6.334 Power Electronics
Spring 2007

Power Electronics Notes - D. Perreault

★★ RESONANT POWER CONVERSION

In many power converter applications, one converts between dc and high-frequency sinusoidal ac (either as an intermediate or final waveform). Applications include:

1. RF Power amplifiers for communications, radar, etc.
2. High frequency inverters for induction heating
3. "Resonant" dc/dc converters using high-frequency sinusoidal intermediate waveforms + energy storage
4. Electronic ballasts for lighting
5. power converters using resonant energy storage/transformer structures (e.g. piezoelectric "electromechanical" transformers)

Here we consider topologies, designs + control methods that are suited to these applications. These applications share certain objectives:
1. High-frequency sinusoidal ac waveforms
2. A desire for high-efficiency power conversion.

But

Different applications have different requirements:
1. Load characteristics (narrow, well-known range vs. wide load variations)
2. Need for or absence of regulation + control of output
3. high sinusoidal waveform purity (required or not)

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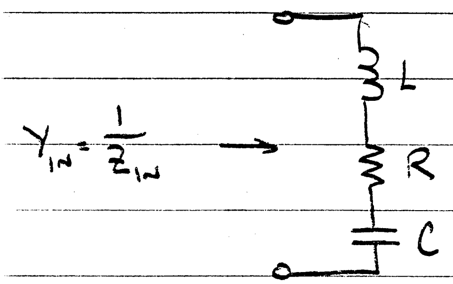
★ Resonant Circuit Review

Second-order resonant circuit example: <u>series resonant tank</u>

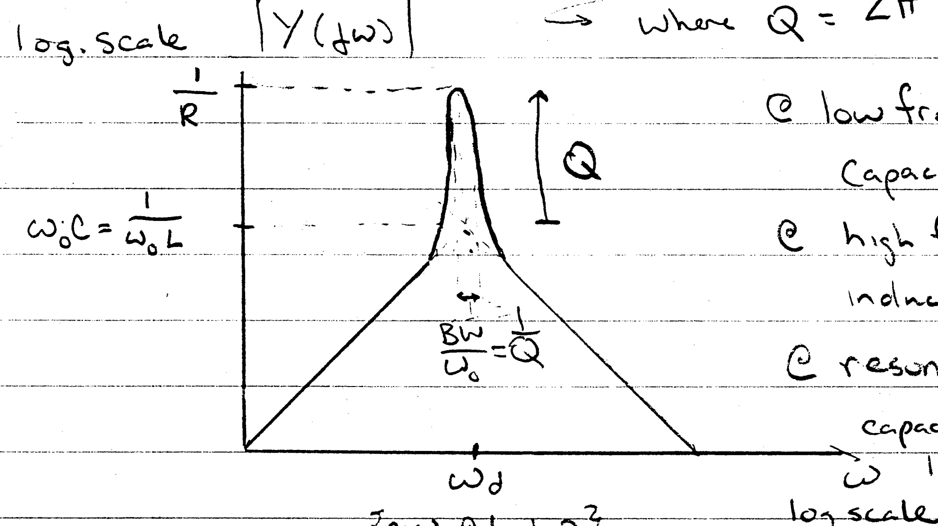


$$Z_{IN} = sL + R + \frac{1}{sC}$$

$$Y_{IN} = \frac{1}{Z_{IN}} = \left(\frac{1}{R}\right)\frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2}$$

where $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$, and $Q = \frac{\omega_0}{2\alpha}$

where $Q = 2\pi \cdot \frac{\text{PEAK ENERGY STORED IN AN AC CYCLE}}{\text{ENERGY DISSIPATED IN AN AC CYCLE}}$



@ low frequency, admittance limited by capacitor

@ high frequency, admittance limited by inductor

@ resonance $\{\omega_d \approx \omega_0\}$ inductor + capacitor <u>impedances</u> cancel, admittance limited only by conductance $1/R$

for high-Q series circuit at resonance:

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \sqrt{L/C}\,/R$$

Reminder: The quality factor $Q = 2\pi \times \frac{(\text{PEAK STORED Energy})}{(\text{Energy dissipated in a cycle})}$

- The admittance of the RLC network at the resonant frequency is a factor of Q higher than that of the inductor or capacitor alone
- The half-power bandwidth of the resonance normalized to the (undamped) resonant frequency is $\frac{1}{Q} = \frac{BW}{\omega_0}$

$\{$@ $\omega \approx \omega_0 \pm \frac{\omega_0}{2Q}$ The power delivered by a voltage source drops to half that at $\omega_0\}$

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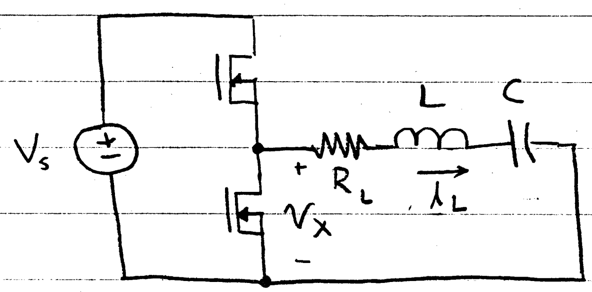
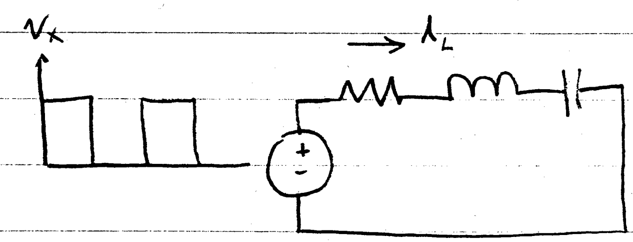
★ "Class-D" Amplifier

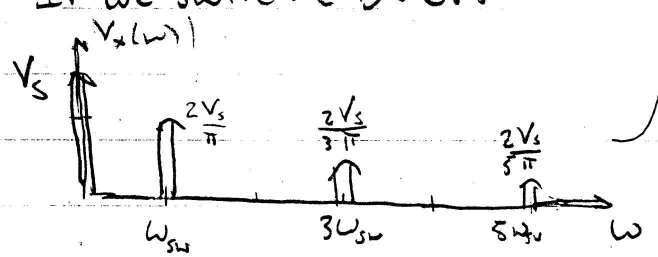
Suppose we drive the series resonant tank (w/ resistive load R_L) from a half-bridge inverter





$$i_L(s) = V_x(s) \cdot Y_{IN}(s)$$

If we switch @ $D \cong 0.5$



@ high Q $\left\{R \ll \sqrt{\frac{L}{C}}\right\}$

ω_{sw} near ω_0 $i_L \sim$ sinusoidal

- In applications where we desire high-purity sine waves (e.g. communications) we might operate very close to resonance
  - → Could do AM by varying V_s externally, FM by varying f_{sw}
- In other applications where we would like to modulate power to the load, we might vary the switching frequency more broadly, above or below the resonant frequency.

→ we can control $|i_L|$ by controlling f_{sw}

1. Above resonance $f_{sw} > \frac{\omega_0}{2\pi}$
2. Below resonance $f_{sw} < \frac{\omega_0}{2\pi}$ (keep $f_{sw} > \frac{3\omega_0}{2\pi}$ to avoid amplifying harmonic content)

⇒ If harmonics well filtered $i_L \sim$ sinusoidal.

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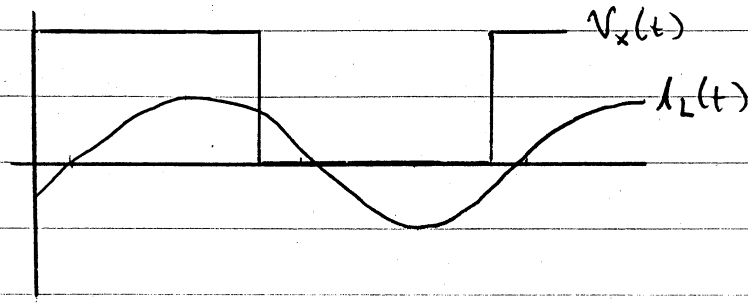
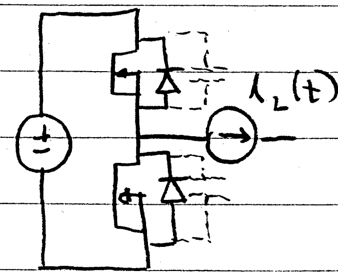
Consider Above Resonance operation





The resonant circuit looks inductive: i_L lags the fundamental of V_x.

So: DIODES TURN OFF @ ZERO VOLTAGE

TRANSISTORS TURN ON @ ZERO VOLTAGE

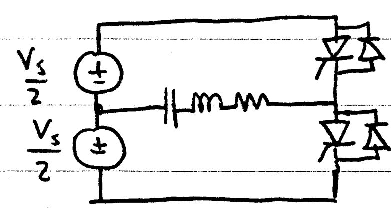
TRANSISTOR TURN OFF + DIODE TURN ON ARE HARD

⇒ BUT IF we utilize/ADD CAPACITANCE IN PARALLEL WITH SWITCHES, (AND PLACE SUFFICIENT DEAD TIMES BETWEEN THE SWITCH SIGNALS), WE CAN GET SOFTER SWITCH TURN OFF + DIODE TURN ON, TOO! (like resonant pole inverter)

⇒ If the tank/load is such that $\frac{dV_x}{dt}$ reaches zero just as V_x charges/discharges to $V_s/0$, the transition is especially soft: class "DE" operation

For operation below resonance, the reverse is true. In this case, natural switch commutation off is achieved (for main switches), but turn on is hard

can operate with thyristors



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In either case:

Device voltages are clamped at V_s, but i_L, V_C can have **LARGE** peak values.

$$\frac{\text{cap voltage fundamental}}{\text{drive voltage fundamental}} = \left.\frac{|V_c|}{|V_{x,1}|}\right|_{\omega=\omega_o} = \left|\left|\frac{\omega_o^2}{s^2+2\alpha s+\omega_o^2}\right|\right|_{s=j\omega_o} = \frac{\omega_o}{2\alpha} = Q$$

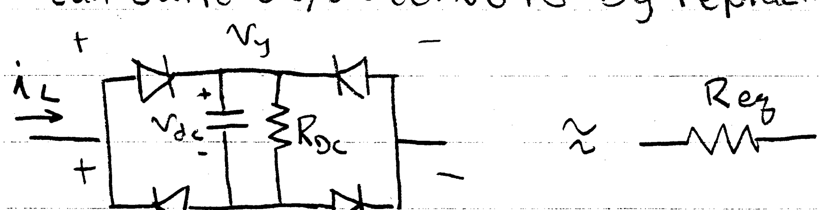
At resonance (worst case):

The capacitor voltage magnitude is larger than the drive voltage fundamental by a factor $Q = \sqrt{\frac{L}{C}}/R$! This can be big!

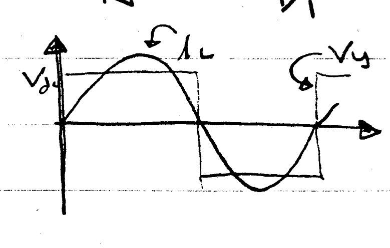
With the series-resonant circuit there are significant concerns about the value of the load resistor (load range):

1. If $R \to$ too small we must move away from resonance, or peak V's, i's become large
2. If $R \to$ too big, the selectivity of the tank becomes poor.
   - → Current not sinusoidal
   - → "soft" switching properties lost

- Can build dc/dc converter by replacing resistor with rectifier



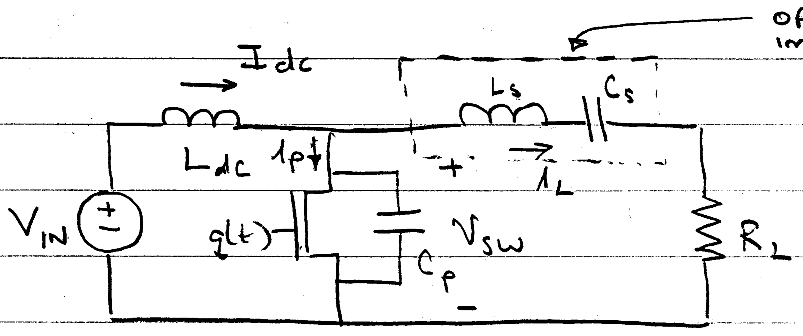
⇒ SERIES RESONANT DC/DC CONVERTER



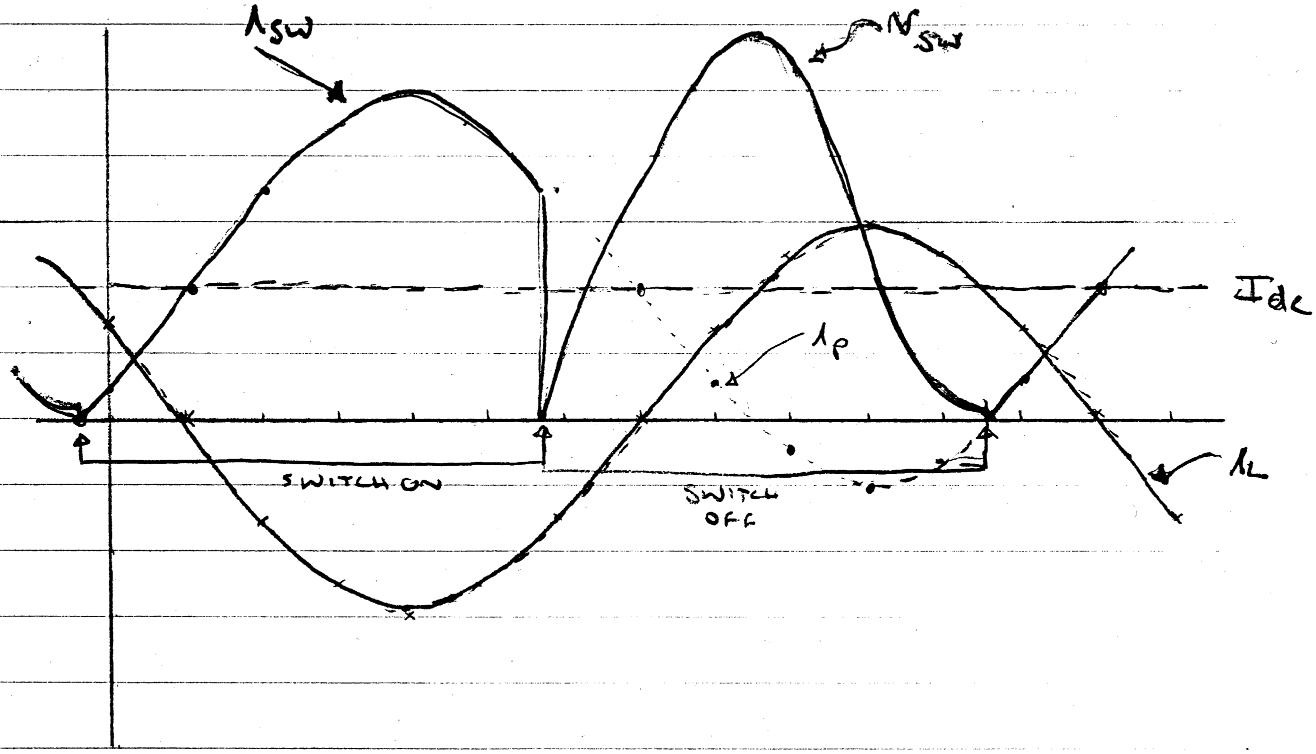
- we then control $|i_L|$, V_{dc} by varying f_{sw}
- control dynamics are challenging

We can also build high-frequency tuned inverters (power amplifiers) with a single switch!

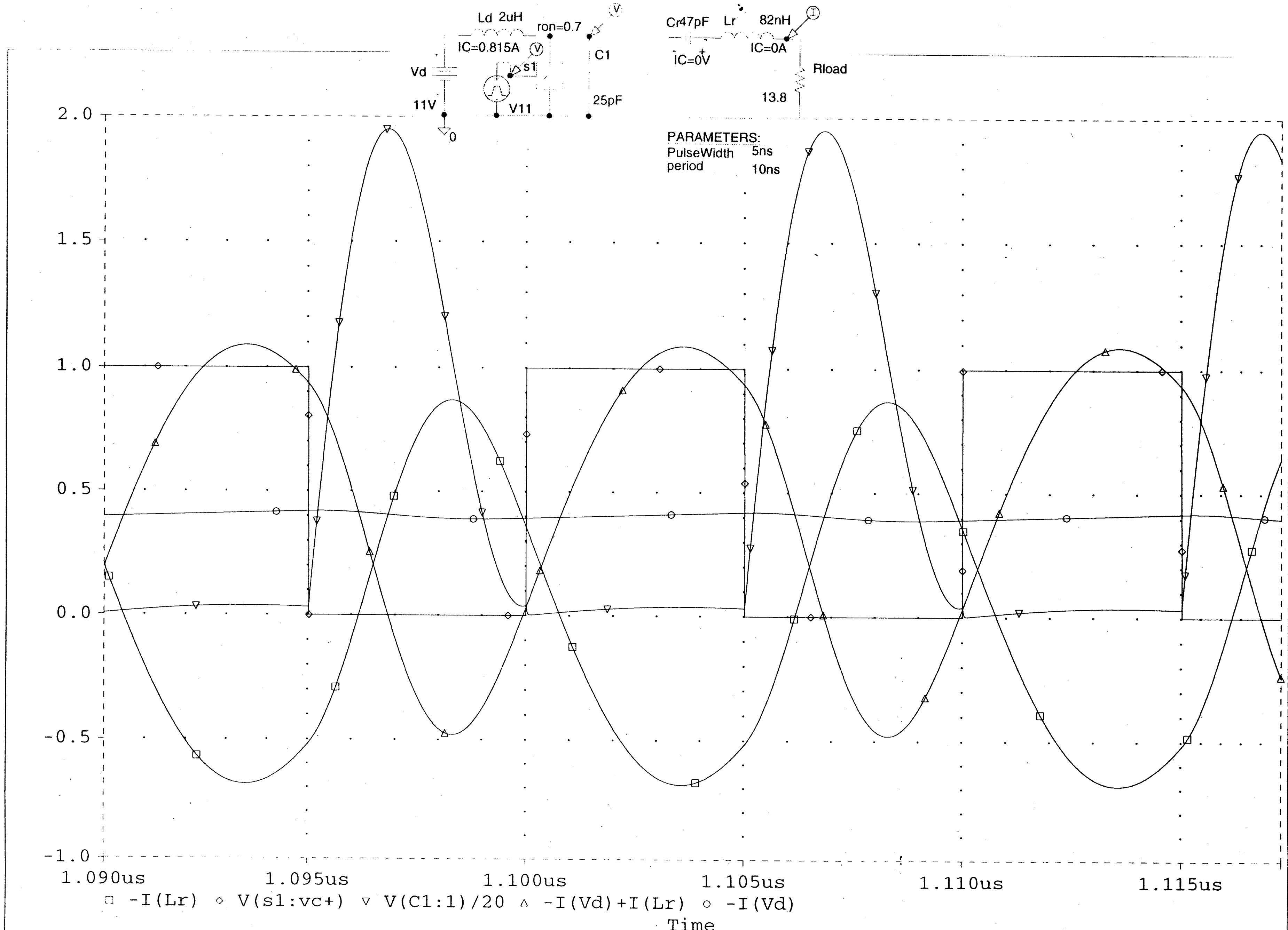
★ "CLASS E" Converter (Developed by an MIT grad, Nat Sokal)



- C_p absorbs (or is) switch capacitance
- Switch above resonance of series tank
- L_{dc} is large enough for smooth ~dc input current



- Network switches between two resonant structures
  - → Component values such that $V_{sw}, \frac{dV_{sw}}{dt} \approx 0$ @ switch turn on (ZVS)
  - → works well only over a narrow range of R_L (~2:1)
  - → C_p makes switch turn-off "soft" ZVS
  - → because of switching conditions, switch gating transition can be a significant fraction of the cycle (~10%)



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- The Class E provides soft-switched (tuned) inversion with only a single ground referenced switch.

- The gate drive can be relatively "slow" without hurting efficiency greatly

- However, we "pay" for the single switch in that the peak switch voltage is 3-4x the dc voltage. This yields worse "switch utilization" than an equivalent class-D converter
  - → high peak voltage can be reduced to ~2X using additional resonances. this is part of the collection of so-called "class F" converters
  - → Class E is still advantageous at high frequencies, where multiple switches become tricky to drive.

- can build class E dc/dc converter by replacing resistor w/rectifier
  - → resonant "Class D" + "class E" rectifiers exist that have time-reversed waveforms from inverters + only require passive devices
  - → frequency control is typical for class-E dc/dc, but load range is somewhat limited by efficiency concerns.
  - → Converter control becomes increasingly challenging at high f.

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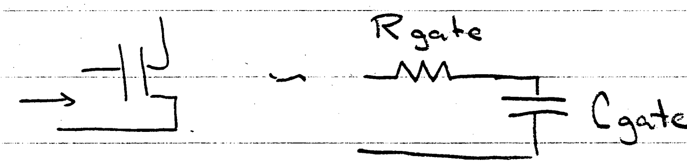
★ RESONANT GATE DRIVES, MULTI-STAGE AMPLIFIERS

In resonant converter circuits we greatly reduce (output) switching losses (ZVS soft switching) and consequently high switching frequencies can be achieved. In this case, gating loss (loss associated with gating the switch on and off) can rapidly become a substantial loss component.

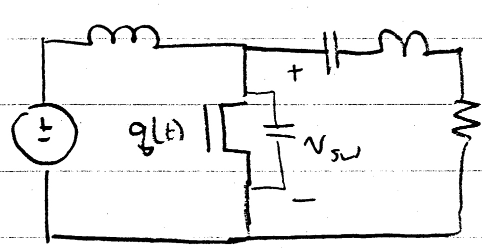
Simplified model

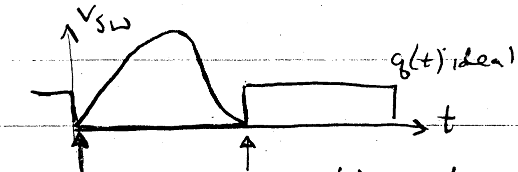


In reality Cgate appears nonlinear (due to capacitance non-linearity and miller effect of gate-drain capacitance).

In most converters, we must gate the switches on + off in a time that is short compared to the switching cycle to maintain low conduction loss. PWM converters: typ 1% of cycle. Resonant converters (e.g., class D, E) are more forgiving, and up to 10% of cycle is sometimes allowable (due to current + voltages @ turn on + off).

e.g.





$\frac{dV_{sw}}{dt}$ at turn off is limited by C_p → slow turn off OK.

V_{sw}, dV_{sw}/dt low @ turn on → slow turn on OK

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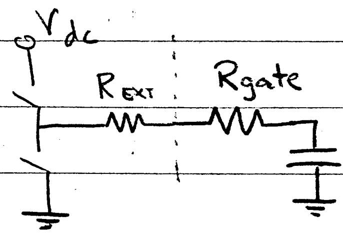
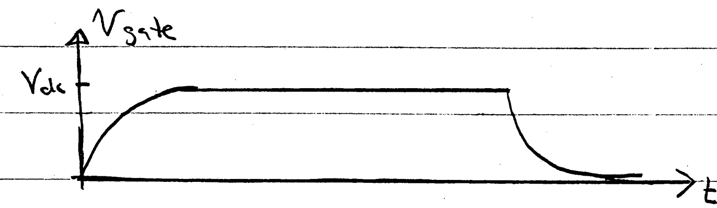
Conventional gate drives are a totem pole to switch fast





<u>Assuming linear C_{gate}</u>, we can calculate gating loss:

$Q_{gate} = C_{gate} V_{dc}$ $\Rightarrow$ We source $W_{source} = Q_{gate} V_{dc} = C_{gate} V_{dc}^2$

we store $W_{STORE} = \frac{1}{2} C_{gate} V_{dc}^2$

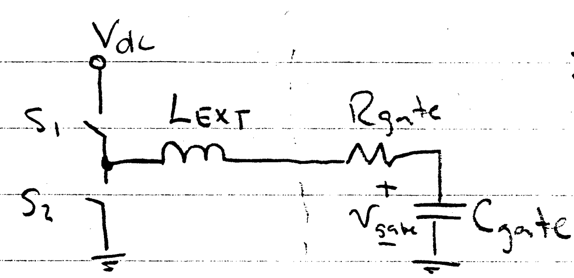
$\Rightarrow$ we lose $\frac{1}{2} C_{gate} V_{dc}^2$ charging the gate (turn on)

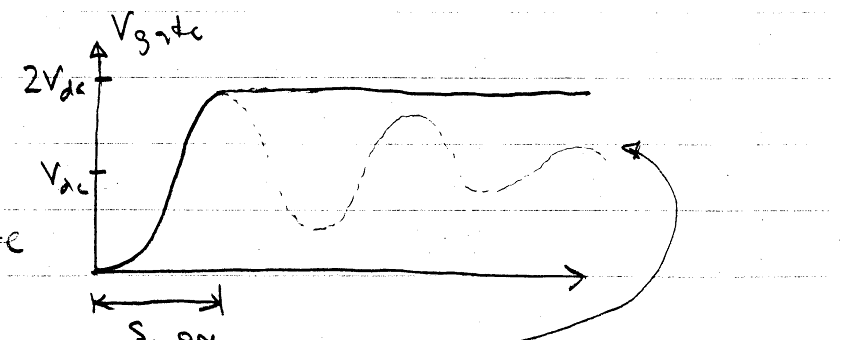
$\Rightarrow$ we lose $\frac{1}{2} C_{gate} V_{dc}^2$ discharging the gate (turn off)

$\therefore$ GATE POWER LOSS: $Q_{gate} V_{dc} f_{sw} = C_{gate} V_{dc}^2 f_{sw}$

These losses rapidly become dominant as frequencies increase (e.g. > 10MHz for many power mosfet designs)

In some cases we can reduce loss with <u>resonant</u> gate drive methods. Suppose we replace R_{EXT} WITH AN inductor:





SOURCE $Q_{gate} V_{dc}$

deliver $\approx \frac{1}{2} C_{gate} (2V_{dc})^2 \approx Q_{gate} V_{dc}$

where $Q_g \approx 2 V_{dc} C_{gate}$

To make this work, we need the Q of the circuit to be high: $\sqrt{\frac{L_{ext}}{C_{gate}}} > R_{gate}$

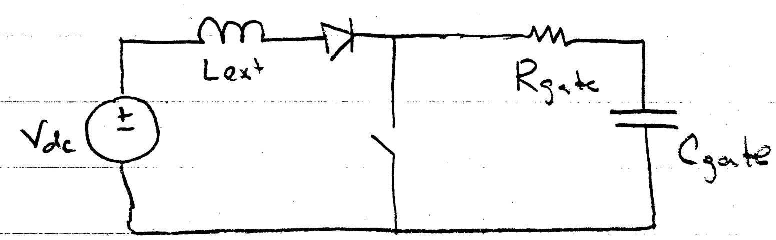
Also, to do this fast relative to a switching cycle, we need

$$\omega_{res} \simeq \frac{1}{\sqrt{L_{ext} C_{gate}}} > 10\,\omega_{sw}$$

So there are design bounds on L_{ext}.

This circuit allows one to control switch duty ratio, and also allows one to recover gate energy at turn off with sufficiently complex switch controls (though this becomes increasingly difficult as $f_{sw} \uparrow$). Many other related possibilities exist. One simple resonant gate drive that allows some range of duty ratio control but does not fully reduce loss

- resonant turn on
- hard turn off



If we do not need to control duty ratio (e.g. in rf power amplifiers, some resonant converters, etc.), we can build the gate drive as an rf amplifier, yielding a <u>multi-stage</u> amplifier system!

$\Rightarrow$ The gate resistance becomes the effective load

$\Rightarrow$ The gate capacitance becomes part of the resonant tank

$\Rightarrow$ <u>Mosfets</u> (for example) can sustain sinusoidal gate waveforms

$\Rightarrow$ Result: Reduced gating loss

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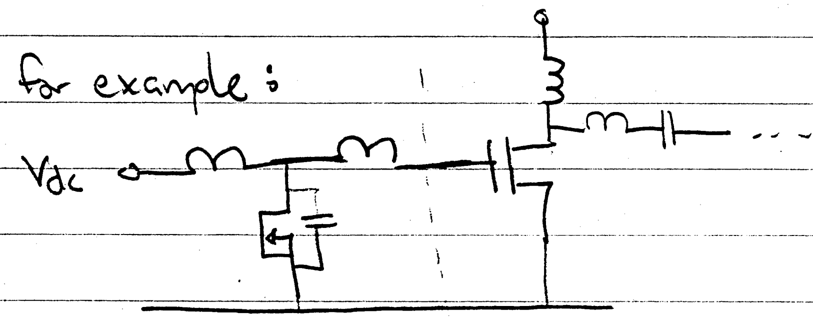
for example:



Class-E gate drive | Class E power stage

⇒ One can build 2-stage, 3-stage, etc. amplifiers to get sufficient output power @ low gating loss!

★ POWER TRANSFER + MATCHING CONSIDERATIONS

As described before, many resonant converter structures only operate well over a relatively narrow (resistive) load range. However it is not necessarily true that the load that presents itself in a problem is well suited to the converter one can build to drive it. (For example, the resonant gate drive one could build might be poorly matched to the gate resistance it drives.)

As a result, we need to concern ourselves with how well the load is matched to a resonant circuit, and to have means to readily adjust this matching.

THE MAXIMUM POWER TRANSFER THEOREM TELLS US HOW WE SHOULD MATCH A DRIVING CIRCUIT TO A LOAD TO MAXIMIZE DELIVERED POWER.

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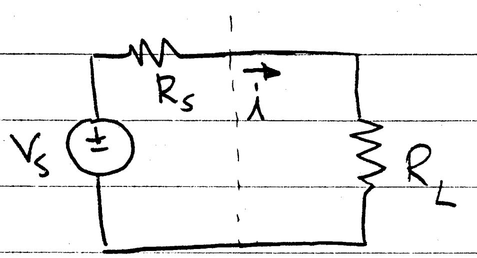
MAXIMUM POWER TRANSFER THEOREM

Suppose we have a source represented by a Thevenin Voltage V_S and Thevenin resistance R_S, which will drive a resistive load R_L. What value of R_L maximizes Power transfer?



@ $R_L = 0$ $\quad i^2 R_L \to 0$ $\therefore$ no power xfer

@ $R_L = \infty$ $\quad i \to 0 \therefore i^2 R_L \to 0 \therefore$ no power

what condition maximizes power?

$$P = \frac{1}{2} i^2 R_L = \frac{1}{2} V_S^2 \frac{R_L}{(R_L + R_S)^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{(R_L+R_S)^2 - 2R_L(R_L+R_S)}{(R_L+R_S)^2} = \frac{R_S^2 - R_L^2}{(R_L+R_S)^2} = 0$$

MAX POWER XFER @ $\underline{R_L = R_S}$

- MORE GENERALLY, @ $Z_S = R_S + jX_S$

  max power @ $\quad Z_L = R_L + jX_L = Z_S^* = R_S - jX_S$

  MAKES LOAD REACTANCE + SOURCE REACTANCE CANCEL, RECOVERING RESISTIVE CASE

  This is the so-called "conjugate impedance match

- If we only control load resistance value, but not phasing, we set the load resistance value to match the source impedance magnitude to maximize power transfer.

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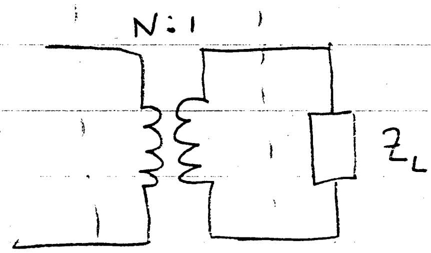
★ **MATCHING NETWORKS**

There is clearly a need to "match" a load to a driving network in many applications (that is, to perform a conversion so that a source can effectively drive a load.)

One way to change a load impedance value is to use a transformer, which can scale impedances seen through it.

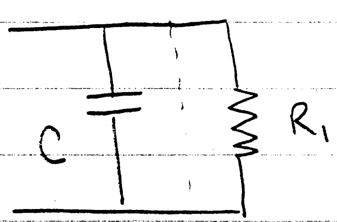
$Z_{IN} = N^2 Z_L$ →



If we are concerned with narrow-band transformation (sinusoids over a narrow frequency range only) we can use a reactive matching network to effect a desired transformation.

Simple example: suppose we have a load R_1, and want to make it "look like" a smaller resistance R_2. We can do this with an LC tank network.

Z_{eq1} →



Place a capacitor in parallel with R_1

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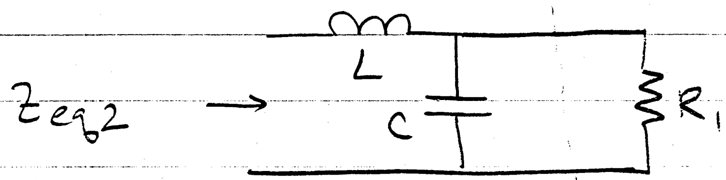
$$Z_{eq1} = \frac{R_1 / j\omega C}{R_1 + 1/j\omega C} = \frac{R_1}{j\omega R_1 C + 1} = \frac{R_1}{1+\omega^2 R_1^2 C^2} - j\,\frac{\omega R_1^2 C}{1+\omega^2 R_1^2 C}$$

Now add an inductance in series



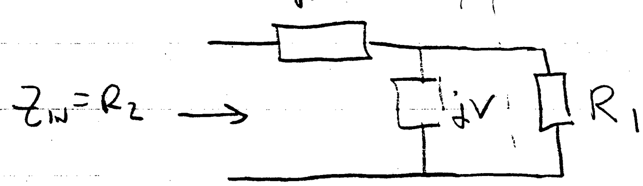
$$Z_{eq2} = \frac{R_1}{1+\omega^2 R_1^2 C} + j\left(\omega L - \frac{\omega R_1^2 C}{1+\omega^2 R_1^2 C^2}\right)$$

By picking C we can make $Re\{Z_{eq2}\} = R_2$ @ ω (desired value)
By picking L we can make $Im\{Z_{eq2}\} = 0$ @ ω (desired value)

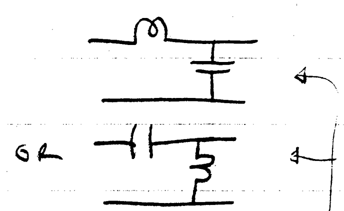
So we effectively transform R_1 into an apparent resistance R_2 at a single frequency ω

→ This is done losslessly through the action of the resonant tank
→ practical conversion limits arise due to parasitics of L, C

In general to "step down" resistance, we need two reactances jV, jW, and <u>V and W must have opposite sign</u> (e.g. 1 inductor, 1 capacitor)



Possibilities



for a specified conversion ratio (at a specified frequency) V + W are determined

These are called "L-section" matching networks

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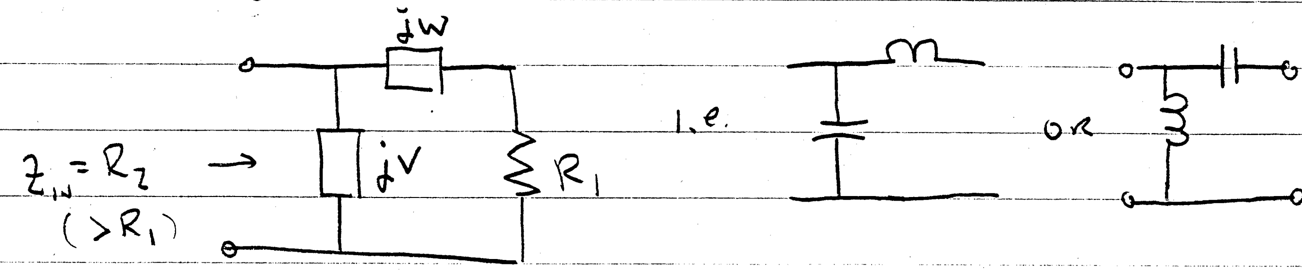
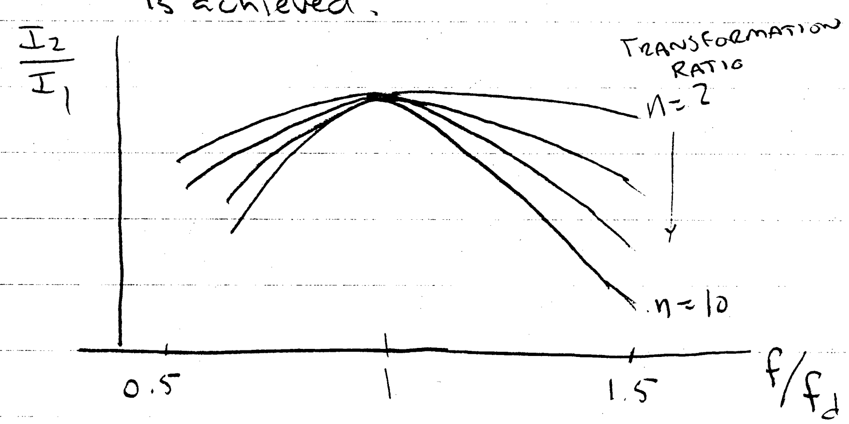
To "step Up" R_1 to a higher value R_2, we first use a series element and then a shunt element (The same thing backwards!)



The bigger the transformation ratio we desire, the harder it is to do in some regards. As transformation ratio ↑

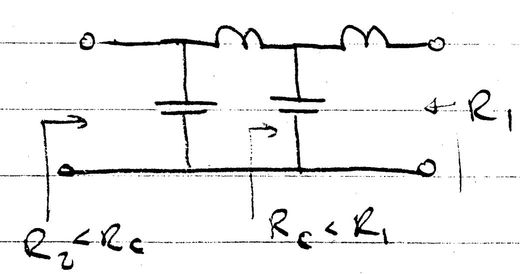
→ we require higher Q matching components

→ the narrower the frequency range over which the match is achieved.



SEE EVERITT + ANNER "COMMUNICATIONS ENGINEERING" 3rd Ed., p.412.

If we need a large transformation we can cascade "L-section" networks.



For limited available component size + Q, a multi-stage network can be a better solution than a single large stage.

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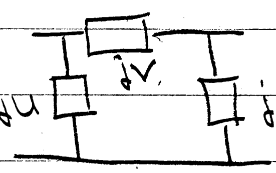
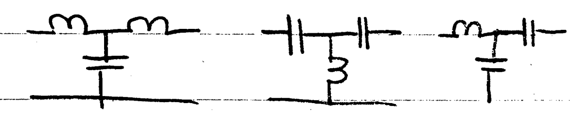
The conversion ratio + frequency completely specify the component values in a single L-section matching network (of the specified flavor).

We can gain additional design flexibility with a "T" or "Π" (pi) stage. In the Π stage one of the reactances must have a sign that is opposite to the other two. e.g.,

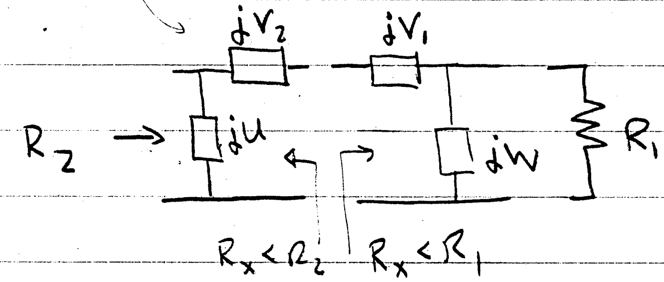




etc.

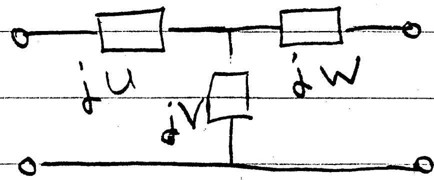
$jV = jV_1 + jV_2$

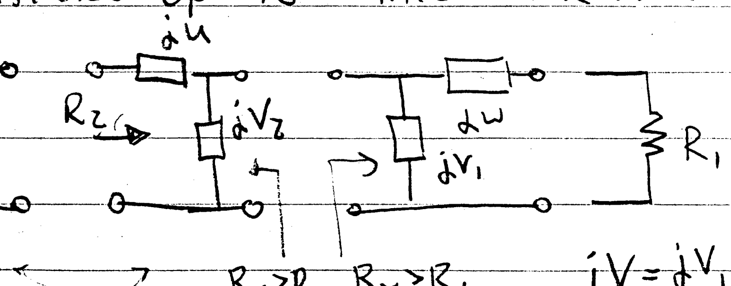
The Π stage can be thought of as two "back-to-back" L sections:



- we use one L section to step R_1 down to an intermediate value R_x (that is less than both R_1, R_2)
- A second L-section steps the intermediate value back up to R_2.

The "T" stage is the dual of the "Π" stage. Again, the sign of one reactance must be opposite that of the other two. Again, it can be thought of as being two back-to-back L-sections, this time stepping resistance "up" to an intermediate value R_y.





$jV = jV_1 \| jV_2$

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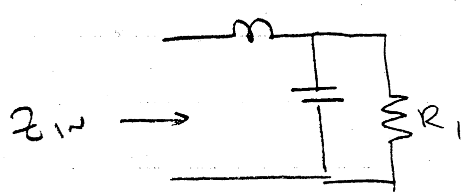
For a given available component Q, an L section will achieve a desired conversion ratio more efficiently than a π or T network. However, a π or T network gives us one additional degree of design freedom we can use to:

- Realize the transformation with smaller or more available components
- Achieve higher frequency selectivity (or Q) of the transformation (useful in communications where signal purity is important.) We can only increase selectivity because we step to larger transformations at the intermediate node.
- We can design the network to achieve a specified phase shift (between the input and output of the matching network).

It is also useful to note how resistance variations affect L-section matching networks. From before



$$Re\{Z_{IN}\} = \frac{R_1}{1+\omega^2 R_1^2 C} = R_2$$

As $R_1 \uparrow$, we find $R_2 \downarrow$ and vice versa

- Similar "inversions" happen in other L-section designs
- T or π networks "double invert", so as $R_1 \uparrow$, $R_2 \uparrow$

$\Rightarrow$ This variation characteristic may lead one to choose a particular number of stages (or L vs T, π)