[X] = \sum_{k=0}^{\infty} \mathbf{P}(X > k).$$