PROFESSOR: OK. Let's take another look at the Monty Hall tree that we worked with to figure out the probability of the switch strategy winning. Now this tree was just absolutely literal and absurdly complicated and large for what we're trying to analyze. That is, we literally thought of each of the three outcomes of whether the prize was behind door one, door two, door three, and then exactly which door the contestant picked next-- either door one, door two, door three, and so on.

And it's clear that this tree has a symmetric structure. That is, these three sub-trees-- whether the prize is behind one, the prize is behind two, or the prize is behind three-- all have the same structure. And we could have gotten away with analyzing one, and deduced that that's what happened with the other two branches. In fact, that really was what we did. But why not make that explicit in our analysis from the first place? Instead of it having this triple tree, let's just look at the tree with one branch.

So there's the central branch that I'm keeping. Namely, we're assuming that the first move is to have the prize at door two, and then the door picked has three choices. And then the door opened has two choices or one, depending on where the prize was placed relative to the contestant's pick here.

If the prize was at two, and the contestant picked door two, then Carol has a choice of two doors-- one or three-- to open. On the other hand, if the prize was at two, and the contestant picked door one, Carol has no choice but to open door three with probability 1.

OK. Now when we're looking at this tree, the first branch is kind of fixed and forced. So we don't really need it as part of the analysis. What we're really looking at is analyzing what happens in the experiment starting at the stage where the contestant picks a door. So let's just enlarge this tree to get a better look at it.

There's the same tree where we're starting off, where we're assuming that the prize is at door two, and then the door picked can be either door one, door two, or door three, and the door opened then can be door three, door one, door three, or door one according to the constraints on Carol.

Now a better way to understand this tree [? as ?] instead of saying the prize is at door two, and this is where the contestant chooses door two, and then have to worry about all the other branches that are symmetrically like this, we could have reformulated the tree model in the first place by saying, let's just consider the cases that wherever the prize is, there are three possibilities. The contestant picks the door where their prize is, or they pick the next door-- let's say in some counter-clockwise direction from where the prize is. Or going around among the doors in a circle, they pick a door that's two doors away from where the prize is.

And if we reformulate it that way, then these are the cases-- either the contestant picks the prize door, or they pick the first door that doesn't have the prize, or they pick the second door that doesn't have the prize. And each of those occur with probability of 1/3. And likewise, once they've picked door number one with no prize, then that means that Carol has the choice of only one door that she can open, because the other unpicked door has a prize. So she's got to open the second no-prize door, because the contestant has picked the first no-prize door.

Likewise, if the contestant picks the prize door, Carol can pick either of the non-prize door-- non-prize one or non-prize two. Both are losses. And likewise here, where Carol's move is forced, and the contestant will win.

So now we're in great shape, because I've really gotten rid of the rest of the tree. It's not as though I'm analyzing 1/3 of it, and the 1/3 analysis is going to apply to the other parts by symmetry. But I've captured the whole story here by simply relativizing the first move instead of it being literally door one, door two, door three.

I don't care what actual door the contestant picks. All I care about is whether the contestant picks the prize door, or the first door that's not the prize door, or the second door that's not the prize door. This would have been a much better tree to start off with the first place, at least for the purpose of analyzing the probability of winning.

Now we're going to get some mileage out of the more complicated tree in a later video segment when we start talking about conditional probabilities of what are the probabilities of things happening at various stages in the experiment. And so we will want to have some of these other vertices that represent stages of the experiment. But if we'd really been thinking solely about how to analyze the probability of winning with the switch strategy, this would've been a much better tree to start off with.

But wait. Let's look at this tree. First of all, there really isn't any need to model this branch of the experiment, because at this point, once we're talking about switching, if the contestant has picked a non-prize door, they win-- period. Carol's move is forced, and it's going to be a win. We might have well just collapsed the win down to say that as soon as they pick a non-prize door they've won, and who cares what happens after that? Same thing down here. So the tree really could've simplified to one where you pick a no-prize door and you win, you pick the other no-prize door and you win, or you pick a prize door, and then Carol has a choice of opening either of the other two no-prize

[INAUDIBLE]

OK. Because after all, what's the point in distinguishing between whether you pick prize door one, or you pick prize door two? You win in both cases. And really, all we care about, we could have condensed the entire tree down to one where either you pick the prize door with probability of 1/3, in which case you're guaranteed to lose no matter what happens. Or you pick the non-prize door, which you do with probability 2/3, in which case you win no matter what happens.

And that is a really simple tree, OK? There it is. And what we can read off immediately is that with the switch strategy the probability of winning is 2/3. So the switch wins if and only if the prize door is not picked. And that means that the probability that switch wins is 2/3, which is what we already figured out using the more complicated tree. But this way of getting at it is a lot clearer.

So the message here is that the tree that you come up with to model the experimental outcomes is really a modeling process. And there may be many models that work to capture a given scenario. And it will often pay off to try to find a simpler tree to make the analysis simpler.