Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
Harvard-MIT Division of Health Science and Technology
6.551J/HST714J Acoustics of Speech and Hearing

Issued Thursday 14-Oct-2004
Due Thursday 21-Oct-2004

Problem 1. Acoustic circuit elements, (Fletcher , p. 128-141; Kinsler et al. p. 231-233; Beranek, p. 128-139)

This problem specifies conditions that allow representation of some acoustic structures by simple lumped elements by developing relationships between structural dimensions and material properties.

a. Acoustic mass

Consider a rigid tube of length *l* and cross-sectional area *A* filled with a fluid of mass density ρ_0. Rigid pistons at each end of the tube move with identical volume velocities *u(t).* Assume that the velocity and mass density are uniform throughout the tube. Assume that the effects of viscosity are ignorable.

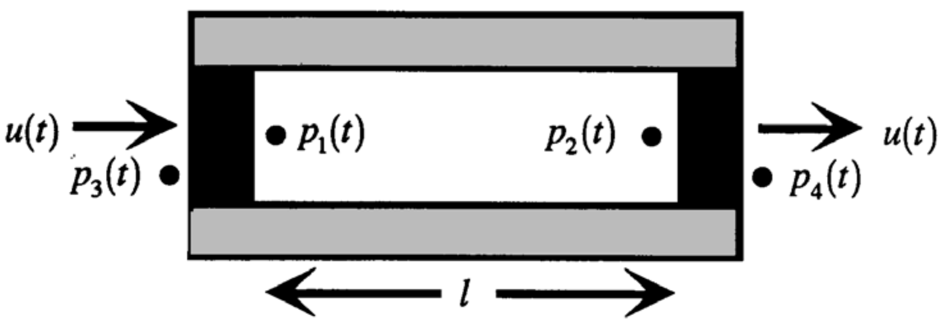

Figure 1: A Uniform Tube of Cross-sectional area *A*

(i) Use Newton's second law to show that the pressure difference between the ends of the tube can be expressed as:

$$p_1(t) - p_2(t) = M^A \frac{du(t)}{dt}, \quad (4.1)$$

where: $M^A = \frac{\rho_0 l}{A} = \frac{M^M}{A^2}$ is the acoustic mass, and

M^M is the mechanical mass of the fluid in the tube.

(ii) In a few sentences explain the relationship between acoustic mass M^A and mechanical mass M^M.

(iii) If each piston has a mechanical mass M_P^M and moves without friction in the tube, determine a relation between $u(t)$ and the pressure difference between the outer surfaces of the pistons $p_3(t)$-$p_4(t)$.

c. Acoustic compliance.

Structures that may be well approximated as acoustic compliances (i.e. volume displacements proportional to sound pressure) include diaphragms (as in microphones, drum heads, tympanic membranes, loudspeaker cones) and volumes of air enclosed by rigid walls (as in nasal sinuses, the mouth, loudspeaker enclosures and middle-ear cavities).

(i.) Loudspeaker cones are often designed to be non-uniform in their elastic properties, so that the central portion is rigid and a narrow annular region at the edge is relatively flexible. Assume that the width of the annular, flexible region is much less than the cone radius *a* and that the stiffness per unit length of the annulus (along the circumference) is *k* Newtons/m^2. Determine the acoustic compliance (i.e. volume displacement per unit pressure difference across the cone) of this structure in terms of *k* and *a* .

(ii.) Consider a rigid walled container of fluid in which the pressure $p_T(t)$ is uniform throughout the volume *V* (Figure 2). A rigid piston (in black) moves into the container with a volume velocity (i.e. volume displacement per time) $u(t)$. Define the acoustic impedance of the closed air space.

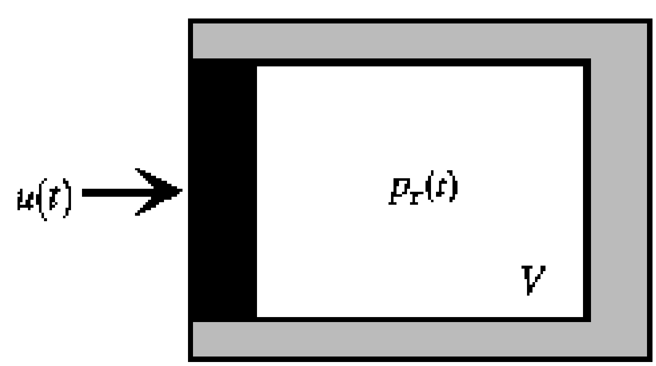

Figure 2: A closed cavity

(iii), Assume the volume of the closed cavity in figure 2 is 1 cc (1 cubic centimeter). Also assume that the piston has a surface area of 1 cm^2, and that the piston displaces in and out in a sinusoidal motion with an amplitude of 1 micrometer ($1x10^{-6}$ meter) and a frequency of 100 Hz. Sketch a drawing of the displacement of the piston in time (you can assume a phase that is convenient). Also sketch drawings of the resultant volume velocity of the piston and the sound pressure within the cavity that results from the piston motion. While your sketches need not be exact they do need to clearly show the limits on the time axis, as well as the amplitude and relative phases of the displacement, volume velocity and sound pressure waveforms.

Problem 2: An Acoustic Filter

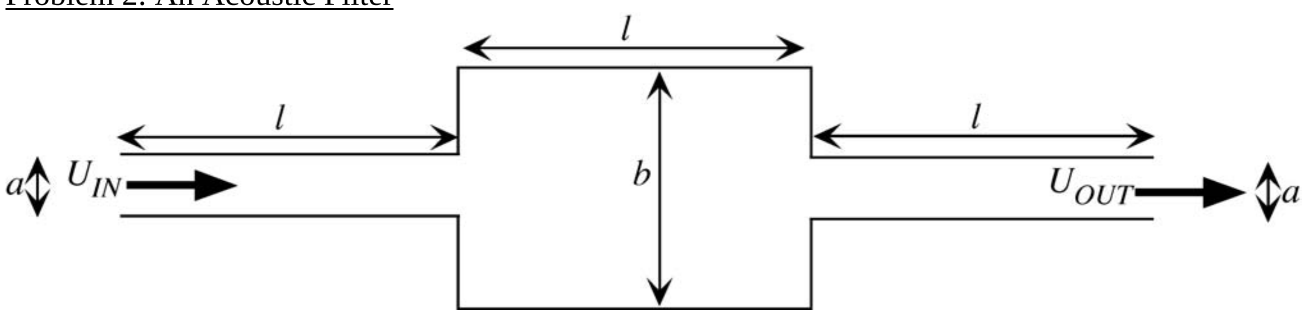

Figure 2

Consider the three coupled cylindrical tubes of Figure 2: where $l = b = 5$ cm and $a = 1$ cm.

i) Construct an electrical analog circuit model of the acoustic system in Figure 2 consisting of three lossless (assume viscosity is negligible) elements.

ii) Write an equation that relates U_{OUT} to U_{IN}. How are U_{OUT} and U_{IN} related when ω is very small? How are U_{OUT} and U_{IN} related when ω is very large?

iii) This system is an acoustic filter (like an automobile's muffler) that passes low frequencies and attenuates high frequencies. Does this fit with your results?

Problem 3: Energy Storage

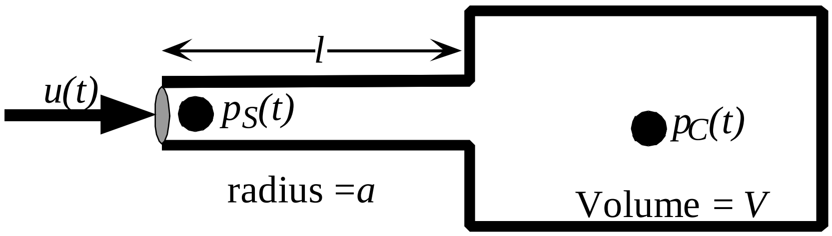

Figure 3.

In Lecture 10 we found that the input impedance of the bottle in Figure 3 could be approximated by the equation below if we assumed losses were negligible:

$$\frac{P_s(s)}{U} = Z_A(s) \approx sM_A + \frac{1}{sC_A} \quad .$$

We also defined an impedance zero at $\omega_0 \approx 1/\sqrt{C_A M_A}$, as well as

the potential energy stored in the compliance, $E_C(t) = \frac{1}{2} C_A p_C^2(t)$, and

the kinetic energy stored in the acoustic mass, $E_M(t) = \frac{1}{2} M_A u^2(t)$.

Using the dimensions used to calculate the Bode plots on page 8 of the lecture 10 notes:

Vol = 0.5 liter = 0.5×10^{-3} m^3 — l = 5 cm = 0.05 m; a =0.005 cm

C_A=Vol/(1.4×10^5)=3.6×10^{-9} Pa/m^3 — M_A=1.18*0.05/(πa^2)=750 kg/m^4

and assuming $u(t)$=1 mm^3/s cos(ωt),

(i) What is the peak value of $E_C(t)$ and $E_M(t)$ at 20 Hz (well below the resonant frequency)

(ii) at 500 Hz (well above the resonant frequency), and

(iii) at 110 Hz (just above the resonant frequency).

(iv) In a paragraph discuss the results of your computations in terms of your understanding of the frequency dependence of the impedance and resonance.

Problem 4: The ideal transformer

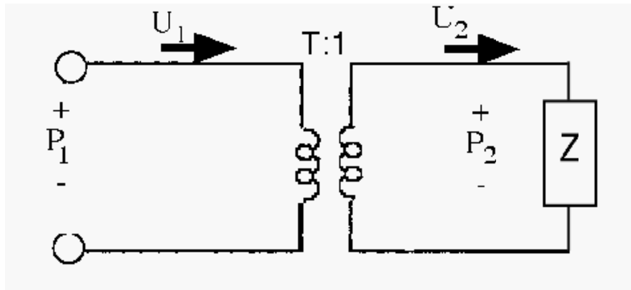

Figure 4:

Consider the ideal 'acoustic transformer' of the analog circuit in figure 4, where $\underline{U}_1$, $\underline{P}_1$, $\underline{U}_2$ and $\underline{P}_2$ are complex amplitudes describing the sinusoidal steady state, and $\underline{P}_2$= 1 Pa.

(i) Define $\underline{U}_2$ in terms of $\underline{P}_2$ and $\underline{Z}$.

(ii) Define $\underline{U}_1$ and $\underline{P}_1$ in terms of $\underline{U}_2$, $\underline{P}_2$ and T.

(iii) Write an equation describing the average power on the right-hand side of the transformer in terms of $\underline{U}_2$ and $\underline{P}_2$.

(iv) Write an equation describing the average power on the left-hand side of the transformer in terms of $\underline{U}_2$ and $\underline{P}_2$.

(v) In a few sentences compare your answers to (iii) and (iv) and comment on power absorption by an ideal transformer.