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6.641 Electromagnetic Fields, Forces, and Motion  
Spring 2005

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## Problem Set 7 - Questions

**Problem 7.1 (W&M Prob 3.3)**

A slab of dielectric slides between plane parallel electrodes as shown. The dielectric obeys the constitutive law  $D = \alpha (E \cdot E) E + \varepsilon_0 E$ , where  $\varepsilon_0$  is the permittivity of free space and  $\alpha$  is a constant. Find the force of electrical origin on the slab. Your answer should take the form  $f^e = f^e(v, x)$ .

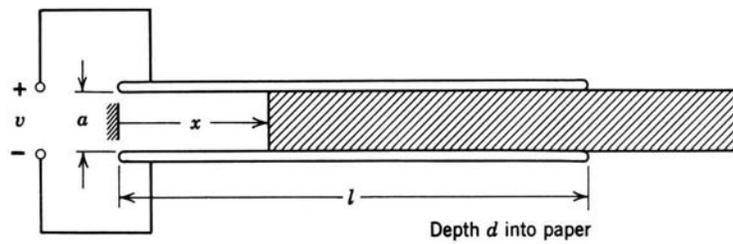
**Fig. 3P.3**

Figure 1: A slab of dielectric sliding between parallel electrodes

Courtesy of Herbert Woodson and James Melcher. Used with permission.  
 Woodson, Herbert H., and James R. Melcher. *Electromechanical Dynamics, Part 1: Discrete Systems*. Malabar, FL: Kreiger Publishing Company, 1968. ISBN: 9780894644597.

### Problem 7.2 (W&M Prob. 3.16)

A plane electrode is free to move into the region between plane-parallel electrodes, as shown in Fig. 3P.16. The outer electrodes are at the same potential, whereas the inner electrode is at a potential determined by the constant voltage source  $V_0$  in series with the output of an amplifier driven by a signal proportional to the displacement of the movable electrode itself. Hence the voltage of the inner electrode relative to that of the outer electrodes is  $v = -V_0 + Ax$ , where  $A$  is a given feedback gain. Find the force of electrical origin  $f^e(x)$ . (Note that this force is only a function of position, since the voltage is a known function of  $x$ .)

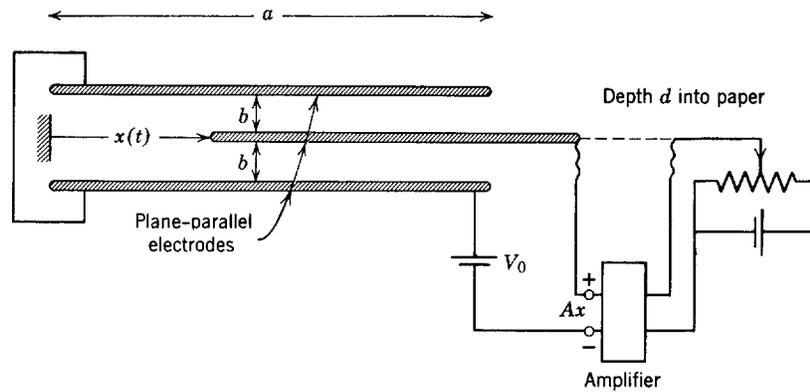


Fig. 3P.16

Figure 2: A plane electrode with position dependent voltage  $v = -V_0 + Ax$  free to move between grounded plane-parallel electrodes.

Courtesy of Herbert Woodson and James Melcher. Used with permission.  
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### Problem 7.3 (W&M Prob. 3.4)

A magnetic circuit, including a movable plunger, is shown in Fig. 3P.4. The circuit is excited by an  $N$ -turn coil and consists of a perfectly permeable yoke and plunger with a variable air gap  $x(t)$  and a fixed nonmagnetic gap  $d$ . The system, with the cross section shown, has width  $w$  into the paper. The following parts lead to a mathematical formulation of the equations of motion for the mass  $M$ , given the excitation  $I(t)$ .

**A**

Find the terminal relation for the flux  $\lambda(i, x)$  linked by the electrical terminal pair. Ignore fringing in the nonmagnetic gaps. Note that the coil links the flux through the magnetic material  $N$  times.

**B**

Find the energy  $W_m(\lambda, x)$  stored in the electromechanical coupling. This should be done by making use of part (a).

**C**

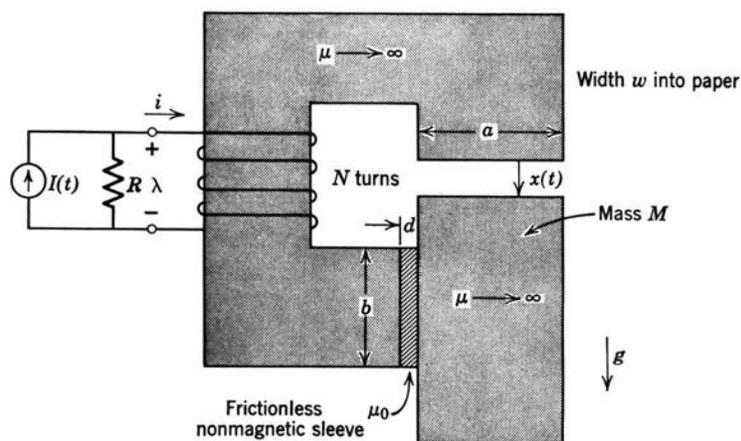
Use the energy function  $W_m(\lambda, x)$  to compute the force of electrical origin  $f^e$  acting on the plunger.

**D**

Write an electrical (circuit) equation of motion involving  $\lambda$  and  $x$  as the only dependent variables and  $I(t)$  as a driving function.

**D**

Write the mechanical equation of motion for the mass. This differential equation should have  $\lambda$  and  $x$  as the only dependent variables, hence taken with the result of (d) should constitute a mathematical formulation appropriate for analyzing the system dynamics.



**Fig. 3P.4**

Figure 3: A magnetic circuit including a movable plunger

Courtesy of Herbert Woodson and James Melcher. Used with permission.  
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### Problem 7.4 (W&M Prob. 3.9)

The electric terminal variables of the electromechanical coupling system shown in Fig. 3P.9 are known to be  $\lambda_1 = ax_1^2i_1^3 + bx_2^2x_1i_2$  and  $\lambda_2 = bx_2^2x_1i_1 + cx_2^2i_2^3$ , where  $a, b,$  and  $c$  are constants.

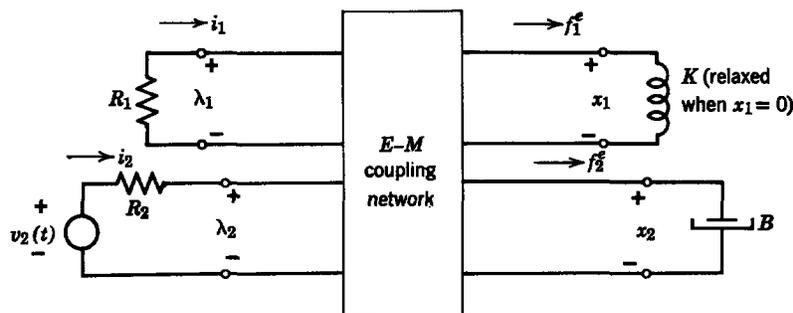


Fig. 3P.9

Figure 4: An electromechanical coupling network

**A**

What is the coenergy  $W'_m(i_1, i_2, x_1, x_2)$  stored in the coupling network?

**B**

Find the forces  $f_1^e$  and  $f_2^e$ .

**C**

Write the *complete* set of equations for the system with the terminal constraints shown.

Courtesy of Herbert Woodson and James Melcher. Used with permission.  
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### Problem 7.5 (W&M Prob. 5.11)

Figure 5P.11 shows a diagrammatic cross section of a two-phase, salient-pole synchronous machine. The windings in an actual machine are distributed in many slots along the periphery of the stator, rather than as shown. The rotor is made of magnetically soft iron which has no residual permanent magnetism. The

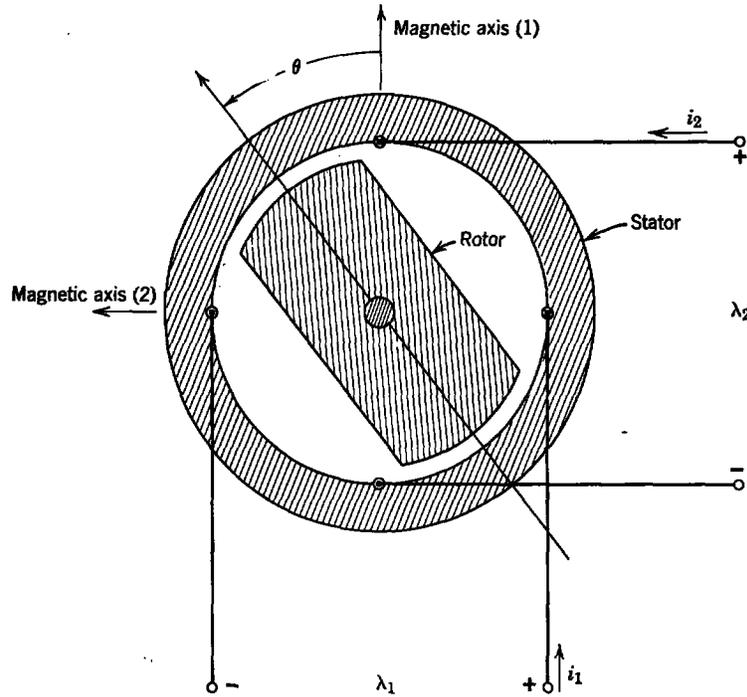


Fig. 5P.11

Figure 5: Cross section of a two-phase, salient-pole synchronous machine

electrical terminal relations are

$$\lambda_1 = (L_0 + M \cos 2\theta)i_1 + M \sin 2\theta i_2,$$

$$\lambda_2 = M \sin 2\theta i_1 + (L_0 - M \cos 2\theta)i_2.$$

**A**

Determine the torque of electrical origin  $T^e(i_1, i_2, \theta)$ .

**B**

Assume that the machine is excited by sources such that  $i_1 = I \cos \omega_s t, i_2 = I \sin \omega_s t$ , and the rotor has the constant angular velocity  $\omega_m$  such that  $\theta = \omega_m t + \gamma$ . Evaluate the instantaneous torque  $T^e$ . Under what conditions is it constant?

**C**

The rotor is subjected to a mechanical torque (acting on it in the  $+\theta$ -direction):  $T = T_0 + T'(t)$ , where  $T_0$  is a constant. The time-varying part of the torque perturbs the steady rotation of (b) so that  $\theta = \omega_m t + \gamma_0 + \gamma'(t)$ .

Assume that the rotor has a moment of inertia  $J$  but that there is no damping. Find the possible equilibrium angles  $\gamma_0$  between the rotor and the stator field. Then write a differential equation for  $\gamma'(t)$ , with  $T'(t)$  as a driving function.

**D**

Consider small perturbations of the rotation  $\gamma'(t)$ , so that the equation of motion found in (c) can be linearized. Find the response to an impulse of torque  $T'(t) = I_0\mu_0(t)$ , assuming that before the impulse in torque the rotation velocity is constant.

**E**

Which of the equilibrium phase angles  $\gamma_0$  found in (c) is stable?

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