# Massachusetts Institute of Technology Department of Electrical Engineering and Computer Science

6.111 - Introductory Digital Systems Laboratory

#### **Problem Set 1 Solutions**

Issued: Lecture 4 Day

#### Problem 1:

Not Graded.

1) a + 0 = a2)  $\bar{a}_{.0} = 0$ 3)  $a + \overline{a} = 1$ 4) a + a = a5) a + ab = a(1 + b) = a6)  $a + \overline{a}b = (a + \overline{a})(a + b) = a + b$ 7)  $a(\overline{a}+b) = a\overline{a}+ab = ab$ 8)  $ab + \overline{a}b = b(a + \overline{a}) = b$ 9)  $(\overline{a} + \overline{b})(\overline{a} + b) = \overline{aa} + \overline{ab} + \overline{b}\overline{a} + \overline{b}b = \overline{a} + \overline{ab} + \overline{a}\overline{b} = \overline{a}(1 + b + \overline{b}) = \overline{a}$ 10) a(a+b+c...) = aa+ab+ac+... = a+ab+ac+... = a11) f(a, b, ab) = a + b + ab = a + b12)  $f(a, b, \overline{ab}) = a + b + \overline{ab} = a + b + \overline{a} = 1$ 13)  $f(a, b, \overline{(ab)}) = a + b + \overline{(ab)} = a + b + \overline{a} + \overline{b} = 1$ 14)  $y + y\overline{y} = y$ 15)  $xy + x\overline{y} = x(y + \overline{y}) = x$ 16)  $\overline{x} + y\overline{x} = \overline{x}(1+y) = \overline{x}$ 17)  $(w + \overline{x} + y + \overline{z})y = y$ 18)  $(x+\overline{y})(x+y) = x$ 19) w + (w + (wx)) = w20) x(x + (xy)) = x21)  $(\overline{x} + \overline{x}) = x$ 22)  $\overline{(x+\overline{x})} = 0$ 23)  $w + (w\overline{x}yz) = w(1 + \overline{x}yz) = w$ 24)  $\overline{w}(\overline{wxyz}) = \overline{w}(\overline{w} + \overline{x} + \overline{y} + \overline{z}) = \overline{w}$ 25)  $xz + \overline{x}y + zy = xz + \overline{x}y$ 26)  $(x+z)(\overline{x}+y)(z+y) = (x+z)(\overline{x}+y)$ 27)  $\overline{x} + \overline{y} + xy\overline{z} = \overline{x} + \overline{y} + \overline{z}$ 

## Problem 2:

1.

$$f = (a + (\bar{b} + \bar{c})) \cdot (\bar{c} + (a + b + d) \cdot (\bar{a} + \bar{b} + \bar{d}))$$

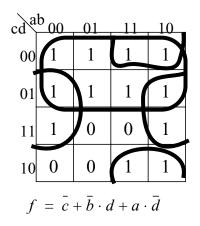
i) truth table

ii) Karnaugh map

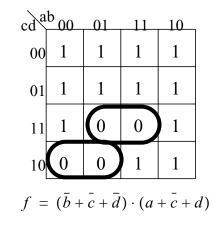
| a | b | c | d | f |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

| cda | b <sub>00</sub> | 01 | 11 | 10 |
|-----|-----------------|----|----|----|
| 00  | 1               | 1  | 1  | 1  |
| 01  | 1               | 1  | 1  | 1  |
| 11  | 1               | 0  | 0  | 1  |
| 10  | 0               | 0  | 1  | 1  |

iii) MPS



iv) MSP



$$f = (\bar{c} + a \cdot b) \cdot (\bar{c} + (a + \bar{d}) \cdot (b + \bar{d})) \cdot (c + (a + \bar{b}) \cdot (b + \bar{d}))$$

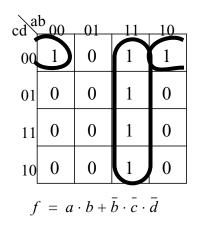
i) truth table

ii) Karnaugh map

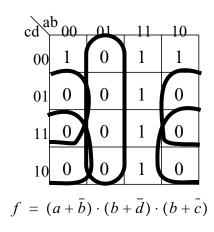
| a | b | c | d | f |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

| cd | b 00                 | 01 | 11 | _10_ |
|----|----------------------|----|----|------|
| 00 | <sup>b</sup> 00<br>1 | 0  | 1  | 1    |
| 01 | 0                    | 0  | 1  | 0    |
| 11 | 0                    | 0  | 1  | 0    |
| 10 | 0                    | 0  | 1  | 0    |
|    |                      |    |    |      |
|    |                      |    |    |      |

iii) MSP



iv) MPS



2.

$$f = \overline{w} \cdot y + w \cdot \overline{x} \cdot y + \overline{w} \cdot x \cdot \overline{z}$$

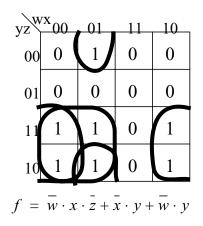
i) truth table

ii) Karnaugh map

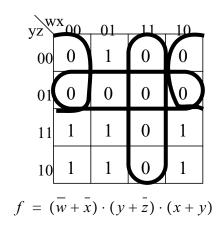
| W | X | У | Z | f |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

yz wx 00 01 11 00 0 

iii) MSP



iv) MPS

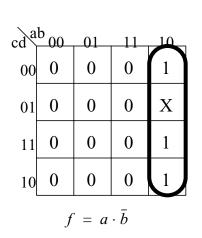


3.

## Problem 3:

#### 1. MSP

a)



b)

| cda                                             | b 00 | 01 | _11_ | _10          |  |  |
|-------------------------------------------------|------|----|------|--------------|--|--|
| 00                                              | 0    | 0  | Х    | 1            |  |  |
| 01                                              | 0    | 0  | 0    | 0            |  |  |
| 11                                              | 0    | 0  | 0    | $\bigcirc 1$ |  |  |
| 10                                              | 0    | 0  | X    | 1            |  |  |
| $f = a \cdot \bar{d} + a \cdot \bar{b} \cdot c$ |      |    |      |              |  |  |

2. MPS

a)

b)

| cd                         | <sup>b</sup> 00 | 01 | -11 | _10_ | cd  | ab 00                        | 01                 | _11 | 10 |
|----------------------------|-----------------|----|-----|------|-----|------------------------------|--------------------|-----|----|
| 00                         | 0               | 0  | 0   | 1    | 00  | 0                            | 0                  | X   | 1  |
| 01                         | 0               | 0  | 0   | Х    | 01  | 0                            | 0                  | 0   | 0  |
| 11                         | 0               | 0  | 0   | 1    | 11  | 0                            | 0                  | 0   | 1  |
| 10                         | 0               | 0  | 0   | 1    | 10  | 0                            | 0                  | X   | 1  |
| $f = a \cdot \overline{b}$ |                 |    |     |      | f = | $a \cdot \overline{b} \cdot$ | $(c+\overline{a})$ | Ī)  |    |

3. The solutions are unique given that we want the minimal equations.

4. The MSP and MPS in part a are equal. The MSP and MPS in part b are not equal as the don't cares in part 1 are assumed to be 1 and the don't cares in part 2 are assumed to be 0.

#### **Problem 4:**

1. 
$$\overline{(\overline{a}+c)} \cdot \overline{(b+c)} = (\overline{a}+c) + (b+c) = \overline{a}+b+c$$

2. 
$$\overline{a \cdot b \cdot c} = \overline{a} + \overline{b} + c$$

3. 
$$\overline{(b+c)} \cdot \overline{(a+c)} \cdot \overline{(a+b)} = (\overline{b} \cdot c) \cdot (a \cdot \overline{c}) \cdot (a \cdot b) = 0$$

### Problem 5:

library ieee; use ieee.std\_logic\_1164.all; entity pset\_1\_problem\_5 is port ( a, b, c, d : in std\_logic; p1, p2 : out std\_logic); end pset\_1\_problem\_5;

architecture structure of pset\_1\_problem\_5 is begin

p1 <= (a and c) or (not a and (b or not c));

p2 <= (not b and not c and d) or (not a and b and d) or (a and not c and d) or (not a and not c and not d);

end structure;