

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
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1. (Example 5.3) A computer executes two types of tasks, priority and non priority, and operates in discrete time units (slots). A priority task arises with probability p at the beginning of each slot, independently of other slots and requires one full slot to complete. A non priority task is always available and is executed at a given slot if no priority task is available. In this context, it may be important to know the probabilistic properties of the time intervals available for non priority tasks.

With this in mind let us call a slot *busy* if within this slot the computer executes a priority task, and otherwise let us call it *idle*. We call a string of idle (or busy) slots, flanked by busy (or idle, respectively) slots an *idle period* or *busy period* respectively. Derive the PMF, mean and variance of the following random variables.

- (a) T = the time index of the first slot;
 - (b) B = the length (number of slots) of the first busy period;
 - (c) I = the length of the first idle period.
 - (d) Z = the number of slots after the first slot of the first busy period up to and including the first subsequent idle slot.
2. (Problem 5.4) Consider a Bernoulli process with probability of success in each trial equal to p .
 - (a) Relate the number of failures before the r th success (sometimes called a **negative binomial** random variable) to a Pascal random variable and derive its PMF.
 - (b) Find the expected value and variance of the number of failures before the r th success.
 - (c) Obtain an expression for the probability that the i th failure occurs before the r th success.
 3. (Problem 5.6) **Sum of a geometric number of independent random variables.** Let $Y = X_1 + X_2 + \dots + X_N$, where the random variables X_i are geometric with parameter p , and N is geometric with parameter q . Assume that the random variables N, X_1, X_2, \dots are independent. Show, without using transforms, that Y is geometric with parameter pq . *Hint:* Interpret the various random variables in terms of a split Bernoulli process.
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