

Introduction to Simulation - Lecture 1

Example Problems and Basic Equations

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Thanks to Deepak Ramaswamy, Michal Rewienski,
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Outline

- Uses For Simulation
 - Engineering Design
 - Virtual Environments
 - Model Verification
- Course Philosophy
- Example Problems
 - Power distribution on an Integrated Circuit
 - Load bearing on a space frame
 - Temperature distribution in a package

Circuit Analysis

- Equations
 - Current-voltage relations for circuit elements (resistors, capacitors, transistors, inductors), current balance equations
- Recent Developments
 - Matrix-Implicit Krylov Subspace methods

Electromagnetic Analysis of Packages

- Equations
 - Maxwell's Partial Differential Equations
- Recent Developments
 - Fast Solvers for Integral Formulations

Structural Analysis of Automobiles

- **Equations**
 - Force-displacement relationships for mechanical elements (plates, beams, shells) and sum of forces = 0.
 - Partial Differential Equations of Continuum Mechanics
- **Recent Developments**
 - Meshless Methods, Iterative methods, Automatic Error Control

Drag Force Analysis of Aircraft

- Equations
 - Navier-Stokes Partial Differential Equations.
- Recent Developments
 - Multigrid Methods for Unstructured Grids

Engine Thermal Analysis

- Equations
 - The Poisson Partial Differential Equation.
- Recent Developments
 - Fast Integral Equation Solvers, Monte-Carlo Methods

Micromachine Device Performance Analysis

- Equations
 - Elastomechanics, Electrostatics, Stokes Flow.
- Recent Developments
 - Fast Integral Equation Solvers, Matrix-Implicit Multi-level Newton Methods for coupled domain problems.

Stock Option Pricing for Hedge Funds



- Equations
 - Black-Scholes Partial Differential Equation
- Recent Developments
 - Financial Service Companies are hiring engineers, mathematicians and physicists.

Virtual Environments for Computer Games

- Equations
 - Multibody Dynamics, elastic collision equations.
- Recent Developments
 - Multirate integration methods, parallel simulation

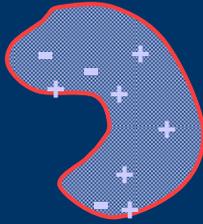
Virtual Surgery

- Equations
 - Partial Differential Equations of Elastomechanics
- Recent Developments
 - Parallel Computing, Fast methods

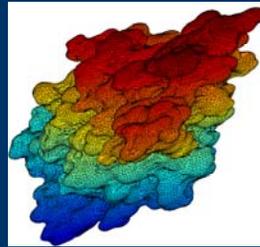
Biomolecule Electrostatic Optimization



Ligand
(drug
molecule)



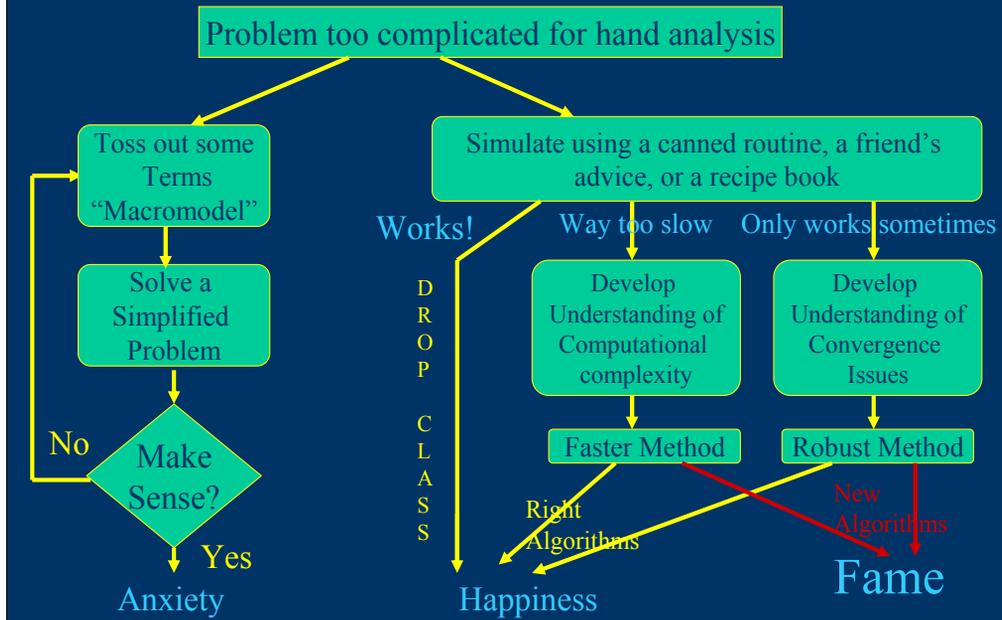
Receptor
(protein
molecule)



Ecm protein

- Equations
 - The Poisson Partial Differential Equation.
- Recent Developments
 - Matrix-Implicit Iterative Methods, Fast Integral Equation Solvers

The Computer Simulation Scenario



Course Philosophy

Examine Several Modern Techniques

Understand, practically and theoretically, how the techniques perform on representative, but real, applications

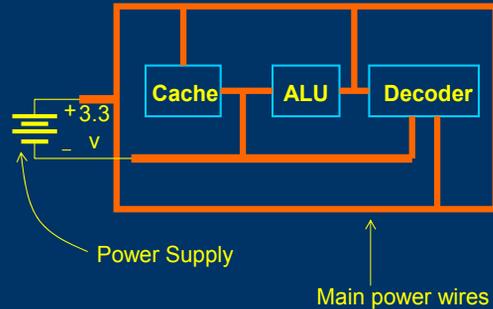
Why Prove Theorems?

Guarantees, given assumptions, that the method will always work.

Can help debug programs.

The theorem proof can tell you what to do in practice.

Power Distribution for a VLSI Circuit



- Is there at least 3v across the ALU ?

One application problem which generates large systems of equations is the problem of distributing power to the various parts of a Very Large Scale Integrated (VLSI) circuit processor.

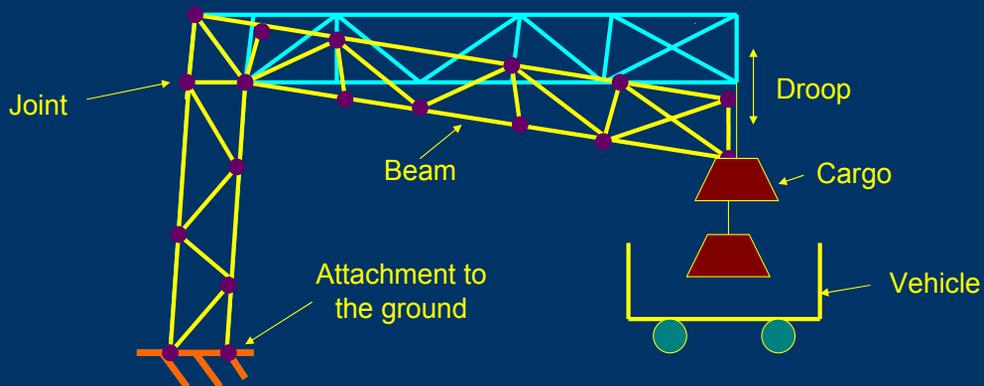
The picture on the left of the slide shows a layout for a typical processor, with different functional blocks noted. The processor pictured has nearly a million transistors, and millions of wires which transport signals and power. All one can really see by eye are the larger wires that carry power and patterns of wires that carry signals, boolean operations such as “and” and “or” .

A typical processor can be divided into a number of functional blocks, as diagrammed on the layout on the left. There are caches, which store copies of data and instructions from main memory for faster access. There are execution units which perform boolean and numerical operations on data, such as “and”, “or” , addition and multiplication. These execution units are often grouped together and referred to as an Arithmetic Logic Unit (ALU). Another main block of the processor is the instruction decoder, which translates instructions fetched from the cache into actions performed by the ALU.

On the right is a vastly simplified diagram of the processor, showing a typical 3.3 volts power supply, the 3 main functional blocks, and the wires (in red) carrying power from the supply to the 3 main functional blocks. The wires, which are part of the integrated circuit, are typically a micron thick, ten microns wide and thousands of microns long (a micron is a millionth of an inch). The resistance of these thin wires is significant, and therefore even though the supply is 3.3 volts, these may not be 3.3 volts across each of the functional blocks.

The main problem we address is whether or not each functional block has sufficient voltage to operate properly.

Load Bearing Space Frame



- Does the Space Frame droop too much under load ! ?

In the diagram is a picture of a space frame used to hold cargo (in red) to be lowered into a vehicle. The space frame is made using steel beams (in yellow) that are bolted together at the purple joints. When cargo is hanging off the end of the space frame, the frame droops.

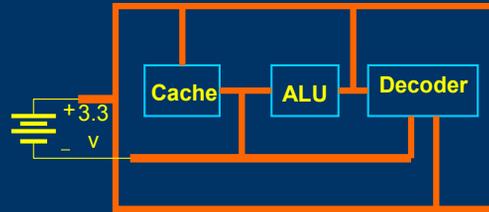
The main problem we will address is how much does the space frame droop under load.

Thermal Analysis

- Does the engine get too hot ! ?

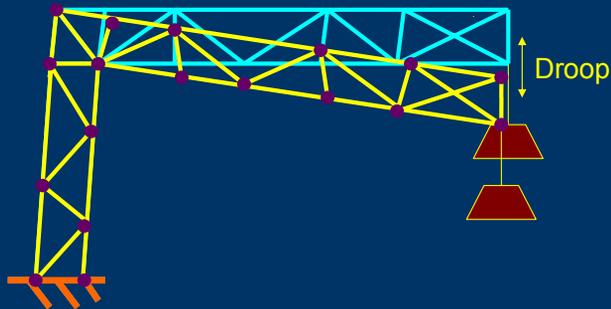
Above is a picture of an engine block, which is typically solid steel or aluminum. The heat generated by the gas burning in the cylinders must be conducted through the engine block to a wide enough surface area that the heat can be dissipated. If not, the engine block temperature will rise too high and the block will melt.

Design Objectives for the VLSI problem



- Select topology and metal widths & lengths so that
 - a) Voltage across every function block > 3 volts
 - b) Minimize the area used for the metal wires

Design Objectives for the Space Frame



Select topology and Strut widths and lengths so that

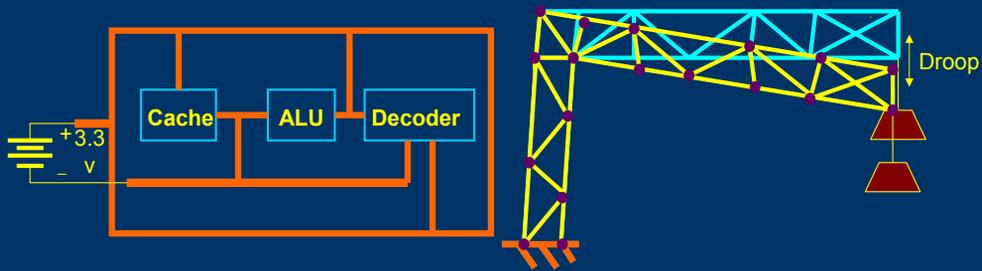
- a) Droop is small enough
- b) Minimize the metal used.

Thermal Analysis

Select the shape so that

- a) The temperature does not get too high
- b) Minimize the metal used.

First Step - Analysis Tools



Given the topology and metal widths & lengths determine

- The voltage across the ALU, Cache and Decoder.
- The droop of the space frame under load.

Who uses VLSI Tools ?

Several big companies

IBM, Motorola, TI, Intel, Compaq, Sony, Hitachi

Non functional prototype costs

- Increases time-to-market
- Design rework costs millions

Once a VLSI circuit is designed, it is fabricated using a sequence of sophisticated deposition and etching processes which convert a wafer of Silicon into millions of transistors and wires. This processing can take more than a month. If the circuit does not function, the design flaw must be found and the fabrication process restarted from the beginning. For this reason, just a few design errors can delay a product for months. In a competitive market, this delay can cost millions in lost revenue in addition to the cost of redesigning the circuit.

In order to avoid fabricating designs with flaws, companies make extensive use of simulation tools to verify design functionality and performance.

Who uses VLSI Tools ?

1000's of small companies

- Small companies make application circuits disk drives, graphics accelerators, CD players, cell phones.
- What is the cost of non-functional prototypes ?
 - Out of business.

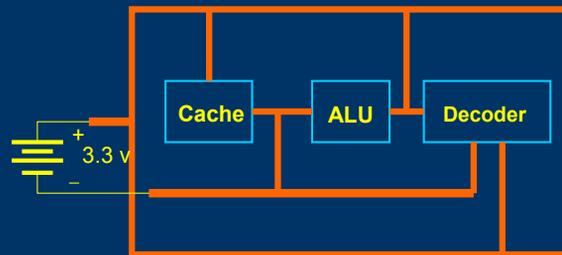
Thousands of small companies design VLSI circuits for applications as diverse as peripherals for personal computers as well as signal processors for audio, video and automotive applications. These small companies cannot afford the cost of fabricating prototype designs that do not function. The very survival of these companies depends on using simulation tools to verify designs before fabrication.

Who makes VLSI Tools ?

Company	employees	sales	Market cap.
Cadence	4,000	1.3 billion	3.8 billion
Synopsis/ Avanti	5,000	1.5 billion	6.9 billion
Mentor Graphics	2,600	.6 billion	1.4 billion

Companies compete by improving analysis efficiency.

Modeling VLSI circuit Power Distribution



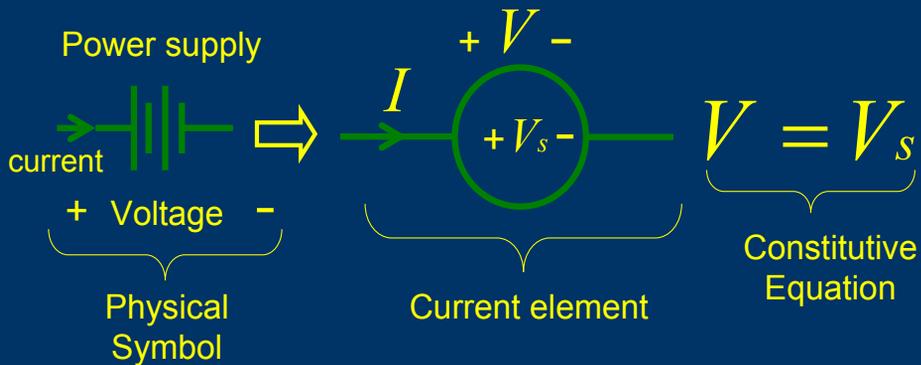
- Power supply provide current at a certain voltage.
- Functional blocks draw current.
- The wire resistance generates losses.

Each of the elements in the simplified layout, the supply, the wires and the functional blocks, can be modeled by relating the voltage across that element to the current that passes through the element. Using these element constitutive relations, we can construct a circuit from which we can determine the voltages across the functional blocks and decide if the VLSI circuit will function.

Modeling the Circuit

Supply becomes

A Voltage Source

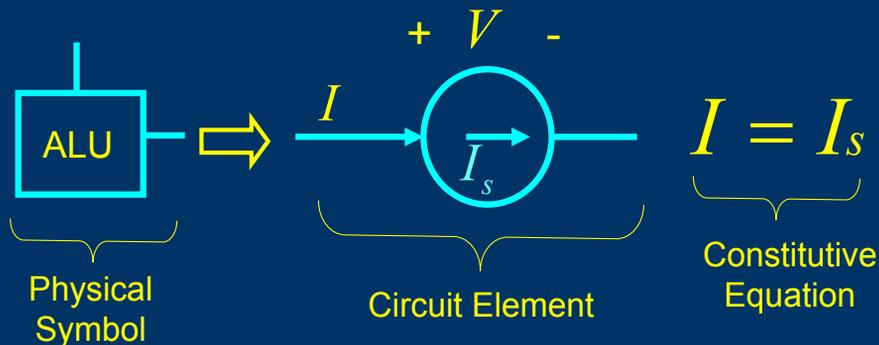


The power supply provides whatever current is necessary to ensure that the voltage across the supply is maintained at a set value. Note that the constitutive equation (in the figure), which is supposed to relate element voltage (V) to element current (I) does not include current as a variable. This should not be surprising since voltage is always maintained regardless of how much current is supplied, and therefore knowing the voltage tells one nothing about the supplied current.

Modeling the Circuit

Functional blocks become

Current Sources

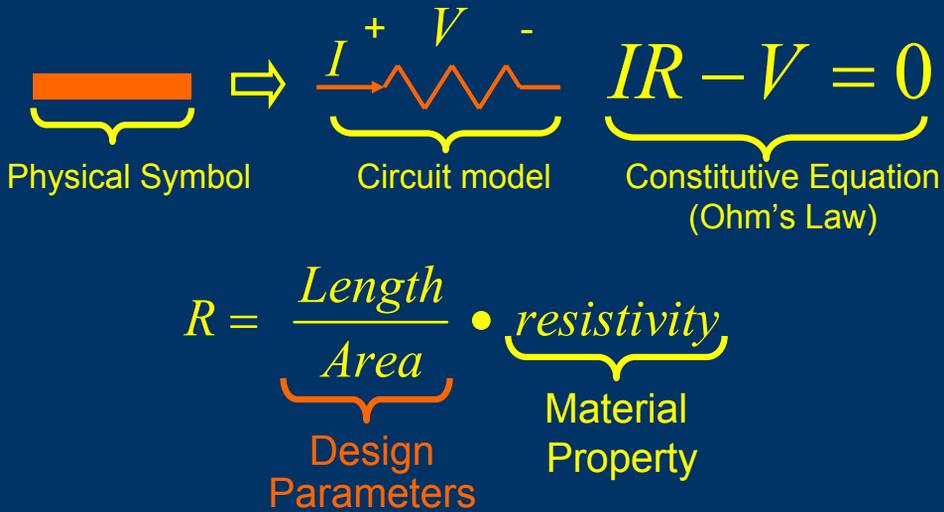


The functional blocks, the ALU, the cache and the decoder are complicated circuits containing thousands of transistors. In order to determine whether the functional block will always have a sufficient voltage to operate, a simple model must be developed that abstracts many of the operating details. A simple “worst-case” model is to assume that each functional block is always drawing its maximum current. Each block is therefore modeled as a current source, although one must assume that the associated currents have been determined by analyzing each functional block in more detail. Note that once again the constitutive equation is missing a variable, this time it is voltage. Since a current source passes the same current independent of the voltage across the source, that V is missing should be expected.

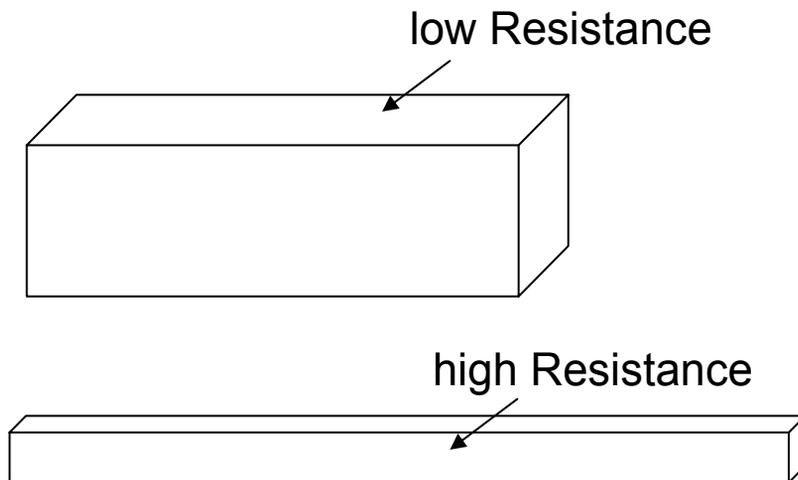
Modeling the Circuit

Metal lines become

Resistors



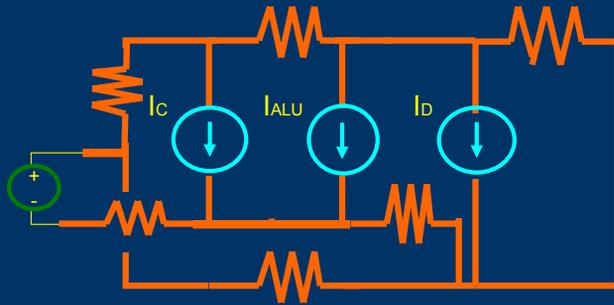
The model for the wires connecting the supply to the functional blocks is a resistor, where the resistance is proportional to the length of the wire (the current has further to travel) and inversely proportional to the wire cross-sectional area (the current has more paths to choose).



That the current through a resistor is proportional to the voltage across the resistor is Ohm's law.

Modeling VLSI Power Distribution

Putting it all together



- Power Supply \Rightarrow voltage source
- Functional Blocks \Rightarrow current sources
- Wires become resistors

Result is a **schematic**

To generate representation which can be used to determine the voltages across each of the functional units, consider each of the models previously described.

First, replace the supply with a voltage source.

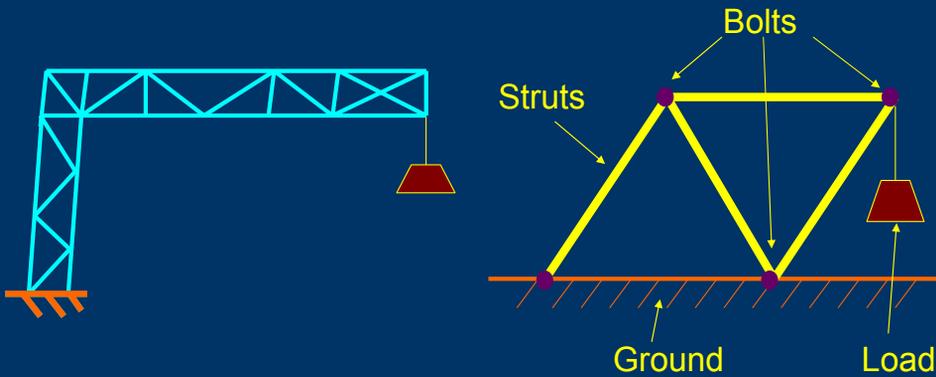
Second, replace each functional block with an associated current source.

Third, replace each section of wire with a resistor.

Note that the resistors representing the wires replace a single section with no branches, though the section can have turns.

The resulting connection of resistors, current sources and voltage sources is called a circuit schematic. Formulating equations from schematics will be discussed later.

Modeling the Space Frame



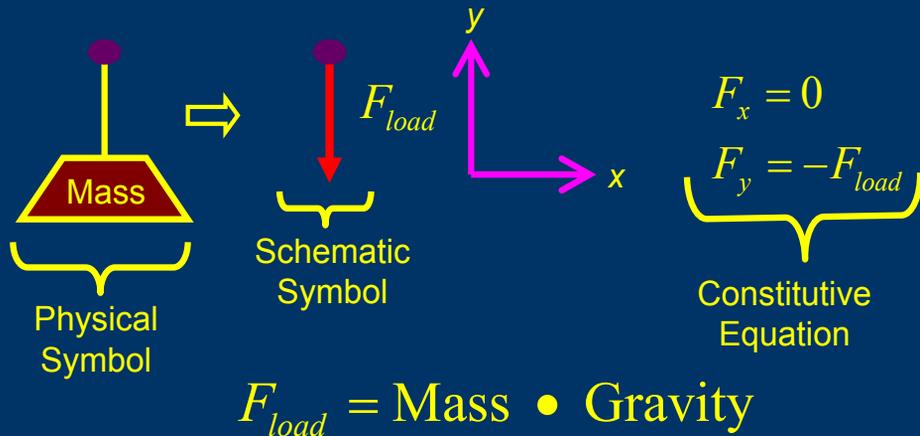
Example is simplified for illustration

In order to examine the space frame, we will consider a simplified example with only four steel beams and a load. Recall that the purple dots represent the points where steel beams are bolted together. Each of the elements in the simplified layout, the beams and the load, can be modeled by relating the relative positions of the element's terminals to the force produced by the element. Using these element constitutive relations, we can construct a schematic from which we can determine the frame's droop.

Modeling the Frame

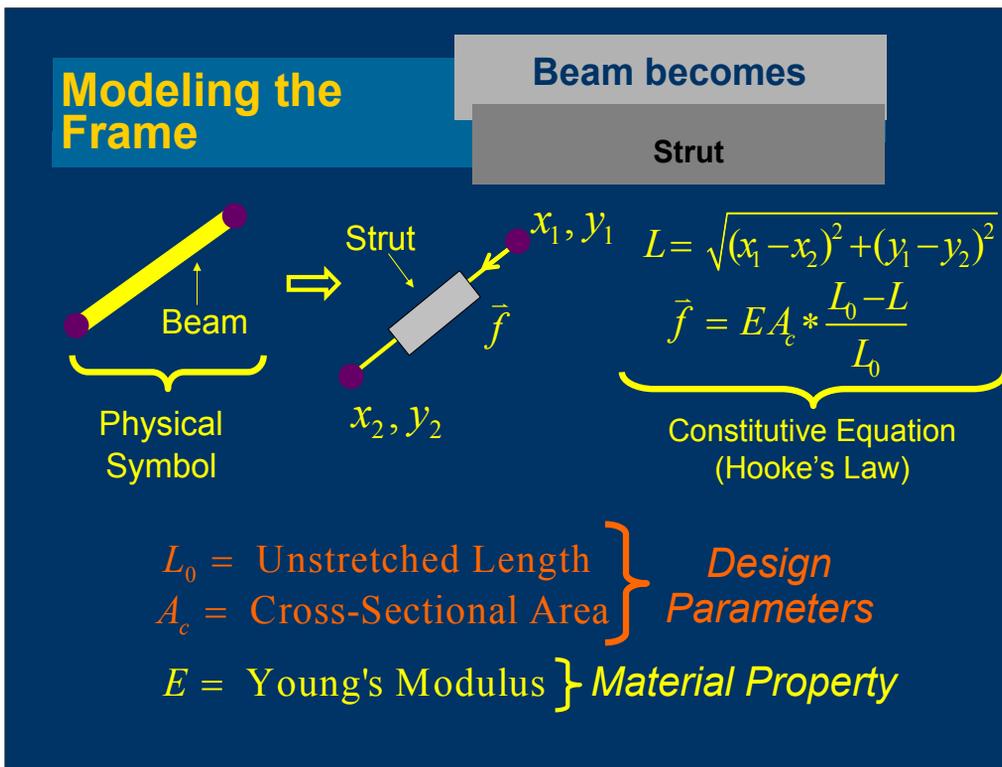
Load becomes

Force Source



The load is modeled as a force pulling in the negative Y direction (Y being vertical, X being horizontal).

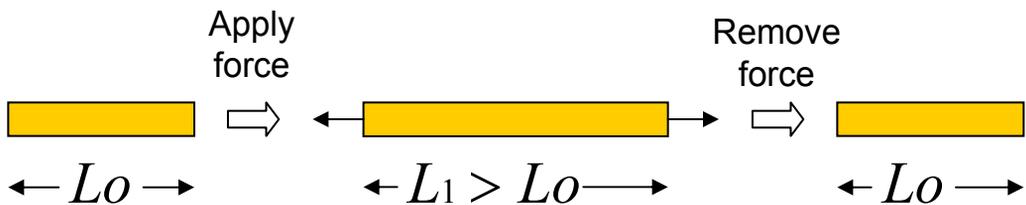
Note that the constitutive equation does not include the variable for the load's position, following from the fact that the load's force is independent of position.



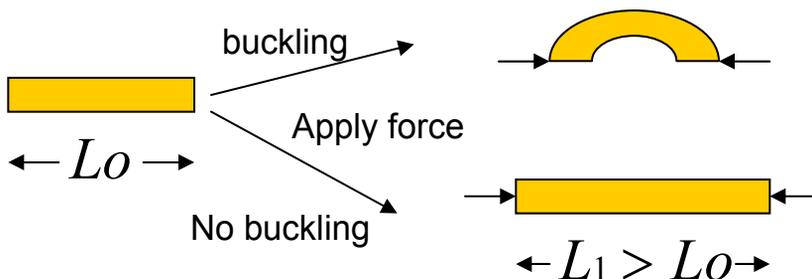
In order to model the steel beams in a space frame, it is necessary to develop a relation between the beam deformation and the restoring force generated by the beam. To derive a formula we will make several assumptions.

- 1) The beam is perfectly elastic.

This means that if one deforms the beam by applying a force, the beam always returns to its original shape after the force is removed.



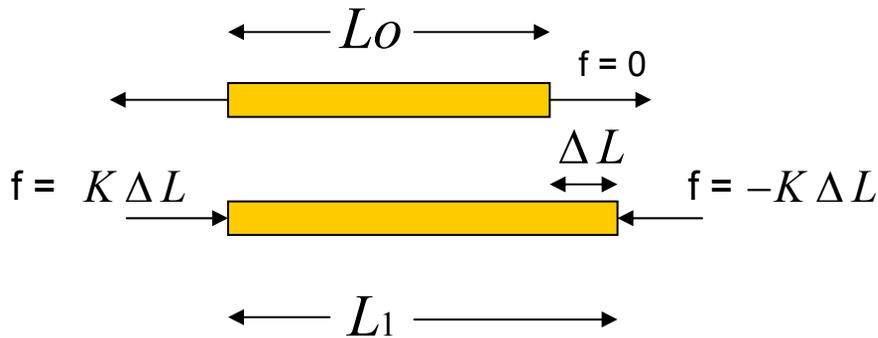
- 2) The beam does not buckle



Buckling is an important phenomenon and ignoring it limits the domain of applicability of this model.

3) The beam is materially linear.

For a beam to be materially linear, the force which acts along the beam is directly proportional to the change in length.



To determine K consider that the force required to stretch a beam an amount

ΔL is
(I) Inversely proportional to its unstretched length (It is easier to stretch a 10 inch rubber band 1 inch than to stretch a 1 inch rubber band 1 inch)

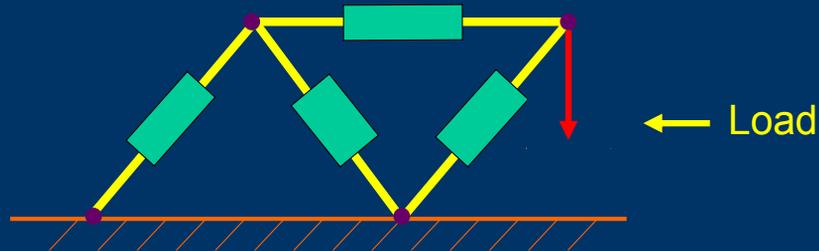
(II) Directly proportional to its cross-sectional area (Imagine 10 rubber bands in parallel)

(III) Dependent on the material (Rubber stretches more easily than steel).

Combining (I), (II) and (III) leads to the formula at the bottom of the slide.

Modeling the Frame

Putting it all together



How much does the load droop?

To generate a representation which can be used to determine the displacements of the beam joints, consider the models previously described.

First, replace the loads with forces.

Second, replace each beam with strut.

Formulating Equations from Schematics

Two Types of Unknowns

Circuit - Node voltages, element currents

Struts - Joint positions, strut forces

Two Types of Equations

Conservation Law Equation

Circuit - Sum of Currents at each node = 0

Struts - Sum of Forces at each joint = 0

Constitutive Equation

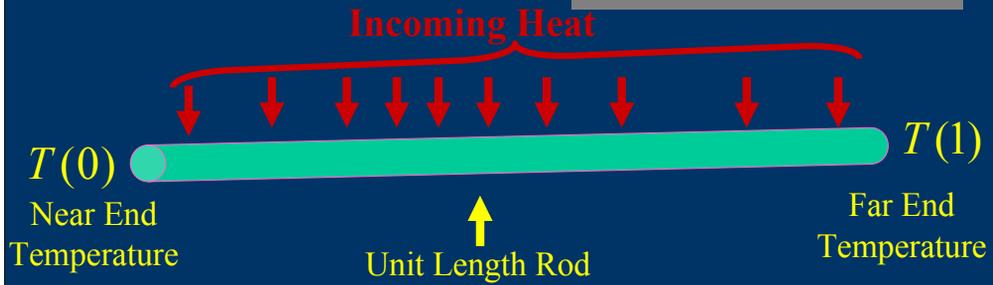
Circuit - element current is related to voltage across the element

Struts - element force is related to the change in element length

Conservation Laws and Constitutive Equations

Heat Flow

1-D Example

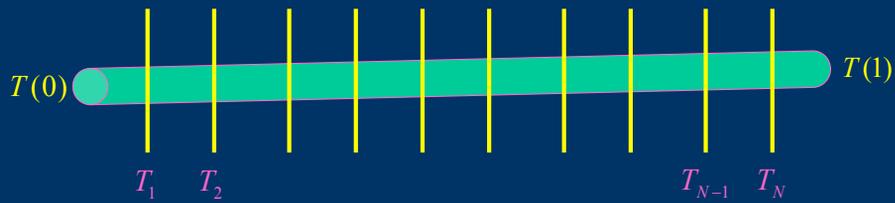


Question: What is the temperature distribution along the bar

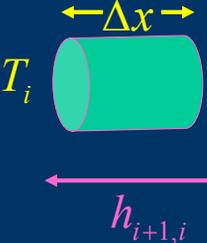


1) Cut the bar into short sections

2) Assign each cut a temperature



Heat Flow through one section

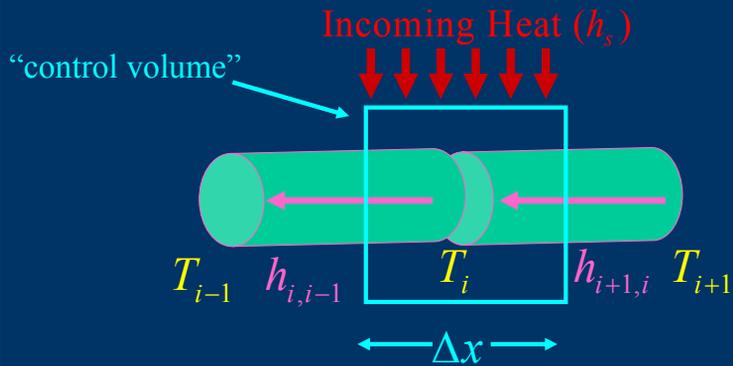


T_i T_{i+1} $h_{i+1,i} = \text{heat flow} = \kappa \frac{T_{i+1} - T_i}{\Delta x}$

Limit as the sections become vanishingly small

$$\lim_{\Delta x \rightarrow 0} h(x) = \kappa \frac{\partial T(x)}{\partial x}$$

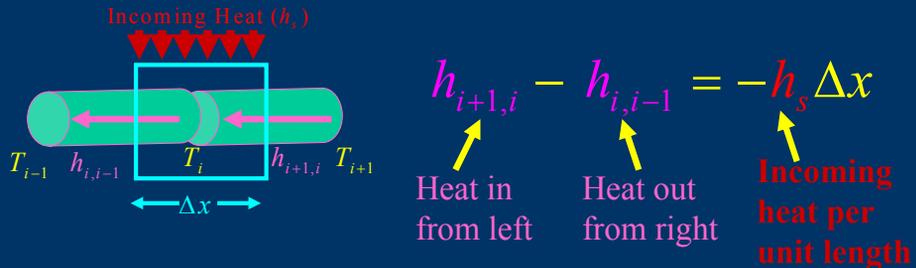
Two Adjacent Sections



Net Heat Flow into Control Volume = 0

SMA-HPC ©2003 MIT $h_{i+1,i} - h_{i,i-1} = -h_s \Delta x$

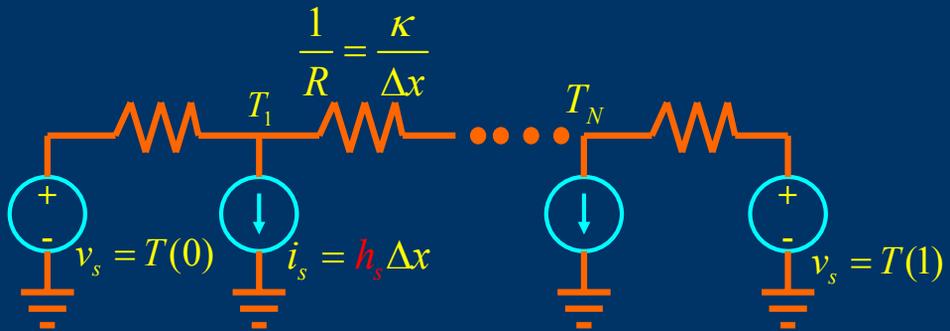
Net Heat Flow into Control Volume = 0



Limit as the sections become vanishingly small

$$\lim_{\Delta x \rightarrow 0} h_s(x) = \frac{\partial h(x)}{\partial x} = \frac{\partial}{\partial x} \kappa \frac{\partial T(x)}{\partial x}$$

Temperature analogous to Voltage
Heat Flow analogous to Current



Formulating Equations

Two Types of Unknowns

- Circuit - Node voltages, element currents
- Struts - Joint positions, strut forces
- Conducting Bar – Temperature, section heat flows

Two Types of Equations

Conservation Law Equation

- Circuit - Sum of Currents at each node = 0
- Struts - Sum of Forces at each joint = 0
- Bar – Sum of heat flows into control volume = 0

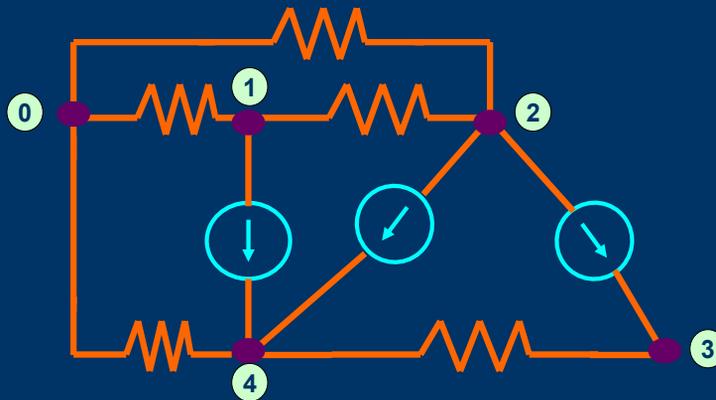
Constitutive Equations

- Circuit – element current related to voltage
- Struts - strut force related to length change
- Bar – section temperature drop related to heat flow

Formulating Equations from Schematics

Circuit Example

Identifying Unknowns

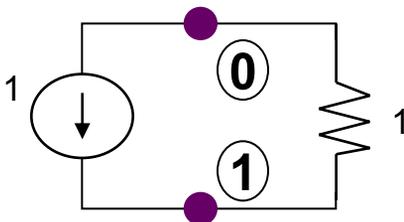


Assign each node a voltage, with one node as 0

Given a circuit schematic, the problem is to determine the node voltages and element currents. In order to begin, one needs labels for the node voltages, and therefore the nodes are numbered zero, one, two, ... N , where $N+1$ is the total number of nodes.

The node numbered zero has a special meaning, it is the reference node. Voltages are not absolute quantities, but must be measured against a reference.

To understand this point better, consider the simple example of a current source and a resistor.

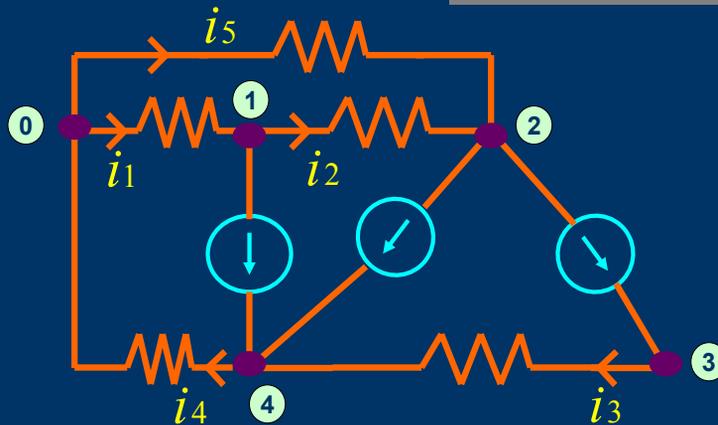


In order for one Amp to flow through the resistor, $V_1 - V_0$ must equal one volt. But does $V_1 = 11$ volts and $V_0 = 10$ volts Or is $V_1 = 101$ volts and $V_0 = 100$ volts ? It really does not matter, what is important is that V_1 is one volt higher than V_0 . So, let V_0 define a reference and set its value to a convenient number, $V_0 = 0$.

Formulating Equations from Schematics

Circuit Example

Identifying Unknowns



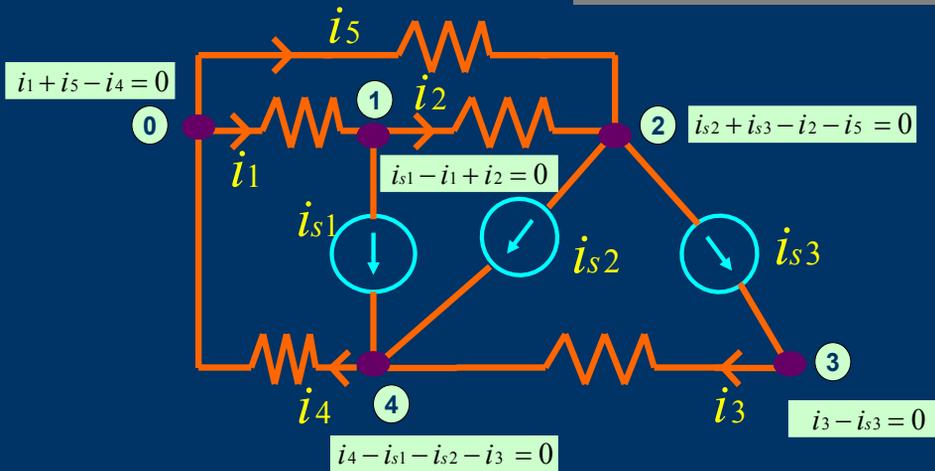
Assign each element except current sources a current

The second set of unknowns are the element currents. Obviously, the currents passing through current sources are already known, so one need only label the currents through resistors and voltage sources. The currents are denoted i_1, i_2, \dots, i_b , where b is the total number of unknown element currents. Since elements connect nodes, in an analogy with graphs, element currents are often referred to as branch currents.

Formulating Equations from Schematics

Circuit Example

Conservation Law



Sum of currents = 0 (Kirchoff's current law)

The conservation law for a circuit is that the sum of currents at each node equals zero. This is often referred to as Kirchoff's current law. Another way to state this law, which more clearly indicates its conservation nature is to say

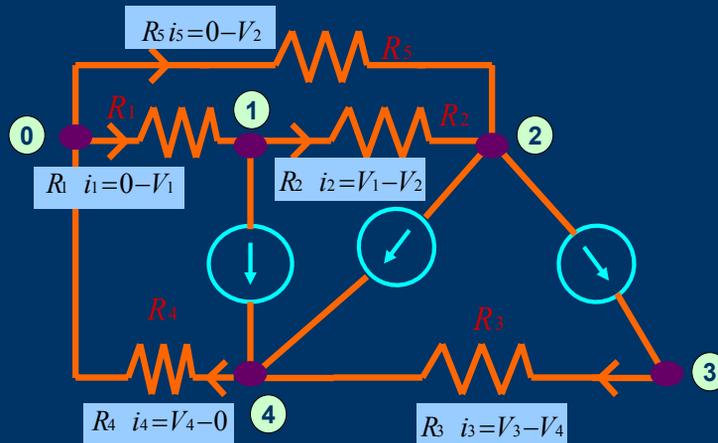
Any current entering a node must leave the node.

The conservation is that no current is lost, what comes in goes out. The green statement also makes it clear that the direction of the current determines its sign when summing the currents. Currents leaving the node are positive terms in the sum and currents entering the node are negative terms (one can reverse this convention but one must be consistent).

Formulating Equations from Schematics

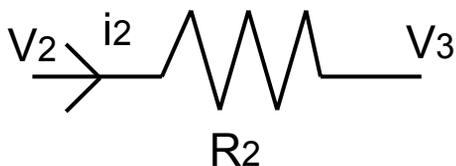
Circuit Example

Constitutive Equations



Use Constitutive Equations to relate branch currents to node voltages (Currents flow from plus node to minus node)

Each element with an unknown branch current has an associated constitutive equation which relates the voltage across the element to the current through the element. For example, consider R_2 in the figure,



The constitutive relation for a resistor is Ohm's law.

$$\frac{1}{R} V = I$$

And in this case $V = V_2 - V_3$ and $I = i_2$.

One should again take note of the direction of the current. If current travels from left node through the resistor to the right node, then the left node voltage will be higher than the right node voltage by an amount $R I$.

Formulating Equations from Schematics

Circuit Example

Summary

Unknowns for the Circuit example

Node voltages (except for a reference)

Element currents (except for current sources)

Equations for the Circuit example

One conservation equation (KCL) for each node
(except for the reference node)

One constitutive equation for each element
(except for current sources)

Note that **# of equations = # of unknowns**

Summary of key points

Many Applications of simulation

Picked Three Representative Examples

Circuits, Struts and Joints, Heat Flow in Bar

Two Types of Equations

Conservation Laws

Circuit - Sum of Currents at each node = 0

Struts - Sum of Forces at each joint = 0

Bar - Sum of heat flows into control volume = 0

Constitutive Equation

Circuit – current-voltage relationship

Struts - force-displacement relationship

Bar - temperature drop-heat flow relationship