$\boxed{\text{Solution}}$

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machinery

Quiz 1 **One Crib Sheet Allowed** **November 6, 2013**

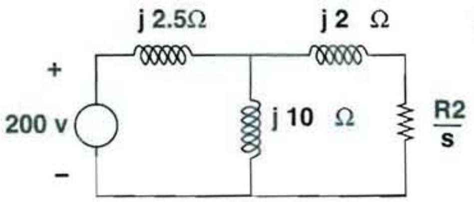
There is space for you to write your answers on this quiz.
There are four problems on this quiz. They have equal weight

Problem 1: Induction Motors

The single phase equivalent circuit of a three-phase, four pole induction motor is shown in Figure 1. For the purposes of this problem we will assume armature resistance is negligible. The machine is connected to a three-phase voltage source with line-neutral voltage of 200 volts, RMS (346 volts, line-line) and a frequency of 400 radians/second. The motor reaches peak torque at a speed of 160 Radians/second (about 1528 RPM).

$$200 \times \frac{10}{12.5} = 160$$

$$10 \| 2.5 = \frac{25}{12.5} = 2$$



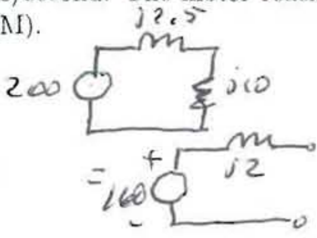


Figure 1: Induction Motor Equivalent Circuit

1. What is the value of that peak torque (in N-m)?

Peak torque => $R_2/s = 4$

$$T = 3\frac{2}{400} \times \frac{160^2 \times 4}{4^2 + 4^2} = \frac{6}{400} \times \frac{160 \times 160 \times 4}{32} = \frac{6 \times 160 \times 160}{100 \times 32}$$

$$= 3\frac{p}{\omega} \times \frac{V^2 \times R_2/s}{X^2 + R_2/s} = \frac{3 \times 1.6 \times 160}{16} = \frac{3 \times 160}{10} = \boxed{48 \text{ NM}}$$

2. What is the value of R_2? (in Ω) $\frac{R_2}{s} = 4$

$$1 - s = \frac{160}{200} = .8 \quad \text{so} \quad s = .2 \qquad \boxed{R_2 = .2 \times 4 = 0.8\,\Omega}$$

Problem 2 DC Machines

A permanent magnet DC motor is connected to a 250 Volt (DC) source. Running 'light' (no mechanical load), it draws negligible current and turns at a speed of 200 Radians/second. (about 1910 RPM). The armature circuit of the machine has a resistance of one Ω. Now the machine is loaded so that it is driving a load torque of 100 N-m, still connected to the 250 VDC source.

1. How much current is it drawing?

$$K = \frac{250}{200} = 1.25 \text{ wb}$$

$$KI = 100 \quad \text{or} \quad \boxed{I = 80A}$$

2. How fast is it turning?

$$K\Omega = 250\,V - RI = 250 - 80 = 170 \text{ V}$$

$$\Omega = \frac{170}{1.25} = 0.8 \times 170 = \boxed{136 \text{ Rad/sec}}$$

$$\begin{array}{r} {}^{5} \\ 170 \\ \underline{.8} \\ 136.0 \end{array}$$

Problem 3 Time constants

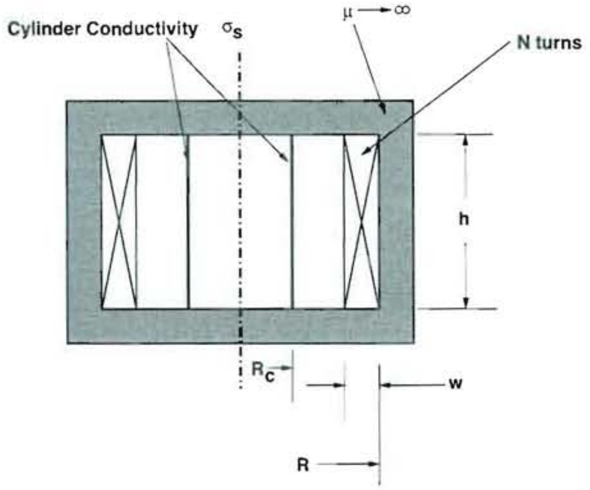


Figure 2: Pot Core with Coil and Shell Inside

Shown in Figure 2 is a pot core with two elements inside: a coil with N turns is located at the outer radius of the cavity within the core. The other thing is a conductive shell of radius R and height h, which just fits inside the axial dimension of the cavity. The shell is thin and has surface conductivity σ_s.

For the purpose of this problem, we are in a universe in which $\frac{1}{\mu_0} = 800,000$. The coil has 1,000 turns, the cylinder radius is $R_c = 10$cm and the coil radius is $R = 20$cm. Height of both elements and of the inside of the pot core is $h = 10$cm.

1. Assuming the coil is radially 'thin', and ignoring the conductive cylinder, what is the coil inductance?

$$L = \frac{\mu_0 A N^2}{h} = \frac{\pi \times .2^2 \times 1000^2}{.1 \times 800{,}000} = \pi \times \frac{.04 \times 10^6}{8 \times 10^4} =$$

$$= \pi \times \frac{4 \times 10^4}{8 \times 10^4} = \frac{\pi}{2} =$$

$$\boxed{L = \frac{\pi}{2}} \approx 1.571 \text{ H}$$

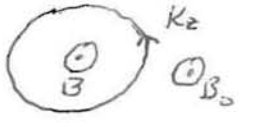
2. Now: assuming the conductive cylinder is 'perfectly diamagnetic', meaning zero flux can penetrate it, as you would have with infinite cylinder conductivity, what is the coil inductance?

For this, $A = .2^2 - .1^2 \times \pi = .03\pi$

Then $L = \frac{3}{4} \times \frac{\pi}{2} = \boxed{\frac{3}{8}\pi}$

3. Assuming that the cylinder has surface conductivity of 80.000 S (reciprocal ohms). and the coil is driven by a step of 10 A. what is the magnetic field *inside* of the cylinder. as a function of time?

Faraday's Law



$$\oint \bar{E}\cdot d\bar{l} = -\iint \frac{dB}{dt}$$

$$\bar{E} = \frac{1}{\sigma}\frac{1}{\mu_0}(B-B_0)$$

$$B-B_0=\mu_0 K_z$$

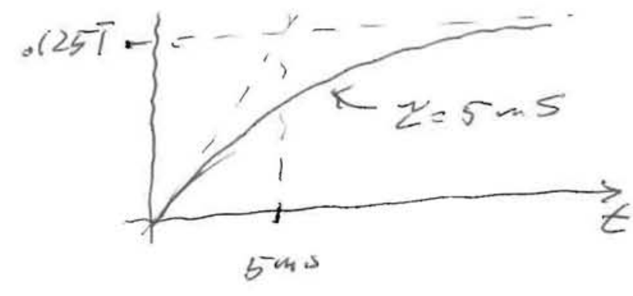
$$\frac{2\cancel{\pi} R_i}{\sigma\mu_0}(B-B_0) = -\cancel{\pi}R_i^2\frac{dB}{dt}$$

$$\frac{\sigma\mu_0 R_i}{2}\frac{dB}{dt} + B = B_0$$

If $B\big|_{t=0} = 0$, then $B = B_0(1-e^{-t/\tau})$ where $\tau = \frac{\sigma\mu_0 R_i}{2}$

$$B_0 = \frac{\mu_0 N i}{h} = \frac{1000\times 10}{800{,}000\times .1} = \frac{10^4}{8\times 10^4} = .125\ T$$

$$\tau = \frac{80{,}000\times .1}{800{,}000\times 2} = \frac{.01}{2} = 5\ mS$$



Problem 4 Synchronous Machine

A three-phase, two pole synchronous machine has the following characteristics:

Rated Power	P_B	3	MVA
Rated Voltage	V_B	1	kV (Peak)
Synchronous Inductance	$L_a - L_{ab}$	2.5	mH
Field to Phase Mutual Inductance	M	25	mH
Synchronous Frequency	ω_0	400	Radians/Second

$$I_B = \frac{3 \times 10^6}{\frac{3}{2} \times 10^3} = 2000 A$$

Assume that armature resistance is negligible.

1. On no-load, open-circuit test at rated speed, what field current is required to produce rated voltage?

$400 \times .025 = 10$

$$1000 = 400 \times .025 \; I_f$$

$$\text{So } \boxed{I_f = 100 A}$$

2. On short-circuit test, what field current is required to produce rated current?

$$2000 = \frac{\omega M}{\omega L} I_f \quad \text{or} \quad I_f = 2000 \times \frac{2.5}{25} = \boxed{200 A}$$

3. Running at rated speed, with field current $I_f = 300A$ and with rated armature terminal voltage (1.000 V, *Peak*, what is the *peak* torque the machine can produce?

$$T_p = \frac{3}{2} \frac{p}{\omega_0} \frac{V E_{af}}{X}$$

$X = 400 \times .0025 = 1$

$I_f = 300 \quad E_{af} = 3000$

$$= \frac{3}{2} \times \frac{1}{400} \times \frac{1000 \times 3000}{1}$$

$$= \frac{9}{2} \times \frac{10^6}{400} = \frac{9}{8} \times 10^4 = \text{[illegible]}$$

Sorry

11,250 N-m

$9 \times .125$

$= .125$
$\;\; 9$
$.325$

$.125$
$\;\; 9$
1.125

4. Running at rated speed with field current $I_f = 300A$ and with the armature driven by a balanced three-phase current of $I_a = 3.000A$, *Peak*, what is the maximum *peak* torque that the machine can produce?

$$T_p = \frac{3}{2} p M I I_f = \frac{3}{2} \times .025 \times 3000 \times 300$$

$$= \frac{3}{2} \times \frac{3000 \times 30}{4} = \frac{27}{8} \times 10^4 = \frac{27}{.8} \times 10^3$$

$$= 1.5 \times \frac{9}{4} \times 10^4 = 3.375 \times 10^4$$

$$= 33,750 \text{ N-m}$$

$.025 \times 300$

$\frac{30}{4}$

$.125$
$\;\; 27$

$\frac{9}{4} = \frac{1}{2} \times 4.5 = 2.25$

$\;\; 1.5$
1125
225
3.375

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