MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Spring Term 2007 — Issued : March 13, 2007
Problem Set 5 — Due : Tuesday, March 20, 2007

Problem 1: For each of the three loop transfer functions $L(s)$ listed below, sketch the Nyquist locus. In each case, determine the range (or ranges) of gain K for stability.

$$L_a(s) = \frac{K(s+10)}{(s+1)(s+100)}$$

$$L_b(s) = \frac{K(s+1)}{s^2(0.1s+1)}$$

$$L_c(s) = \frac{Ke^{-s}}{s+1}$$

Problem 2: For each of the two loop transfer functions $L(s)$ listed below, sketch the Nyquist locus. In each case, label the regions of stability appropriately (you don't have to calculate ranges of K).

$$L_d(s) = \frac{K(s+1)}{s^2(0.1s+1)^2}$$

$$L_e(s) = \frac{K(s+100)^2}{(s+1)^3(s+1000)}$$

Problem 3: What is the pole-zero plot that correponds to the Nyquist Plot below?

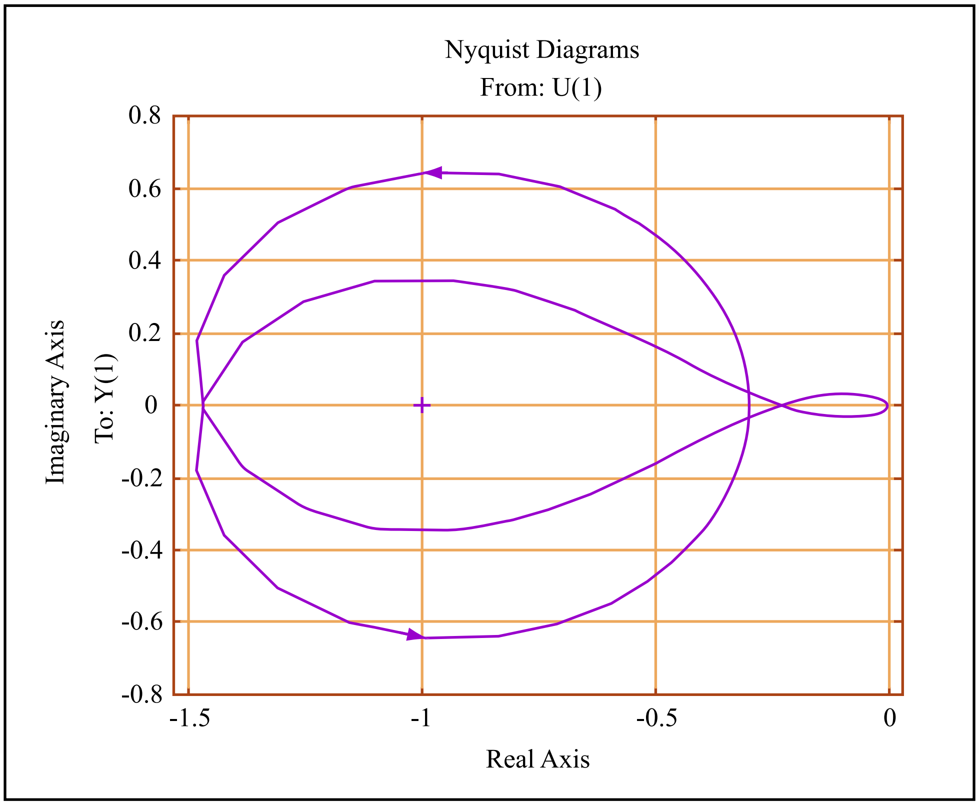
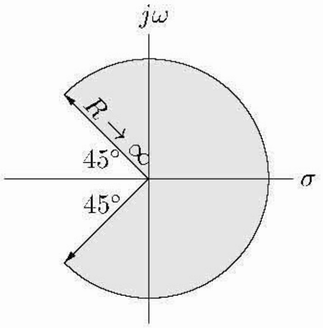
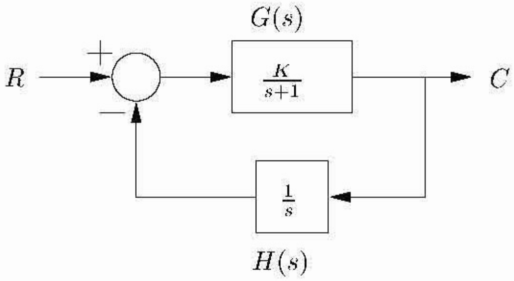


Figure by MIT OpenCourseWare.

Problem 4: One of the most attractive features of the Nyquist criterion is that it provides a convenient method for predicting what values of K will fall in a certain region of the complex plane. In most circumstances, we are only interested in knowing when the closed loop poles leave the left half-plane. Rather than restrict ourselves to this mundane application of such a powerful tool, consider the following problem. In designing a feedback system, your specifications require that the system have a damping ratio less than $\zeta = 0.707$. Use the figures below.

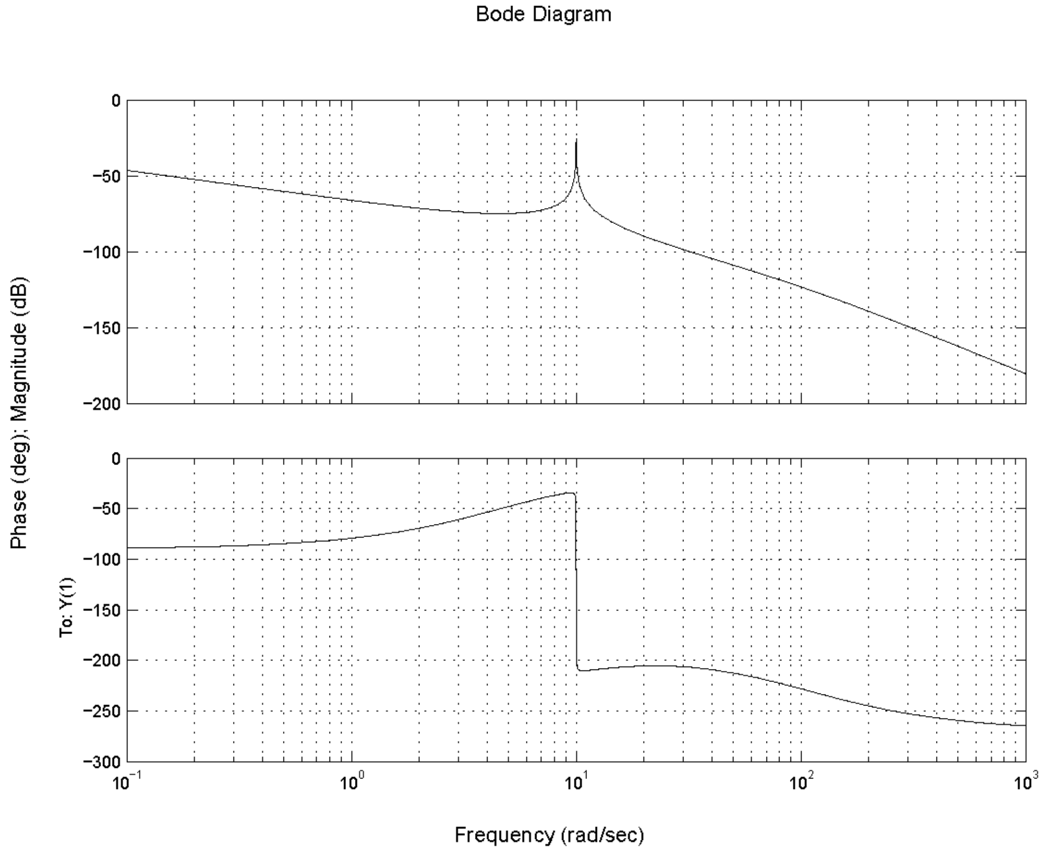




(a) Draw a Nyquist plot for the loop transfer function $L(s) = G(s)H(s)$ and the Nyquist contour as given above. Determine the values of K (both positive and negative) for which the poles of the closed loop system lie outside the shaded region. For these corresponding values of K, determine how many poles lie in the shaded region from the encirclement information in the diagram.

(b) Plot the response of $c(t)$ for $r(t) = t$, i.e. a unit ramp and the value of K being such that the poles of the closed loop system lie just on the border of the shaded region. Assume that $K > 0$. What is the value of t_p for this particular $c(t)$?

Problem 5: One advantage of the Nyquist criterion over the Routh test or root locus is that you don't need a rational transfer function to study stability. This can come in handy when, given frequency response data for an open loop system $L(j\omega)$, you want to learn about stability issues that may arise when closing a feedback loop around that $L(s)$. One such set of data is given below; using this plot, sketch the Nyquist locus for $L(s)$ under the assumption that you are using unity feedback.



Problem 6: The gain-phase plane plot of $L(s)$ for a unity feedback system is shown in the plot attached to the end of this problem set. The plot approaches $+\infty$ at $-90°$.

(a) What is M_p?

(b) What is ω_p (peak frequency)?

(c) What is ω_c (crossover frequency)?

(d) What is the gain margin?

(e) What is ϕ_M (phase margin)?

(f) It is desired to increase M_p to 1.6. By what factor must the gain of the system K be increased?

(g) If K is increased as in part f, what is the new value of ω_p?

(h) What is the new ϕ_M?

(i) What is the new ω_c?

Put Gould's hand-drawn Nichols Chart here.