

6.041SC Probabilistic Systems Analysis and Applied Probability, Fall 2013 Transcript – Recitation: Bernoulli Process Practice

Hi everyone. Today I'm going to talk about Bernoulli process practice number one. In this problem, you are visiting a rain forest. But unfortunately you have run out of insect repellent.

As a result, the probability of you getting mosquito bites is really high. At each second, the probability that a mosquito will land on your neck is 0.5. If a mosquito lands on your neck, the probability that it will bite you is 0.2. And the probability that it will never bother you is 0.8. All of this happens independently among all mosquitoes.

For part A of the problem, we're interested in finding the expected value of the time between successive mosquito bites and the variance of the time between successive mosquito bites. From the problem statement we know that the probability distributions of getting mosquito bites at different times are identically distributed and independent. Therefore, the mosquito bites occur as a Bernoulli process with parameter p , where p represents the probability of getting a mosquito bite at each second.

And p can be calculated as the probability that a mosquito lands on your neck at each second multiplied by the probability that a mosquito will bite you, given that it has landed on your neck. And this is equal to 0.5 times 0.2, which is equal to 0.1.

Next let us define x as the time between successive mosquito bites. Because of the memory-less property of the Bernoulli process, which means the probability of getting mosquito bites at different times are independent, x is equivalent to the time until the next mosquito bite. And x is a geometrical random variable whose PMF is like the following.

For all x , let's say equal to 0, the probabilities are equal to 0. For x equal to 1, the probability that it takes 1 second to the next mosquito bite is simply equal to p . And for x equal to 2, the probability that it takes 2 seconds until the next mosquito bite is equal to $1 - p$ times p . And for x equal to 3, the probability that it takes 3 seconds until the next mosquito bite is equal to $1 - p$ to the power of 2 times p .

Similarly, for x equal to k , the probability that it takes k seconds until the next mosquito bite is equal to $1 - p$ to the power of $k - 1$ times p . Therefore the expected value of x is equal to 1 over p , which is equal to 1 over 0.1, which is equal to 10. And the variance of x is equal to $1 - p$ over p squared, which is equal to $1 - 0.1$ over 0.1 squared, which is equal to 90.

For part B of the problem, we're considering another type of bug. Similar to the case as the mosquitoes, here at each second the probability that a tick will land on your neck is equal to 0.1. And if a tick lands on your neck, the probability that it will bite you is equal to 0.7. And the probability that it will never bother you is equal to 0.3. And all this happens independently among all ticks and all mosquitoes.

So similar to the case as part A, where mosquito bites occurs as a Bernoulli process with parameter p equal to 0.1, here the tick bites also across a Bernoulli process with parameter q equal to 0.1 times 0.7, which is equal to 0.07. And q is the probability of getting a tick bite at each second.

Therefore, the bug bites occurs as a merged process from the mosquito bites and the tick bites. And let r represent the parameter for the bug bites. So here r is equal to the probability of getting either a mosquito bite or a tick bite. And this is equivalent to 1 minus the probability of getting no mosquito bite and no tick bite.

Because the mosquito bites and the tick bites happens independently, therefore this can be written as 1 minus the probability of no mosquito bites times the probability of no tick bites at each second. And this is equal to 1 minus p times 1 minus q , which is p plus q minus pq , which is equal to 0.1 plus 0.7 minus 0.1 times 0.7. Which is equal to 0.163.

Next, let us define y as the time between successive bug bites. So similar as x in part a, here y is a geometric distribution with parameter r . And therefore the expected value of y is equal to 1 over r , which is equal to 1 over 0.163. That is approximately 6.135.

And the variance of y is equal to 1 minus r over r squared, which is equal to 1 minus 0.163 over 0.163 squared. And this is approximately 31.503. So this gives us the expected value of the time between successive bug bites and the variance of the time between successive bug bites. And this concludes our two days practice on Bernoulli process.

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