MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Spring Term 2007 — Issued : March 20, 2007
Problem Set 6 — Due : *Thursday*, April 5, 2007

Problem 1: For the following loop transfer functions in unity feedback, use approximate asymptotic methods to find the magnitude of the closed-loop frequency response.

$$L_a(s) = \frac{0.25s}{(s+1)(0.01s+1)^2}$$

$$L_b(s) = \frac{25s}{(s+1)(0.01s+1)^2}$$

$$L_c(s) = \frac{\pi(s+1)}{s^2(0.1s+1)(0.01s+1)}$$

Problem 2: Construct asymptotic Bode plots (magnitude and phase) for the following loop transfer functions and show the location of the applicable stability margins (gain margin, phase margin, ω_ϕ, and ω_c). Construct a table listing the stability margins.

$$L_a(s) = \frac{20}{0.2s+1}$$

$$L_b(s) = \frac{20}{(0.2s+1)(0.1s+1)}$$

$$L_c(s) = \frac{100}{s(0.1s+1)(0.001s+1)}$$

$$L_d(s) = \frac{100}{s(0.1s+1)(0.01s+1)}$$

$$L_e(s) = \frac{500(0.2s+1)}{s^2(0.01s+1)(0.001s+1)}$$

Problem 3: For each of the loop transfer functions in Problem 3, sketch the Nyquist plots and relate the gain margin and phase margin from the Bode plot to the appropriate locations on the Nyquist plot.

Problem 4: Are each of the loop transfer functions in Problem 3 stable?

Problem 5: You are given a set of performance specifications for a unity-gain feedback system. They are as follows:

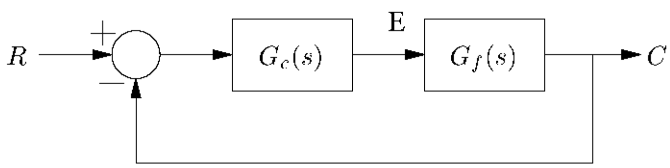
- Crossover frequency of $\omega_c = 100$ rps with a slope of -1 at crossover.
- Zero steady state error for a ramp input.
- $\left|\frac{E(j\omega)}{R(j\omega)}\right| < 0.005$ for sinusoidal inputs over the range $0 < \omega < 1$ rps.
- For sensor noise variations in the frequency range $\omega > 1000$ rps, $\left|\frac{C(j\omega)}{N(j\omega)}\right| < 0.1$.
- To obtain an adequate margin of stability, the frequency range of the -1 slope around crossover should be at least 2 decades.
- The phase margin should be at least $60°$.

(a) Draw the Bode obstacle course for these specifications.

(b) Construct an asymptotic Bode magnitude curve which will meet these specifications with as little extra magnitude as possible. At the corners of your diagram label the corner frequencies and the values of the magnitudes of $L(j\omega)$ at each corner. Give an expression for a rational function $L(s)$ which will exhibit the same asymptotic curve that you have just constructed.

(c) Based upon the asymptotic magnitude curve of part (b), draw the approximate asymptotic Bode magnitude curve of the closed loop response $\left|\frac{E(j\omega)}{R(j\omega)}\right|$.

Problem 6: For the following feedback system and plant transfer function $G_f(s)$



$$G_f(s) = \frac{1.44 \times 10^4}{s(s+3)(s+200)}$$

(a) Construct a Bode obstacle course that meets the following specifications:

- For the time to the first peak to be less than 0.085 second, $\omega_d > 37$ rps. We can thus require that magnitude crossover occur at 40 rps.
- For the peak overshoot to be less than 30%, $\zeta = 0.36$ or $M_p = 1.5$. (verify this fact)
- For an error of less than 0.0023 radians for a velocity input of 1 rps, the magnitude of $L(s)$ at low frequencies must behave at least as $\frac{435}{\omega}$.
- The system must be stable.

(b) Choose an asymptotic representation for the Bode magnitude response of $L(j\omega)$ and determine a compensation function $G_c(s)$ which will ensure that the closed-loop system meets **all** of the specifications.

(c) If high frequency noise is present at the output, over what frequency range will the error due to noise be less than 1% of the noise based on your choice of $G_c(s)$?