# MASSACHUSETTS INSTITUTE OF TECHNOLOGY <br> Department of Electrical Engineering and Computer Science 

6.630 Electromagnetics

Quiz No. 2
Time: $\quad 3: 00 \mathrm{pm}-5: 00 \mathrm{pm}$
Problem 1 (8\%)
Close one end of a charged transmission line at $t=0$ with $V(z)=1$ as shown in Fig. 1. Determine $V(z)$ at times $t=0, \frac{\ell}{2 v}, \frac{\ell}{v}, \frac{3 \ell}{2 v}$.


Fig. 1

## Problem 2 (6\%)

Find the radiation pattern for the six-dipoles array as shown in Fig. 2. What are the unit pattern, group pattern, and resultant pattern?
$\frac{\lambda}{2} \quad \frac{\lambda}{2}$
$\lambda$


Problem 3 (10\%)
Consider the periodic structure shown in Fig. 3. Find $\mu_{e f f}$. When is $\mu_{e f f}<0$ ?


## Problem 4 (20\%)

The result of a measurement of the voltage standing wave pattern on a transmission line with characteristic impedance $Z_{o}=100 \Omega$ is shown in Fig. 4.
(a) What is the wavelength $\lambda$.
(b) Calculate the VSWR.
(c) Calculate the reflection coefficient $\Gamma_{L}$.
(d) Determine the load impedance $Z_{L}$.


Fig. 4

## Problem 5 (30\%)

Consider a $\frac{\lambda}{4}$ long transmission line, with characteristic impedance $50 \Omega$, as shown in Fig. 5. One end is connected to a voltage source $V_{g}=100 \sin \omega t$, which has a source impedance $Z_{g}=50 \Omega$, while the other end connected to a load impedance $Z_{L}=j 50 \Omega$.
(a) Write out the complex expression for $V(z), I(z)$.
(b) Solve the instantaneous power and time-averaged power dissipated in $Z_{L}$.
(c) Find $Z_{A}$, which is the input impedance at $z=-\lambda / 4$.


Fig. 5

## Problem 6 (26\%)

Cosider a perfectly conducting parallel-plate waveguide with the plates seperated by $d$. The guided TM waves propagate in the $\hat{z}$ direction. The operating frequency is 10 GHz .

$z=0$


Fig. 6
(a) What's the relationship between $d$ and the highest TM mode which can be guided in this waveguide?
(b) If $d$ is reduced to ensure that only one TM mode exists in this waveguide, write down the condition for $d$.
(c) Under the condition where only one TM mode is propagating in this waveguide, the diffraction pattern is shown on a screen at $z=\ell(\ell \gg d)$. The first null on the screen is at $x=s$. Write out $s$ in terms of $d$ and $\ell$.

