6.776
High Speed Communication Circuits
Lecture 10
Noise Modeling in Amplifiers

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Notation for Mean, Variance, and Correlation

- **Consider random variables x and y with probability density functions $f_x(x)$ and $f_y(y)$ and joint probability function $f_{xy}(x,y)$**
  - **Expected value (mean) of x is**

$$\overline{x} = E(x) = \int_{-\infty}^{\infty} x f_x(x) dx$$

    - Note: we will often abuse notation and denote $\overline{x}$ as a random variable (i.e., noise) rather than its mean
  - **The variance of x (assuming it has zero mean) is**

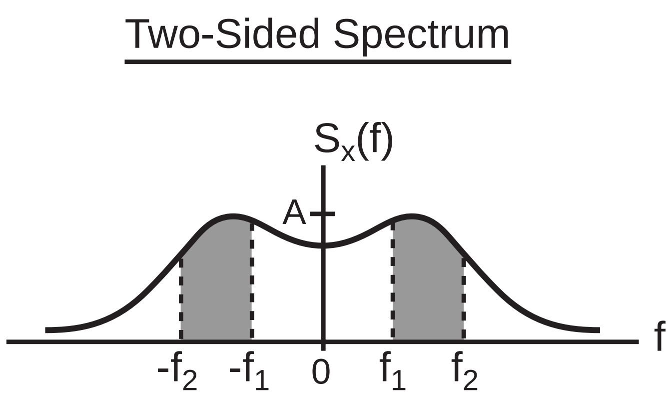
$$\overline{x^2} = E(x^* x) = \int_{-\infty}^{\infty} x^* x f_x(x) dx$$

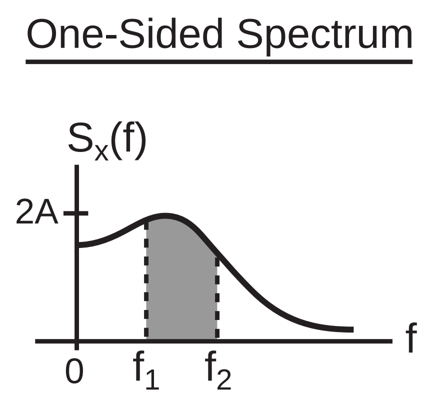
  - **A useful statistic is**

$$\overline{xy} = E(xy) = \int_{-\infty}^{\infty} xy f_{xy}(x,y) dx dy$$

    - If the above is zero, x and y are said to be uncorrelated

Relationship Between Variance and Spectral Density





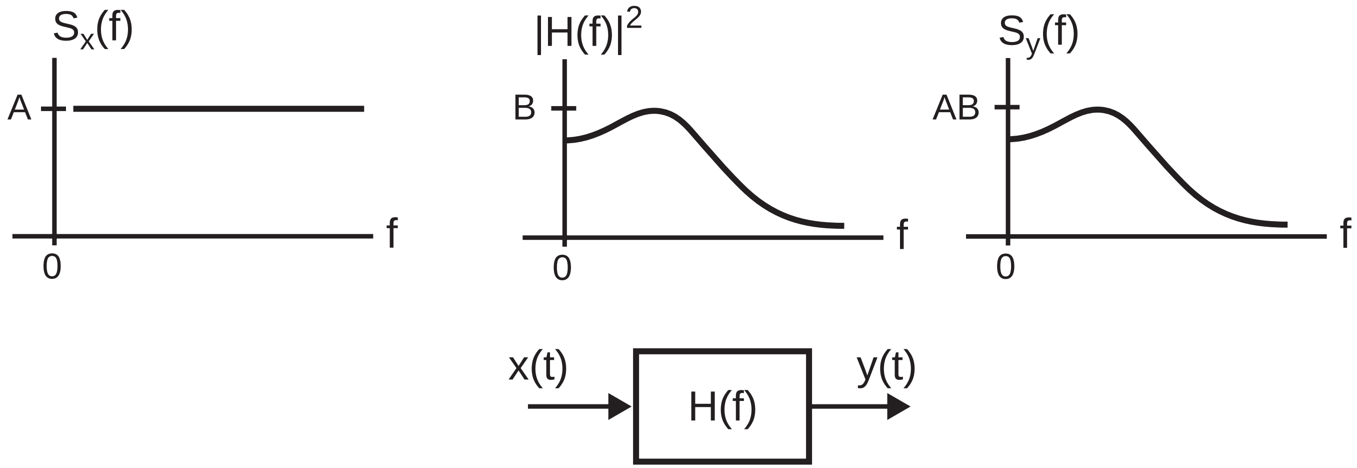
- **Two-sided spectrum**

$$\overline{x^2} = \int_{-f_2}^{-f_1} S_x(f)df + \int_{f_1}^{f_2} S_x(f)df$$

  - **Since spectrum is symmetric** $\Rightarrow \overline{x^2} = 2\int_{f_1}^{f_2} S_x(f)df$

- **One-sided spectrum defined over positive frequencies**
  - **Magnitude defined as twice that of its corresponding two-sided spectrum**
- **In the next few lectures, we assume a one-sided spectrum for all noise analysis**

The Impact of Filtering on Spectral Density



- **For the random signal passing through a linear, time-invariant system with transfer function H(f)**

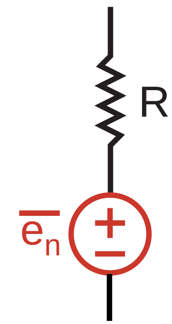
$$S_y(f) = |H(f)|^2 S_x(f)$$

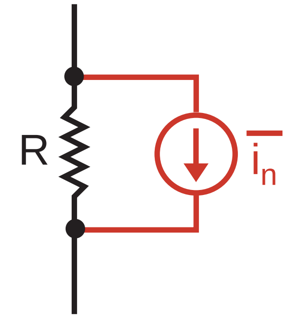
  - **We see that if x(t) is amplified by gain A, we have**

$$S_y(f) = A^2 S_x(f) \;\;\Rightarrow\;\; \overline{y^2} = A^2 \overline{x^2}$$

Noise in Resistors

- **Can be described in terms of either voltage or current**





$$\overline{e_n^2} = 4kTR\Delta f \qquad \overline{i_n^2} = 4kT\frac{1}{R}\Delta f$$

- **k is Boltzmann's constant**

$$k = 1.38 \times 10^{-23} J/K$$

- **T is temperature (in Kelvins)**
  - **Usually assume room temperature of 27 degrees Celsius**

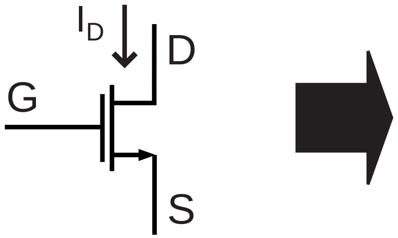
$$\Rightarrow \; T = 300K$$

Noise In Inductors and Capacitors

- **Ideal capacitors and inductors have no noise!**

- **In practice, however, they will have parasitic resistance**
  - **Induces noise**
  - **Parameterized by adding resistances in parallel/series with inductor/capacitor**
    - Include parasitic resistor noise sources

Noise in CMOS Transistors (Assumed in Saturation)



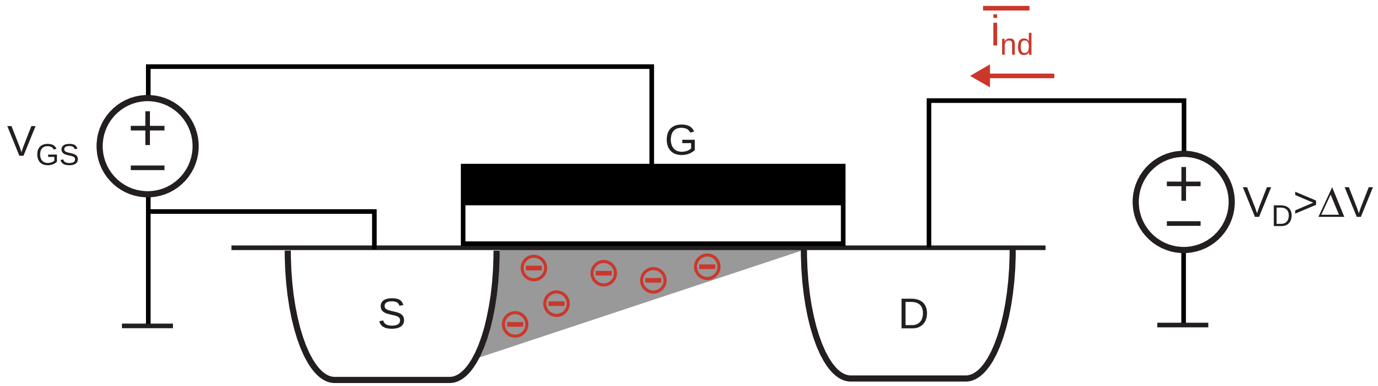
Transistor Noise Sources

Drain Noise (Thermal and 1/f)

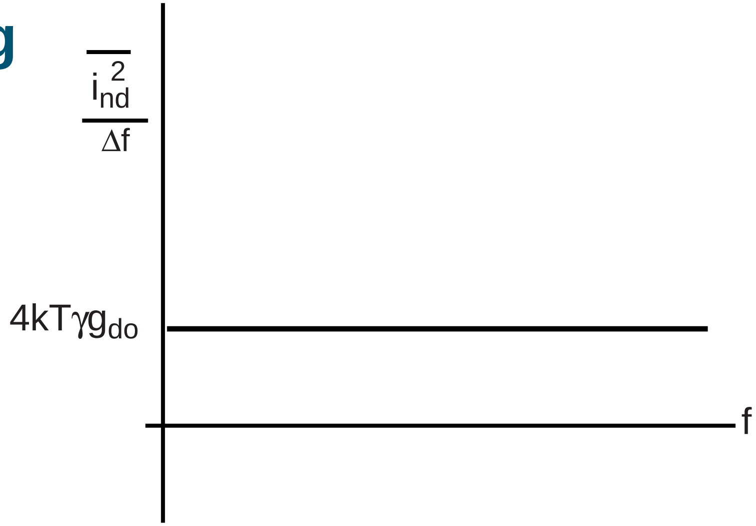
Gate Noise (Induced and Routing Parasitic)

- **Modeling of noise in transistors must include several noise sources**
  - **Drain noise**
    - Thermal and 1/f – influenced by transistor size and bias
  - **Gate noise**
    - Induced from channel – influenced by transistor size and bias
    - Caused by routing resistance to gate (including resistance of polysilicon gate)
      - Can be made negligible with proper layout such as fingering of devices

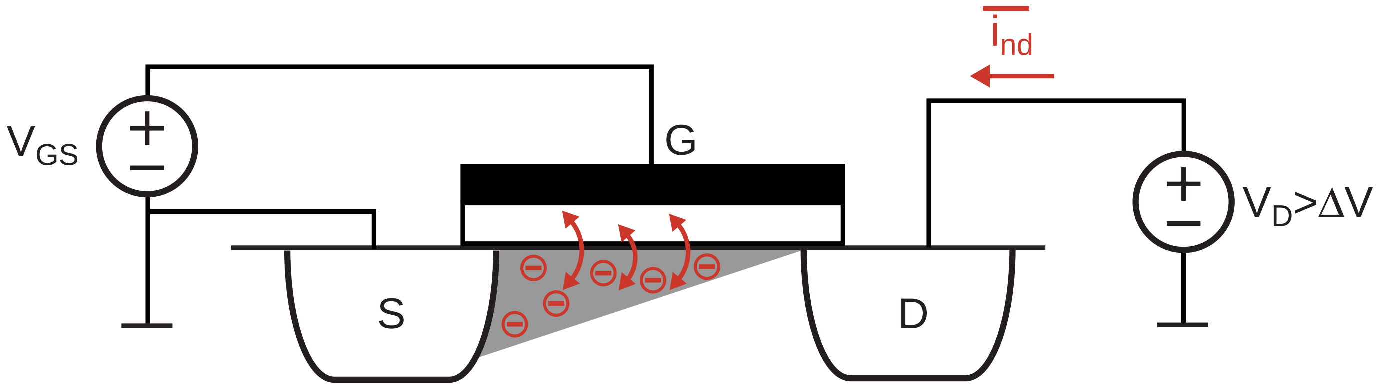
Drain Noise – Thermal (Assume Device in Saturation)



- **Thermally agitated carriers in the channel cause a randomly varying current** $\left.\overline{i_{nd}^2}\right|_{th} = 4kT\gamma g_{do}\Delta f$
  - **γ is called excess noise factor**
    - = 2/3 in long channel
    - = 2 to 3 (or higher!) in short channel NMOS (less in PMOS)
  - **g_{do} will be discussed shortly** (Note: $g_{do} = g_m/\alpha$)



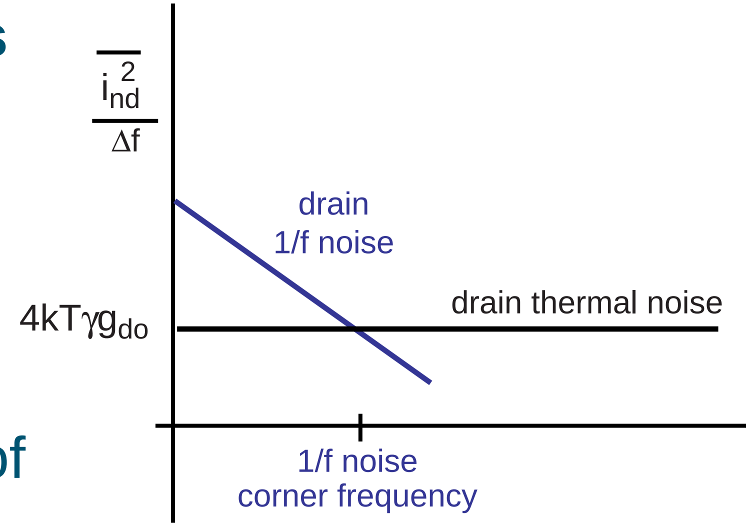
Drain Noise – 1/f (Assume Device in Saturation)



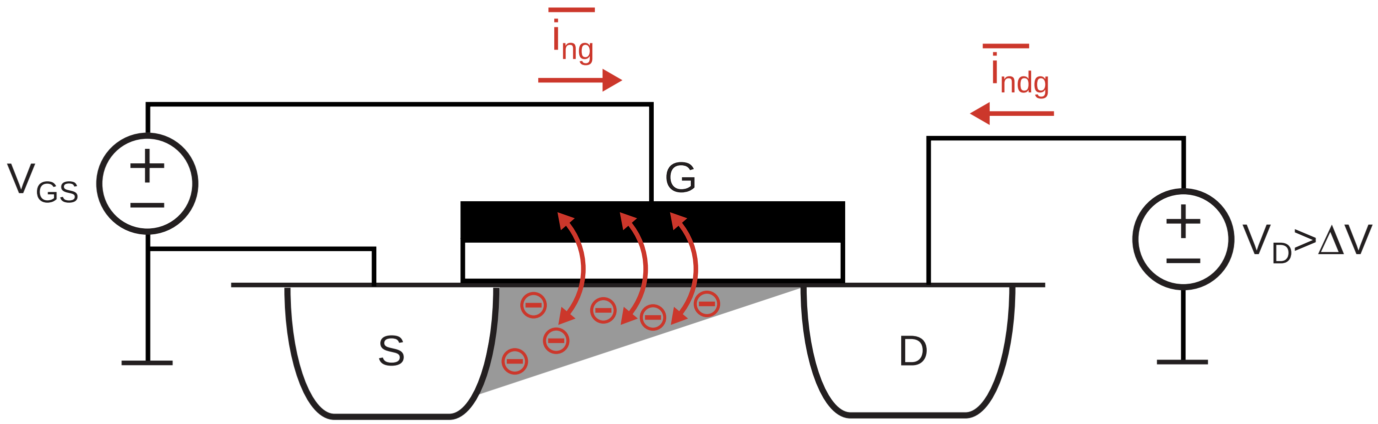
- **Traps at channel/oxide interface randomly capture/release carriers**

$$\left.\overline{i_{nd}^2}\right|_{1/f} = \frac{K_f}{f^n}\Delta f \approx \frac{K}{f}\frac{g_m^2}{WLC_{ox}^2}\Delta f$$

  - Parameterized by K_f and n
    - Provided by fab (note n ≈ 1)
    - Currently: K_f of PMOS << K_f of NMOS due to buried channel
  - **To minimize: want large area (high WL)**

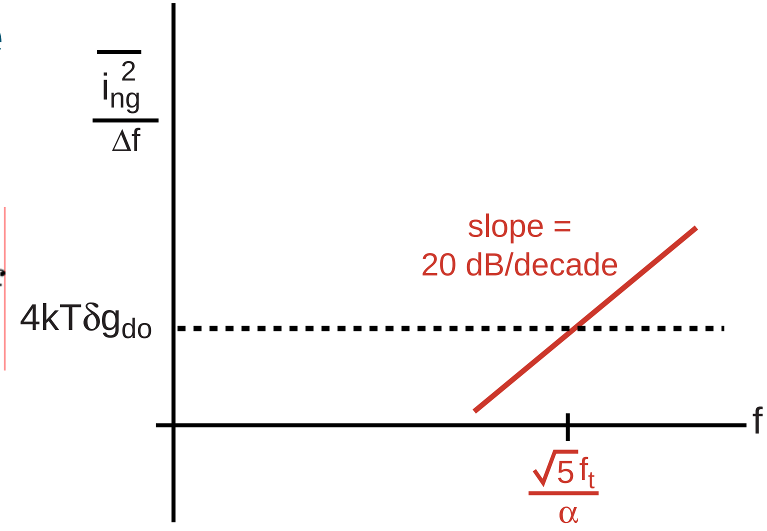


Induced Gate Noise (Assume Device in Saturation)



- **Fluctuating channel potential couples capacitively into the gate terminal, causing a noise gate current**

$$\overline{i_{ng}^2} = 4kT\delta g_{do}\left(\frac{2\pi f}{\sqrt{5}/\alpha(g_m/C_{gs})}\right)^2\Delta f$$



  - **δ is gate noise coefficient**
    - Typically assumed to be 2γ
  - **Correlated to drain noise!**

(Note: $\alpha = g_m/g_{do}$)

Useful References on MOSFET Noise

- **Thermal Noise**
  - **B. Wang et. al., "MOSFET Thermal Noise Modeling for Analog Integrated Circuits", JSSC, July 1994**
- **Gate Noise**
  - **Jung-Suk Goo, "High Frequency Noise in CMOS Low Noise Amplifiers", PhD Thesis, Stanford University, August 2001**
    - http://www-tcad.stanford.edu/tcad/pubs/theses/goo.pdf
  - **Jung-Suk Goo et. al., "The Equivalence of van der Ziel and BSIM4 Models in Modeling the Induced Gate Noise of MOSFETS", IEDM 2000, 35.2.1-35.2.4**
  - **Todd Sepke, "Investigation of Noise Sources in Scaled CMOS Field-Effect Transistors", MS Thesis, MIT, June 2002**

Drain-Source Conductance: g_{do}

- **g_{do} is defined as channel resistance with V_{ds}=0**
  - **Transistor in triode, so that**

$$I_d = \mu_n C_{ox} \frac{W}{L}\left((V_{gs} - V_T)V_{ds} - \frac{V_{ds}^2}{2}\right)$$

$$\Rightarrow \boxed{g_{do} = \left.\frac{dI_d}{dV_{ds}}\right|_{V_{ds}=0} = \mu_n C_{ox}\frac{W}{L}(V_{gs} - V_T)}$$

  - **Equals g_m for long channel devices**
- **Key parameters for 0.18μ NMOS devices**

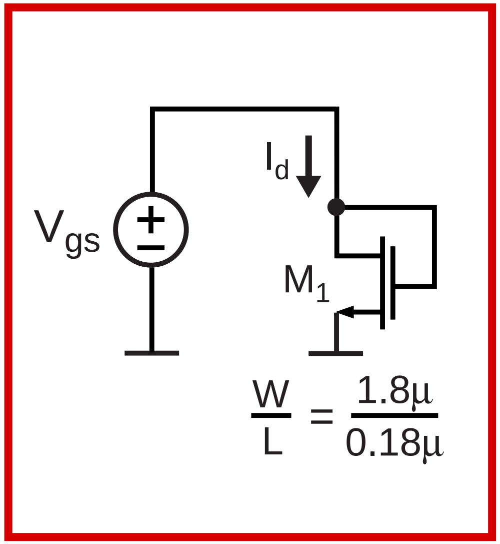
$$\mu_n = 327.4 \text{ cm}^2/(V \cdot s)$$

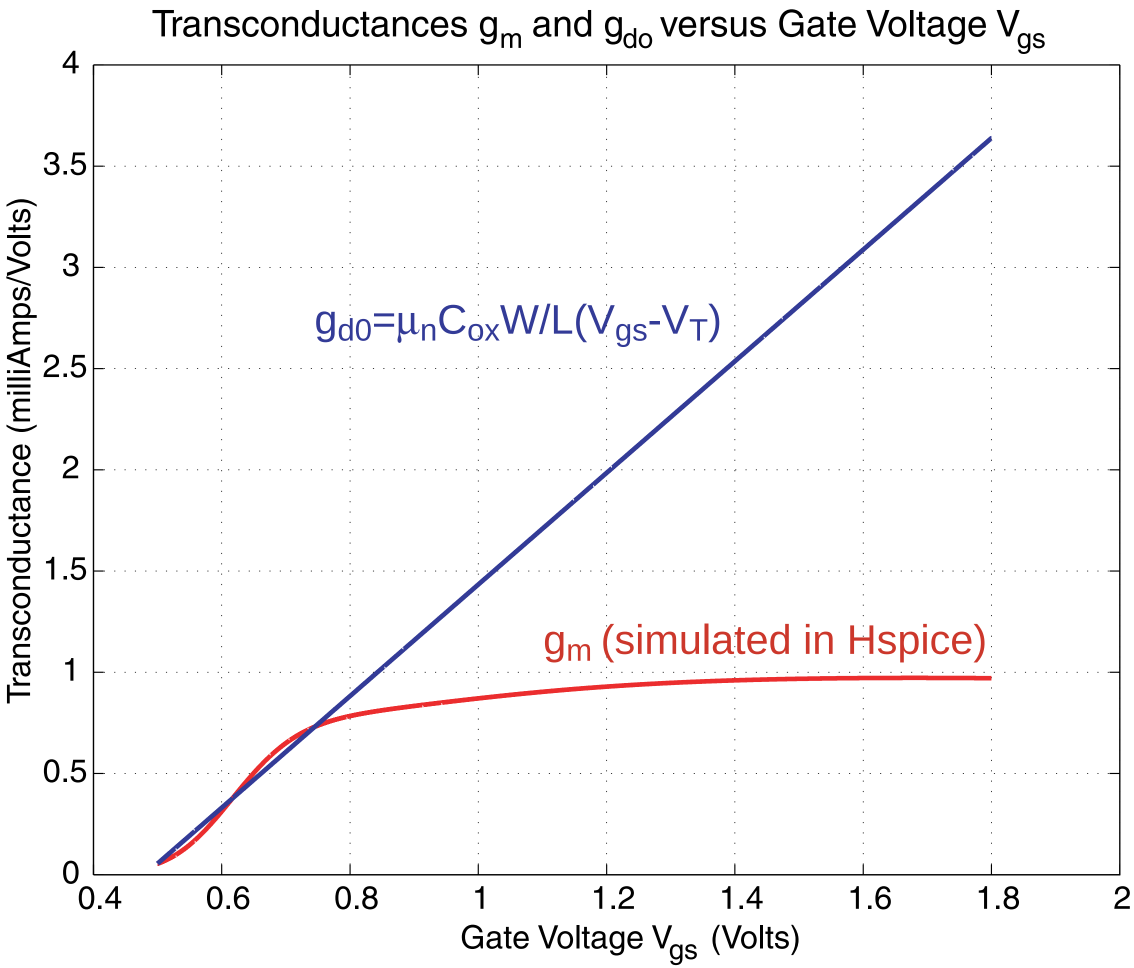
$$t_{ox} = 4.1 \times 10^{-9} \text{ m} \quad \epsilon_o = 3.9(8.85 \times 10^{-12}) \text{ F/m}$$

$$\Rightarrow \mu_n C_{ox} = \mu_n \frac{\epsilon_o}{t_{ox}} = 275.6 \times 10^{-6} \text{ F}/(V \cdot s)$$

$$V_T = 0.48 \text{ V}$$

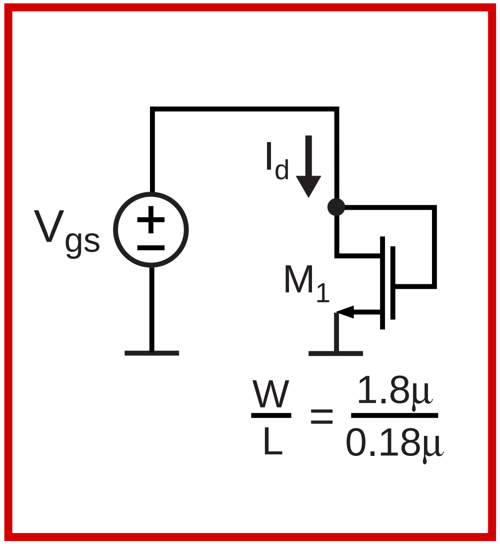
Plot of g_m and g_{do} versus V_{gs} for 0.18μ NMOS Device

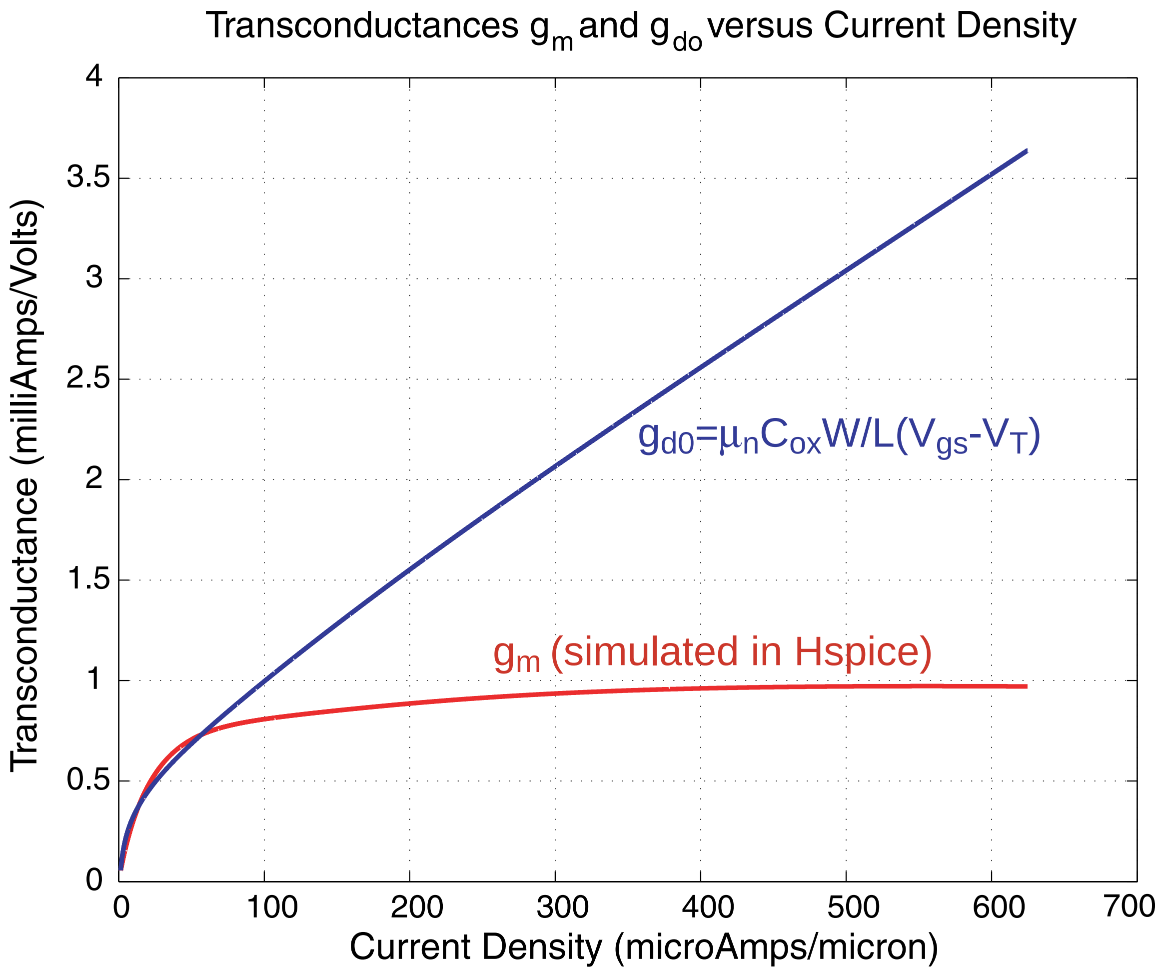




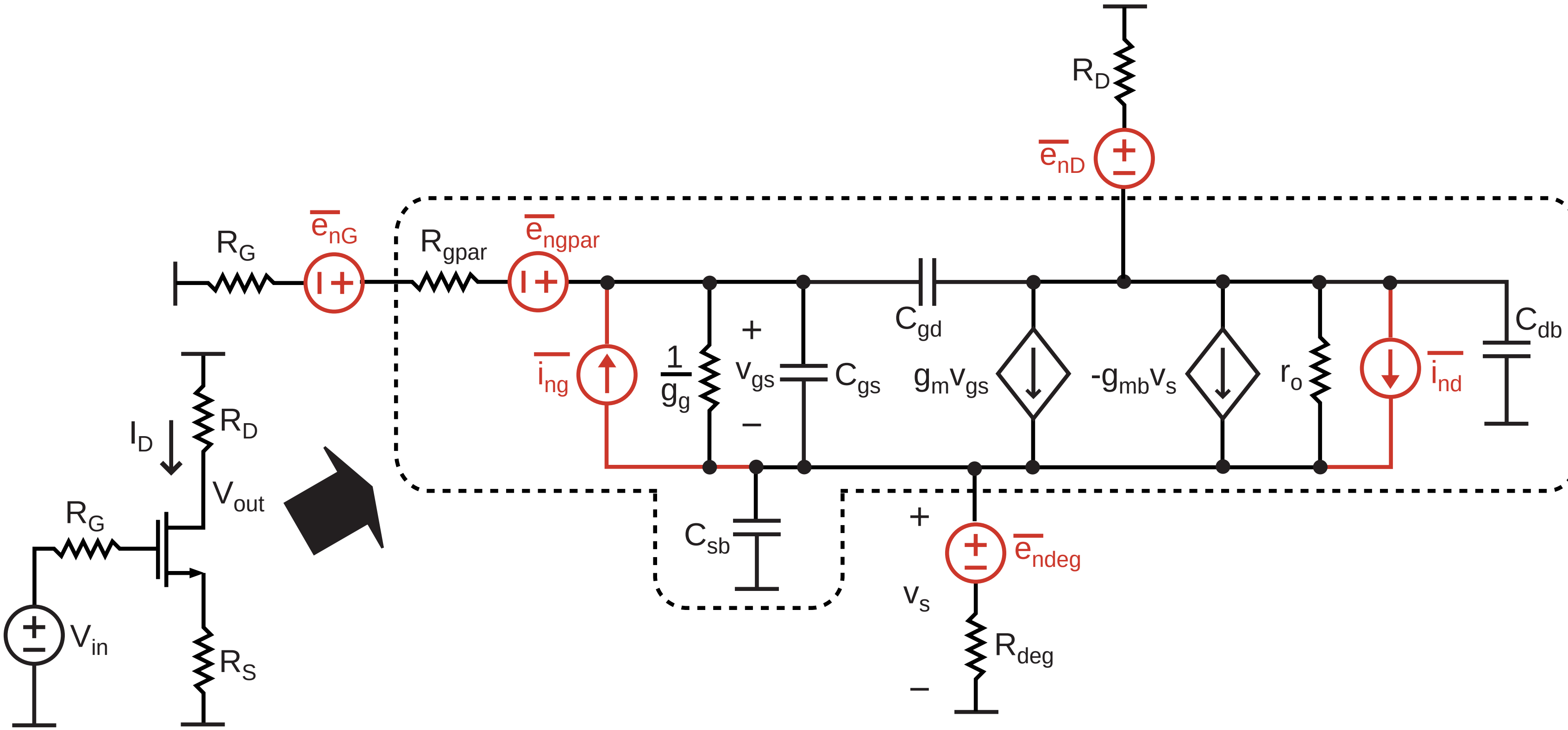
- **For V_{gs} bias voltages around 1.2 V:** $\alpha = \frac{g_m}{g_{do}} \approx \frac{1}{2}$

Plot of g_m and g_{do} versus I_{dens} for 0.18μ NMOS Device





Noise Sources in a CMOS Amplifier



$\overline{e_{nG}}, \overline{e_{nD}}, \overline{e_{ndeg}}$: noise sources of external resistors

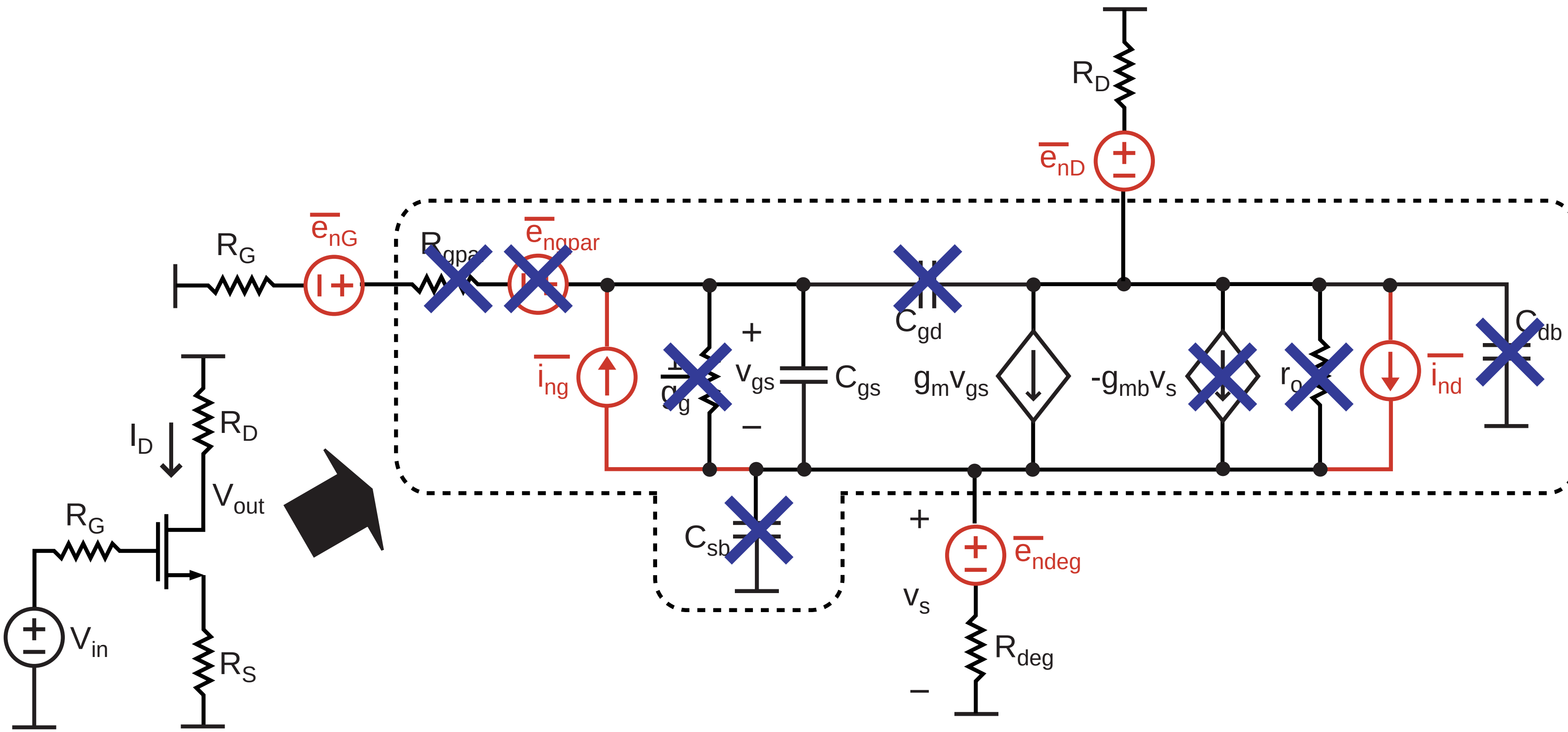
$R_{gpar}, \overline{e_{ngpar}}$: parasitic gate resistance and its noise

$\overline{i_{ng}}$: induced gate noise,

g_g : caused by distributed nature of channel $\left(g_g = \frac{w^2 C_{gs}^2}{5 g_{d0}}\right)$

$\overline{i_{nd}}$: drain noise (thermal and 1/f)

Remove Model Components for Simplicity



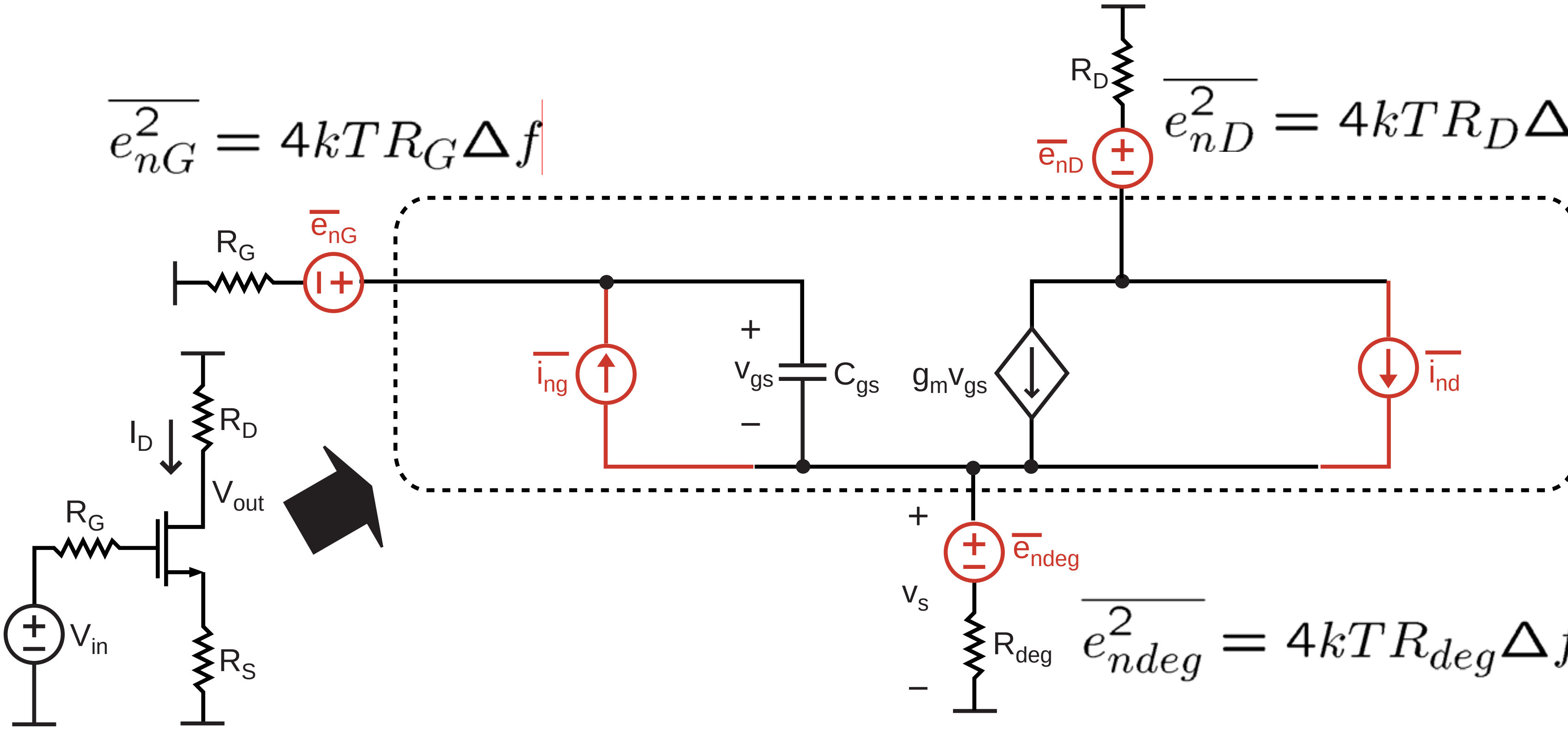
R_{gpar}, $\overline{e_{ngpar}}$: can make negligible with proper layout

g_g : assume to be neglible (for $w \ll w_t$)

C_{sb}, C_{gd}, C_{db}, g_{mb} : too painful to include for calculations

r_o : impact is minor since R_D is small (for high bandwidth)

Key Noise Sources for Noise Analysis



$$\overline{e_{nG}^2} = 4kTR_G\Delta f$$

$$\overline{e_{nD}^2} = 4kTR_D\Delta f$$

$$\overline{e_{ndeg}^2} = 4kTR_{deg}\Delta f$$

- **Transistor gate noise**

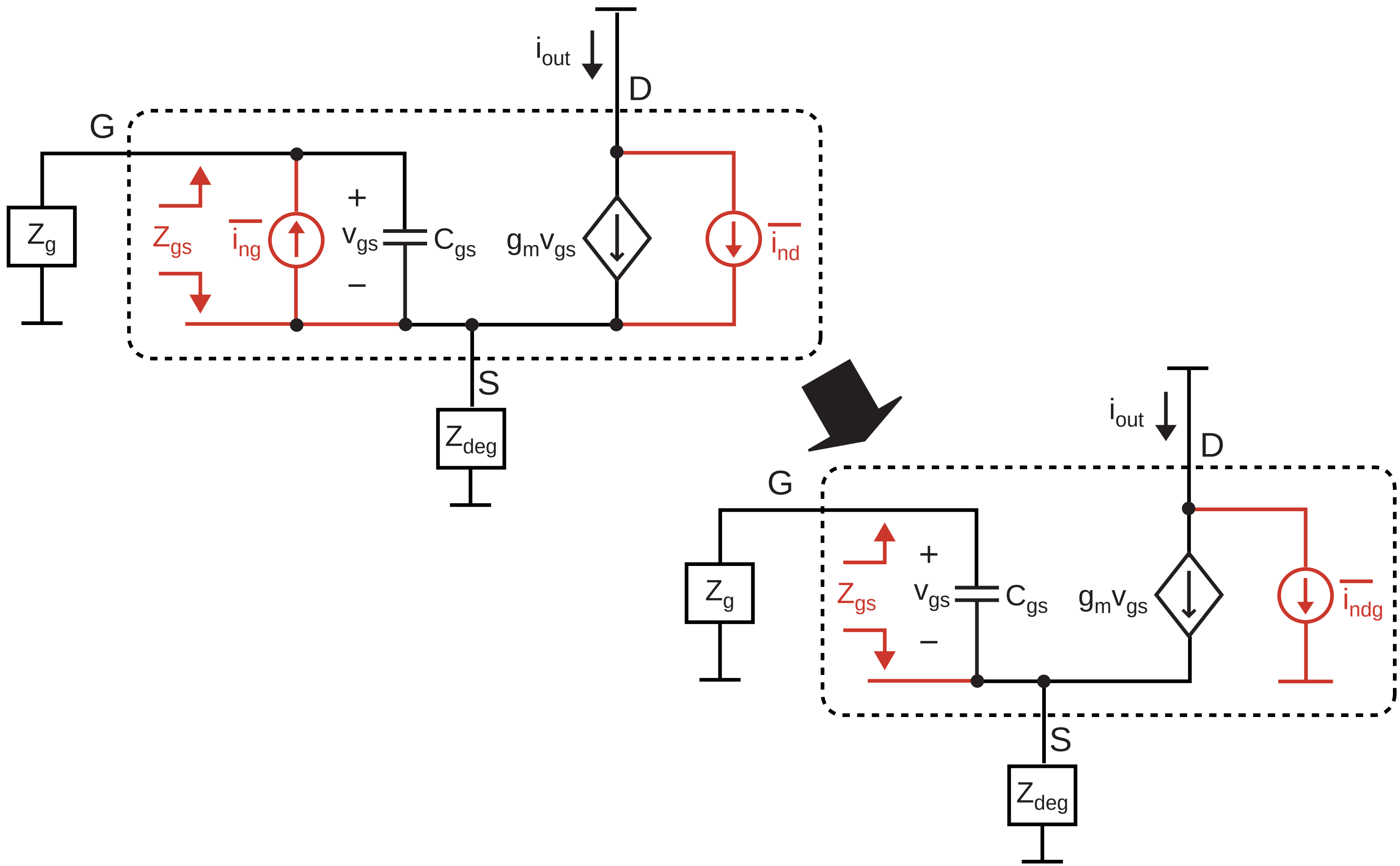
$$\overline{i_{ng}^2} = 4kT\delta g_g\Delta f,$$

where $g_g = \frac{w^2C_{gs}^2}{5g_{d0}}$

- **Transistor drain noise**

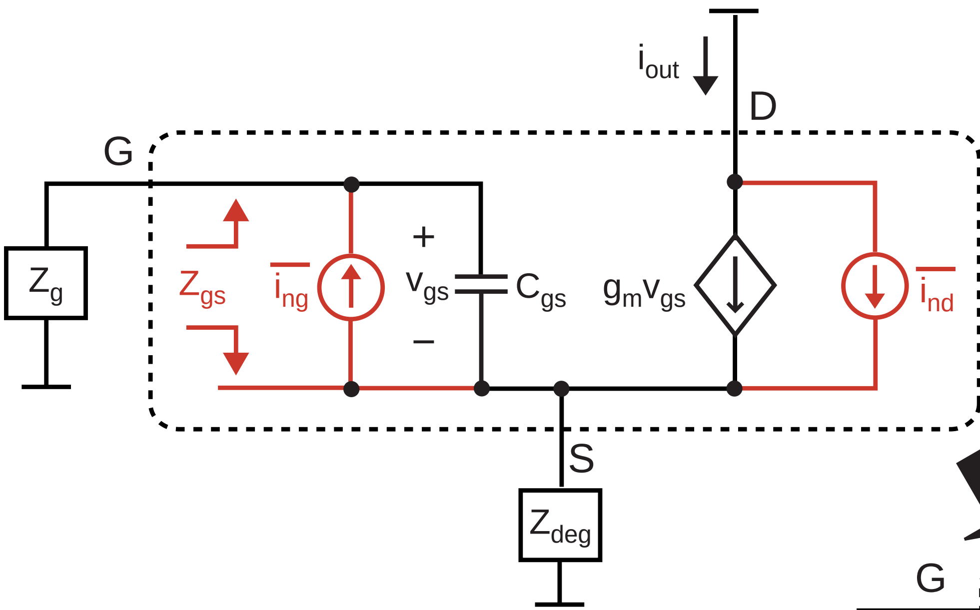
$$\overline{i_{nd}^2} = \underbrace{4kT\gamma g_{do}\Delta f}_{\text{Thermal noise}} + \underbrace{\frac{K_f}{f^n}\Delta f}_{\text{1/f noise}}$$

Apply Thevenin Techniques to Simplify Noise Analysis

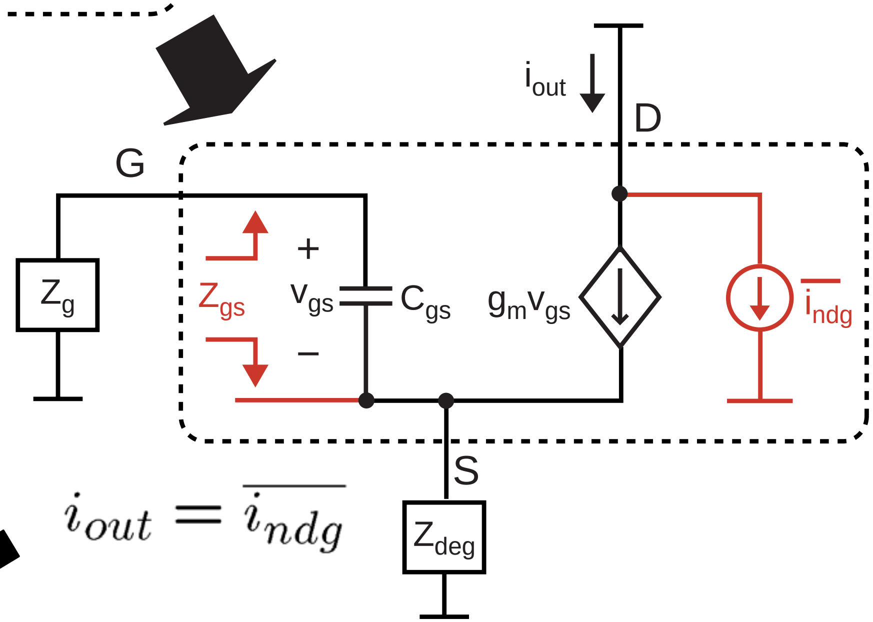


- **Assumption: noise independent of load resistor on drain**

Calculation of Equivalent Output Noise for Each Case



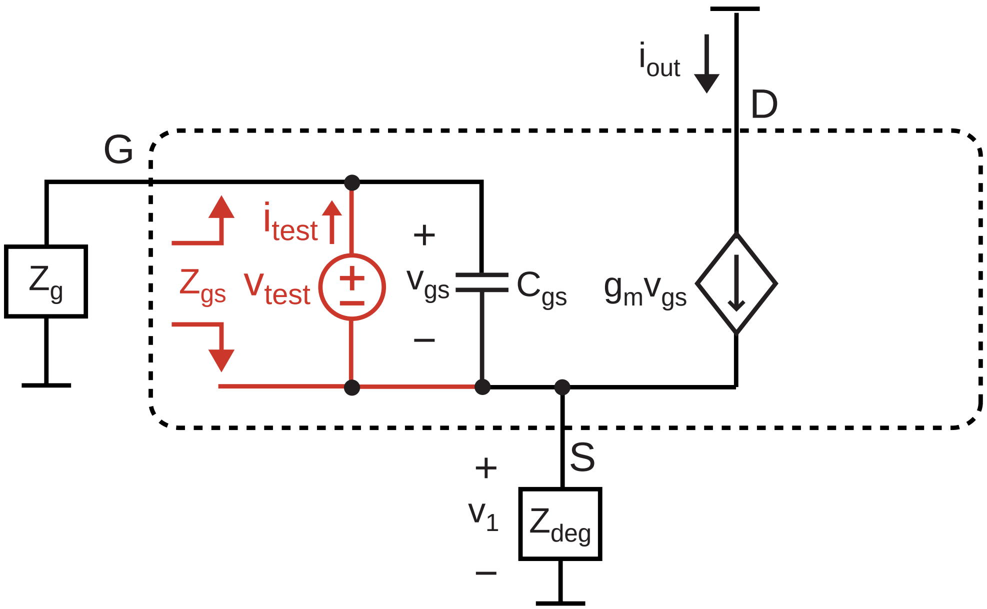
$$i_{out} = g_m Z_{gs} \overline{i_{ng}} + \eta \overline{i_{nd}}$$



$$i_{out} = \overline{i_{ndg}}$$

$$\overline{i_{ndg}} = g_m Z_{gs} \overline{i_{ng}} + \eta \overline{i_{nd}}$$

Calculation of Z_{gs}



- **Write KCL equations**

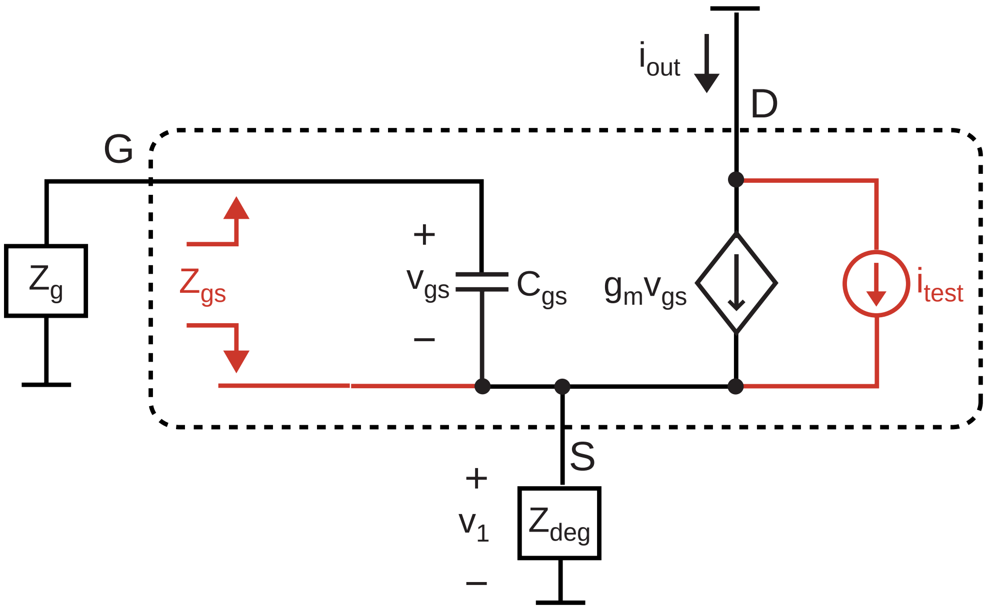
$$(1)\quad -i_{test} + \frac{v_{test}}{1/(sC_{gs})} + g_m v_{test} = \frac{v_1}{Z_{deg}}$$

$$(2)\quad \frac{v_{test} + v_1}{Z_g} + \frac{v_1}{Z_{deg}} = g_m v_{test}$$

- **After much algebra:**

$$Z_{gs} = \frac{v_{test}}{i_{test}} = \boxed{\frac{1}{sC_{gs}} \,\Big|\Big|\, \frac{Z_{deg} + Z_g}{1 + g_m Z_{deg}}}$$

Calculation of η



- **Determine V_{gs} to find i_{out} in terms of i_{test}**

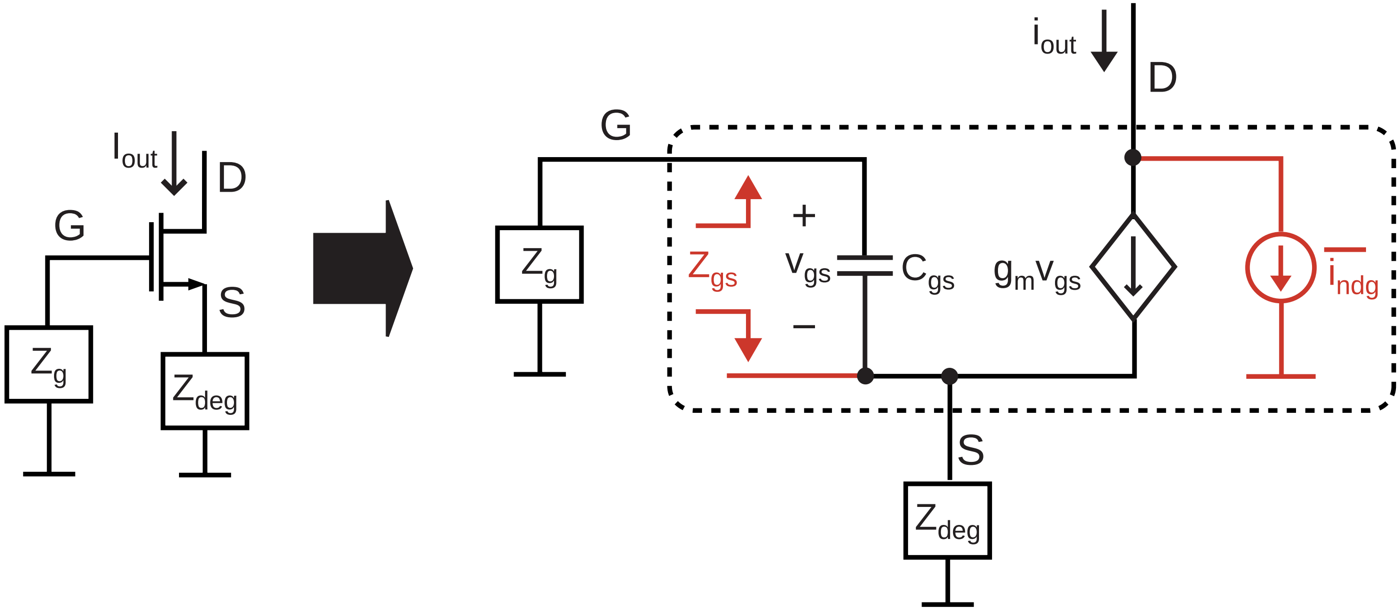
$$(1)\ i_{out} = i_{test} + g_m v_{gs} \qquad (2)\ v_{gs} = -v_1 \frac{1/(sC_{gs})}{1/(sC_{gs}) + Z_g}$$

$$(3)\ v_1 = i_{out}(Z_{deg}||(\frac{1}{sC_{gs}} + Z_g))$$

- **After much algebra:**

$$\eta = \frac{i_{out}}{i_{test}} = \boxed{1 - \left(\frac{g_m Z_{deg}}{Z_{deg} + Z_g}\right) Z_{gs}}$$

Calculation of Output Current Noise Variance (Power)



$$i_{out} = \overline{i_{ndg}} = \eta\overline{i_{nd}} + g_m Z_{gs}\overline{i_{ng}}$$

- **To find noise variance:**

$$\overline{i_{ndg}^2} = \overline{i_{ndg}^* i_{ndg}} = \overline{(\eta^* i_{nd}^* + g_m Z_{gs}^* i_{ng}^*)(\eta i_{nd} + g_m Z_{gs} i_{ng})}$$

Variance (i.e., Power) Calc. for Output Current Noise

- **Noise variance calculation**

$$\overline{i_{ndg}^2} = |\eta|^2 \overline{i_{nd} i_{nd}^*} + \overline{i_{nd}^* i_{ng}} g_m \eta^* Z_{gs} + \overline{i_{nd} i_{ng}^*} (g_m \eta Z_{gs})^* + \overline{i_{ng} i_{ng}^*} |g_m Z_{gs}|^2$$

$$= |\eta|^2 \overline{i_{nd}^2} + 2Re\{\overline{i_{nd}^* i_{ng}} g_m \eta^* Z_{gs}\} + \overline{i_{ng}^2} |g_m Z_{gs}|^2$$

$$= |\eta|^2 \overline{i_{nd}^2} + 2Re\{\frac{\overline{i_{nd}^* i_{ng}}}{\sqrt{\overline{i_{nd}^2}\ \overline{i_{ng}^2}}} \sqrt{\overline{i_{nd}^2}\ \overline{i_{ng}^2}} g_m \eta^* Z_{gs}\} + \overline{i_{ng}^2} |g_m Z_{gs}|^2$$

- **Define correlation coefficient c between i_{ng} and i_{nd}**

$$c = \frac{\overline{i_{nd}^* i_{ng}}}{\sqrt{\overline{i_{nd}^2}\ \overline{i_{ng}^2}}} \Rightarrow \overline{i_{ndg}^2} = |\eta|^2 \overline{i_{nd}^2} + 2Re\{c\sqrt{\overline{i_{nd}^2}\ \overline{i_{ng}^2}} g_m \eta^* Z_{gs}\} + \overline{i_{ng}^2} |g_m Z_{gs}|^2$$

$$\overline{i_{ndg}^2} = \overline{i_{nd}^2} \left(|\eta|^2 + 2Re\left\{ c\sqrt{\frac{\overline{i_{ng}^2}}{\overline{i_{nd}^2}}} g_m \eta^* Z_{gs} \right\} + \frac{\overline{i_{ng}^2}}{\overline{i_{nd}^2}} g_m^2 |Z_{gs}|^2 \right)$$

Parameterized Expression for Output Noise Variance

- **Key equation from last slide**

$$\overline{i_{ndg}^2} = \overline{i_{nd}^2}\left(|\eta|^2 + 2Re\left\{c\sqrt{\frac{\overline{i_{ng}^2}}{\overline{i_{nd}^2}}}g_m\eta^* Z_{gs}\right\} + \frac{\overline{i_{ng}^2}}{\overline{i_{nd}^2}}g_m^2|Z_{gs}|^2\right)$$

- **Solve for noise ratio**

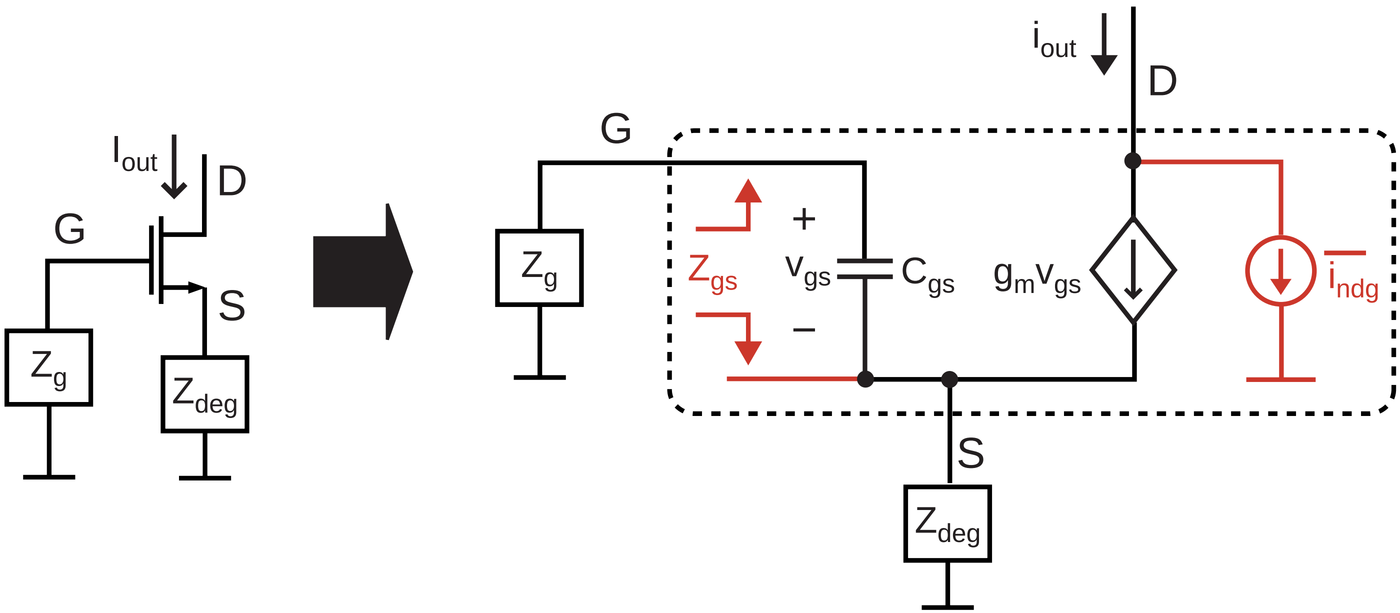
$$\sqrt{\frac{\overline{i_{ng}^2}}{\overline{i_{nd}^2}}}g_m = g_m\sqrt{\frac{4kT\delta(wC_{gs})^2/(5g_{do})}{4kT\gamma g_{do}}} = \frac{g_m}{g_{do}}\sqrt{\frac{\delta}{5\gamma}}(wC_{gs})$$

- **Define parameters Z_{gsw} and χ_d**

$$Z_{gsw} = wC_{gs}Z_{gs}, \quad \chi_d = \frac{g_m}{g_{do}}\sqrt{\frac{\delta}{5\gamma}}$$

$$\Rightarrow \quad \overline{i_{ndg}^2} = \overline{i_{nd}^2}\left(|\eta|^2 + 2Re\{c\chi_d\eta^* Z_{gsw}\} + \chi_d^2|Z_{gsw}|^2\right)$$

Small Signal Model for Noise Calculations

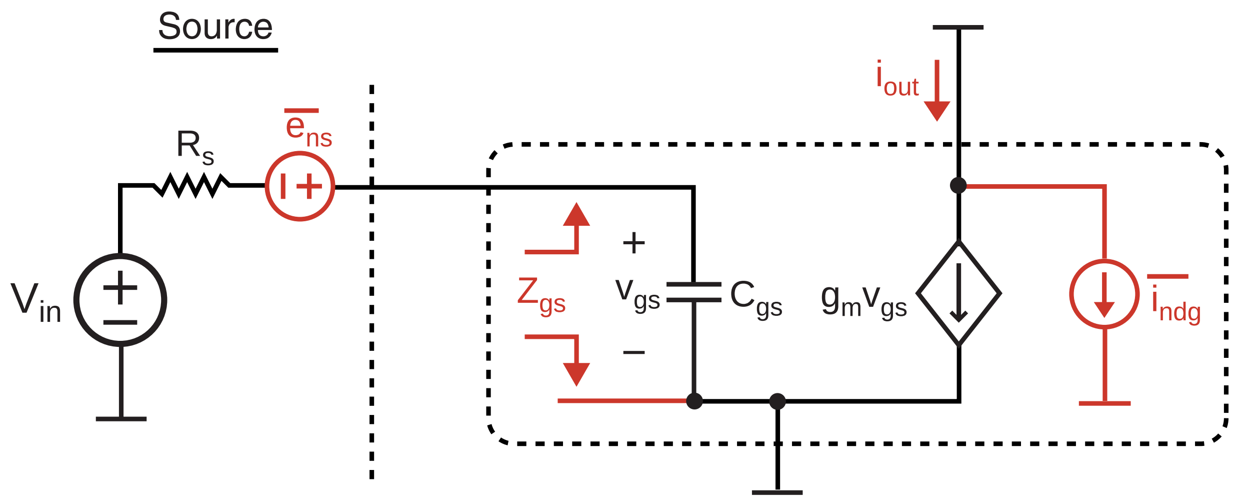


$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(|\eta|^2 + 2Re\left\{c\chi_d\eta^* Z_{gsw}\right\} + \chi_d^2|Z_{gsw}|^2\right)$$

where: $\frac{\overline{i_{nd}^2}}{\Delta f} = 4kT\gamma g_{do}, \ \chi_d = \frac{g_m}{g_{do}}\sqrt{\frac{\delta}{5\gamma}}, \ Z_{gsw} = wC_{gs}Z_{gs}$

$$Z_{gs} = \frac{1}{sC_{gs}} \Bigg|\Bigg| \frac{Z_{deg} + Z_g}{1 + g_m Z_{deg}} \qquad \eta = 1 - \left(\frac{g_m Z_{deg}}{Z_{deg} + Z_g}\right) Z_{gs}$$

Example: Output Current Noise with $Z_s = R_s$, $Z_{deg} = 0$



- **Step 1: Determine key noise parameters**
  - **For 0.18μ CMOS, we will assume the following**

$$c = -j0.55, \quad \gamma = 3, \quad \delta = 2\gamma = 6, \quad \frac{g_m}{g_{do}} = \frac{1}{2} \Rightarrow \chi_d = 0.32$$

- **Step 2: calculate η and Z_{gsw}**

$$\eta = 1, \quad Z_{gsw} = wC_{gs}\left(R_s || \frac{1}{jwC_{gs}}\right) = \frac{wC_{gs}R_s}{1 + jwC_{gs}R_s}$$

Calculation of Output Current Noise (continued)

- **Step 3: Plug values into the previously derived expression**

$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 + 2Re\left\{-j|c|\chi_d Z_{gsw}\right\} + \chi_d^2 |Z_{gsw}|^2\right)$$

Drain Noise Multiplying Factor

$$Z_{gsw} = wC_{gs}\left(R_s || \frac{1}{jwC_{gs}}\right) = \frac{wC_{gs}R_s}{1 + jwC_{gs}R_s}$$

  - **For w << 1/($R_s C_{gs}$):**

$$Z_{gsw} \approx wC_{gs}R_s \quad \Rightarrow \quad \frac{\overline{i_{ndg}^2}}{\Delta f} \approx \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 + \chi_d^2 (wC_{gs}R_s)^2\right)$$

Gate noise contribution

Calculation of Output Current Noise (continued)

- **Step 3: Plug values into the previously derived expression**

$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 + 2Re\left\{-j|c|\chi_d Z_{gsw}\right\} + \chi_d^2 |Z_{gsw}|^2\right)$$

Drain Noise Multiplying Factor

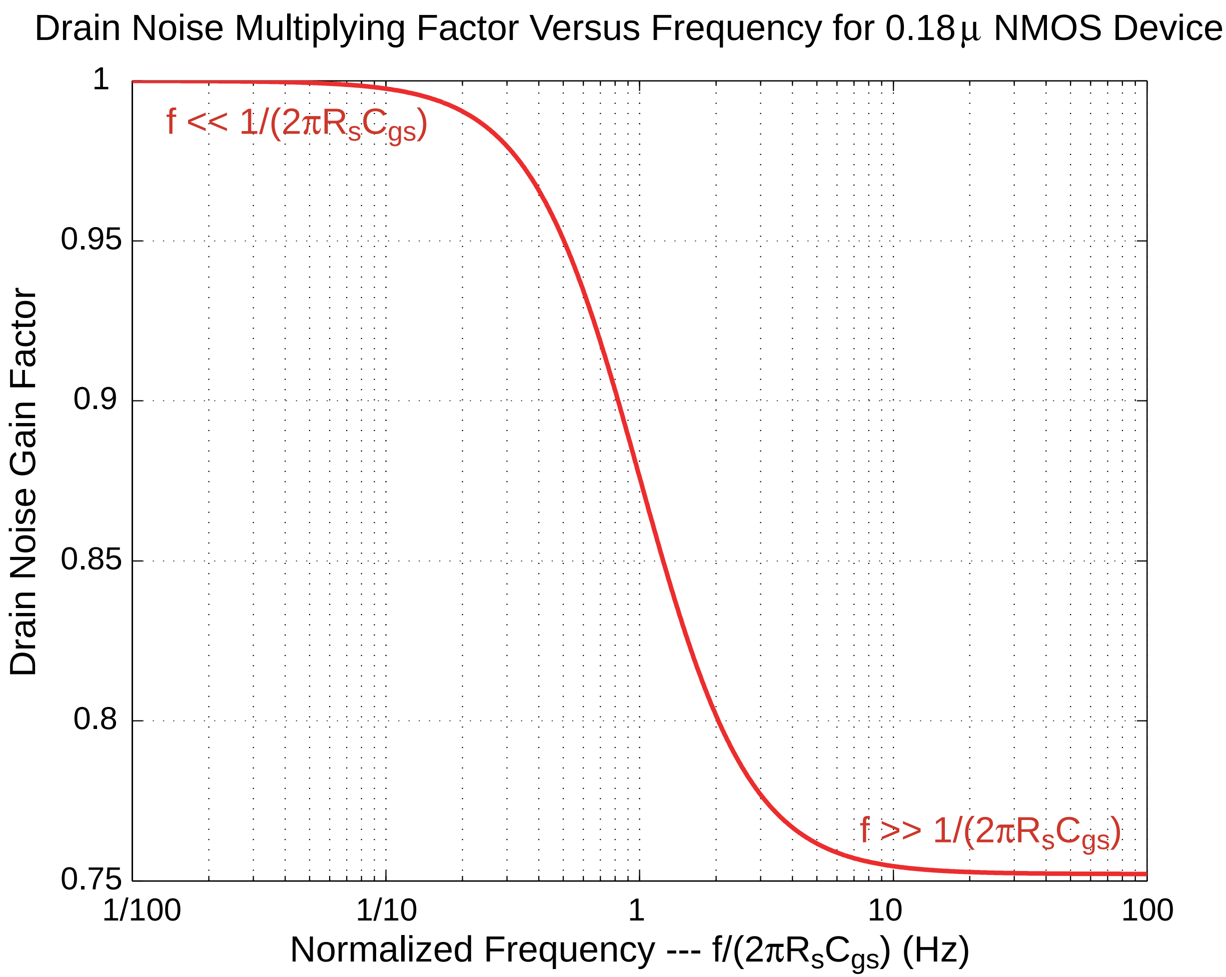
$$Z_{gsw} = wC_{gs}\left(R_s || \frac{1}{jwC_{gs}}\right) = \frac{wC_{gs}R_s}{1 + jwC_{gs}R_s}$$

- **For w >> 1/(R_sC_{gs}):**

$$Z_{gsw} \approx 1/j \qquad \Rightarrow \qquad \frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 - 2|c|\chi_d + \chi_d^2\right)$$

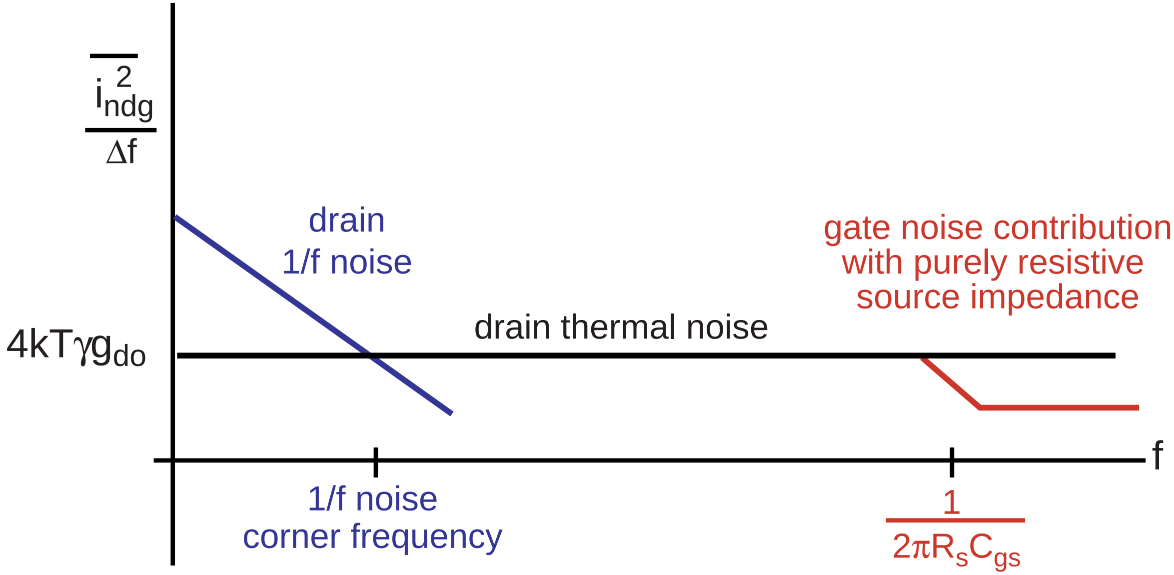
Gate noise contribution

Plot of Drain Noise Multiplying Factor (0.18μ NMOS)



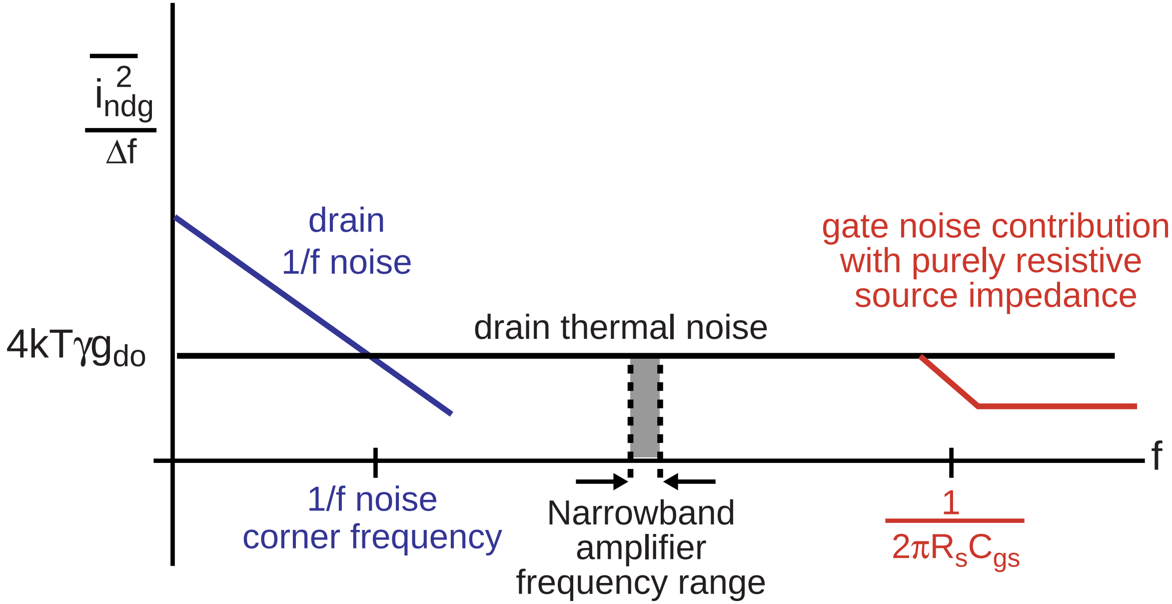
- **Conclusion: gate noise has little effect on common source amp when source impedance is purely resistive!**

Broadband Amplifier Design Considerations for Noise



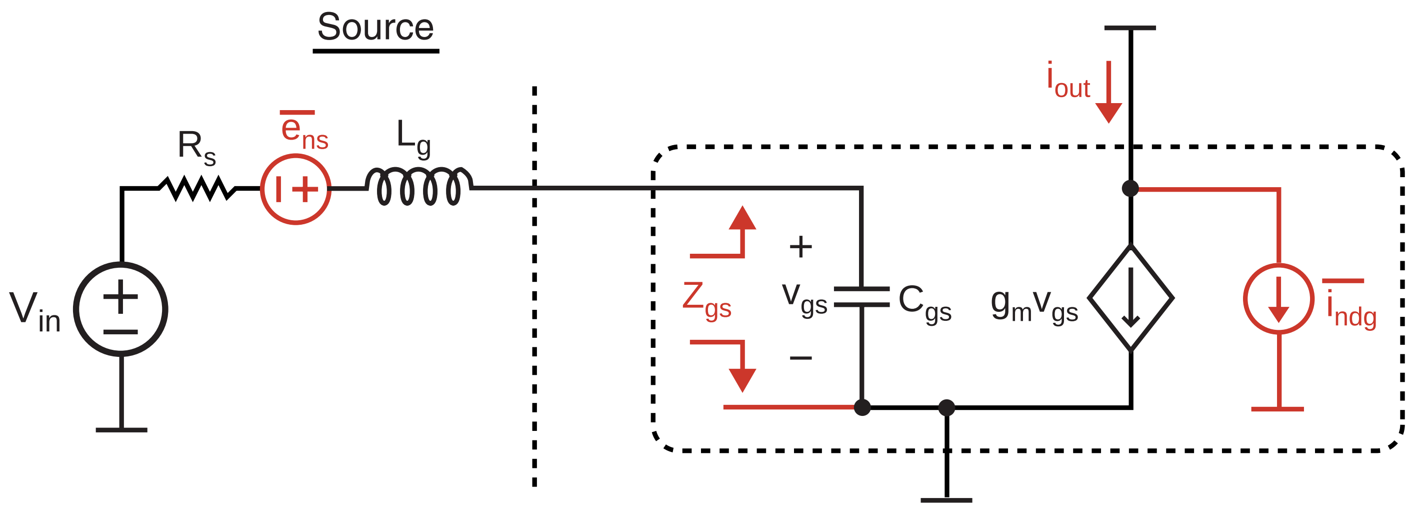
- **Drain thermal noise is the chief issue of concern when designing amplifiers with > 1 GHz bandwidth**
  - **1/f noise corner is usually less than 1 MHz**
  - **Gate noise contribution only has influence at high frequencies**
- **Noise performance specification is usually given in terms of input referred voltage noise**

Narrowband Amplifier Noise Requirements



- **Here we focus on a narrowband of operation**
  - **Don’t care about noise outside that band since it will be filtered out**
- **Gate noise is a significant issue here**
  - **Using reactive elements in the source dramatically impacts the influence of gate noise**
- **Specification usually given in terms of Noise Figure**

The Impact of Gate Noise with $Z_s = R_s + sL_g$



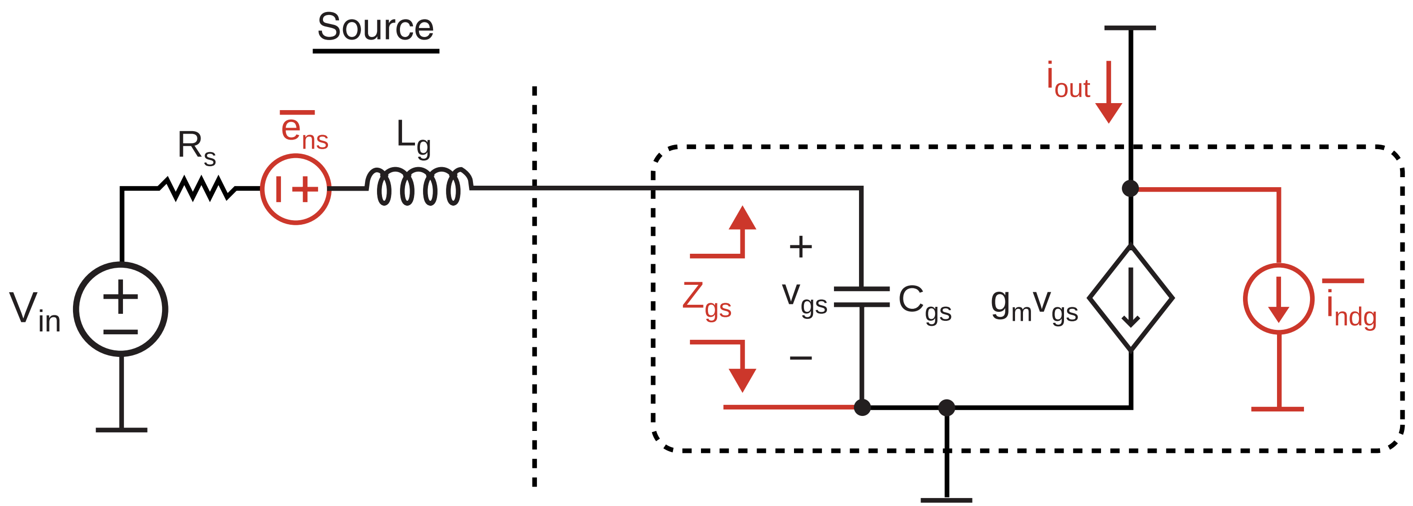
- **Step 1: Determine key noise parameters**
  - **For 0.18μ CMOS, again assume the following**

$$c = -j0.55, \quad \gamma = 3, \quad \delta = 2\gamma = 6, \quad \frac{g_m}{g_{do}} = \frac{1}{2} \Rightarrow \chi_d = 0.32$$

- **Step 2: Note that η =1 , calculate Z_{gsw}**

$$Z_{gsw} = wC_{gs}\left((R_s + jwL_g)||\frac{1}{jwC_{gs}}\right) = \frac{wC_{gs}(R_s + jwL_g)}{1 - w^2 L_g C_{gs} + jwC_{gs}R_s}$$

Evaluate Z_{gsw} At Resonance



- **Set L_g such that it resonates with C_{gs} at the center frequency (w_o) of the narrow band of interest**

$$\Rightarrow \quad \frac{1}{\sqrt{L_g C_{gs}}} = w_o \qquad \text{Note: } Q = \frac{1}{w_o C_{gs} R_s} = \frac{w_o L_g}{R_s}$$

- **Calculate Z_{gsw} at frequency w_o**

$$Z_{gsw} = \frac{w_o C_{gs}(R_s + j w_o L_g)}{1 - w_o^2 L_g C_{gs} + j w_o C_{gs} R_s} = w_o C_{gs}(Q^2 R_s - j\sqrt{L_g/C_{gs}})$$

$$= \boxed{Q - j}$$

The Impact of Gate Noise with $Z_s = R_s + sL_g$ (Cont.)

- **Key noise expression derived earlier**

$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 + 2Re\left\{-j|c|\chi_d Z_{gsw}\right\} + \chi_d^2 |Z_{gsw}|^2\right)$$

- **Substitute in for Z_{gsw}**

$$2Re\left\{-j|c|\chi_d Z_{gsw}\right\} = 2Re\left\{-j|c|\chi_d (Q - j)\right\} = -2|c|\chi_d$$

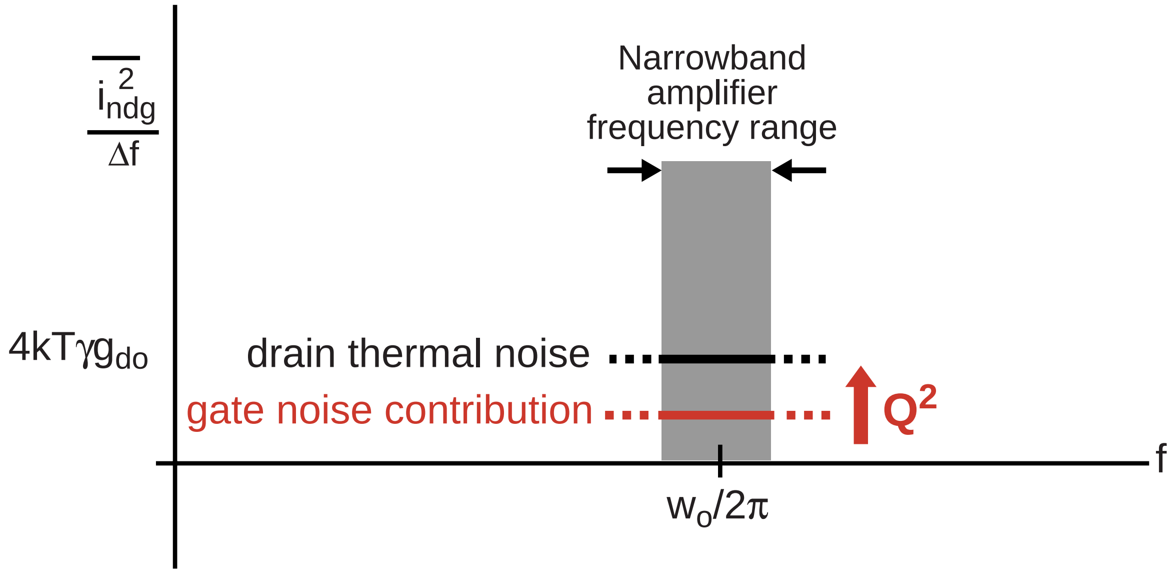
$$\chi_d^2 |Z_{gsw}|^2 = \chi_d^2 |Q - j|^2 = \chi_d^2 (Q^2 + 1)$$

$$\Rightarrow \quad \frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 - 2|c|\chi_d + \chi_d^2 (Q^2 + 1)\right)$$

Gate noise contribution

- **Gate noise contribution is a function of Q!**
  - **Rises monotonically with Q**

At What Value of Q Does Gate Noise Exceed Drain Noise?



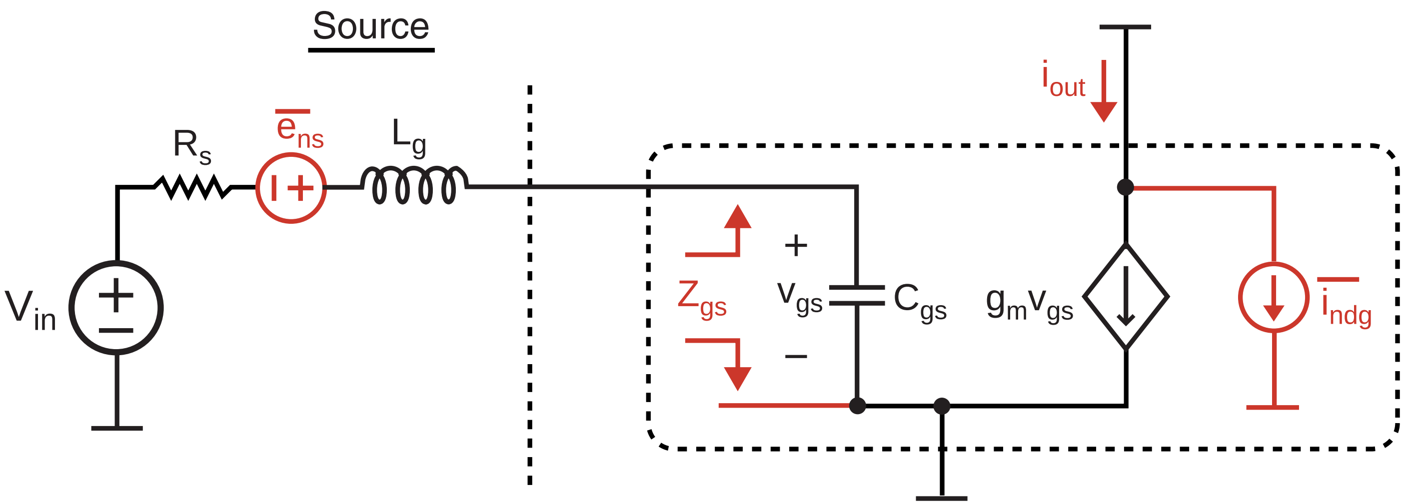
- Determine crossover point for Q value

$$\frac{\overline{i_{ndg}^2}}{\Delta f} = \frac{\overline{i_{nd}^2}}{\Delta f}\left(1 - 2|c|\chi_d + \chi_d^2(Q^2+1)\right) = \frac{\overline{i_{nd}^2}}{\Delta f}\,1$$

$$\Rightarrow \quad Q = \sqrt{1/\chi_d^2 - 1 + 2|c|/\chi_d} \quad (= 3.5 \text{ for } 0.18\mu \text{ specs})$$

  - Critical Q value for crossover is primarily set by process

Calculation of the Signal Spectrum at the Output



- **First calculate relationship between v_{in} and i_{out}**

$$i_{out,sig} = g_m v_{gs} = g_m \frac{1}{1 - w^2 L_g C_{gs} + jwR_s C_{gs}} V_{in}$$

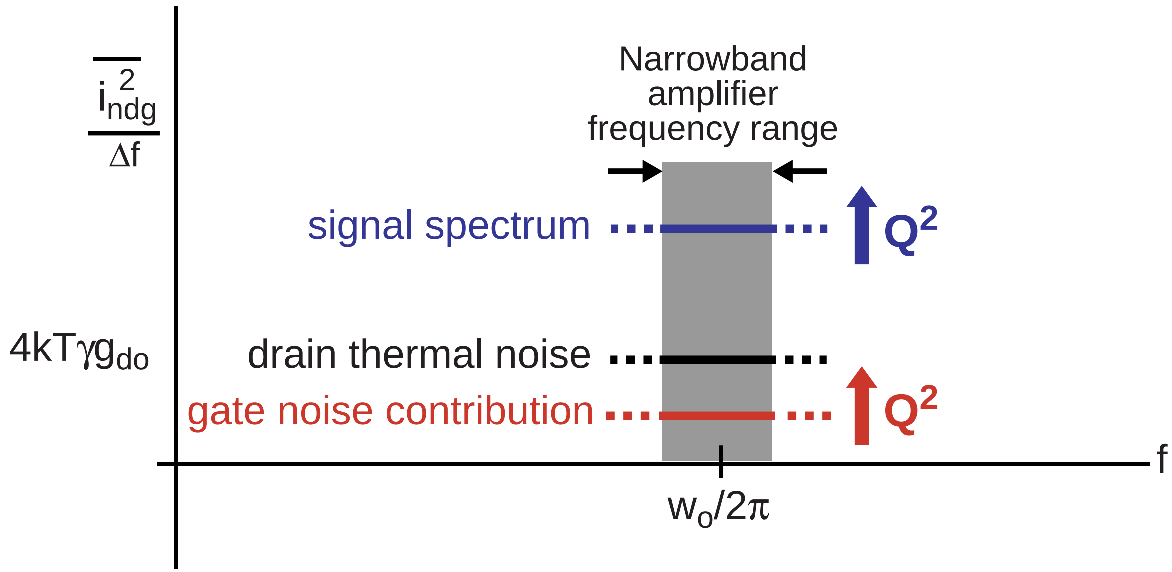
- **At resonance:**

$$i_{out,sig} = g_m v_{gs} = g_m \frac{1}{jw_o R_s C_{gs}} v_{in} = g_m(-jQ)v_{in}$$

- **Spectral density of signal at output at resonant frequency**

$$S_{iout,sig}(f) = |g_m(-jQ)|^2 S_{in}(f) = \boxed{(g_m Q)^2 S_{in}(f)}$$

Impact of Q on SNR (Ignoring R_s Noise)

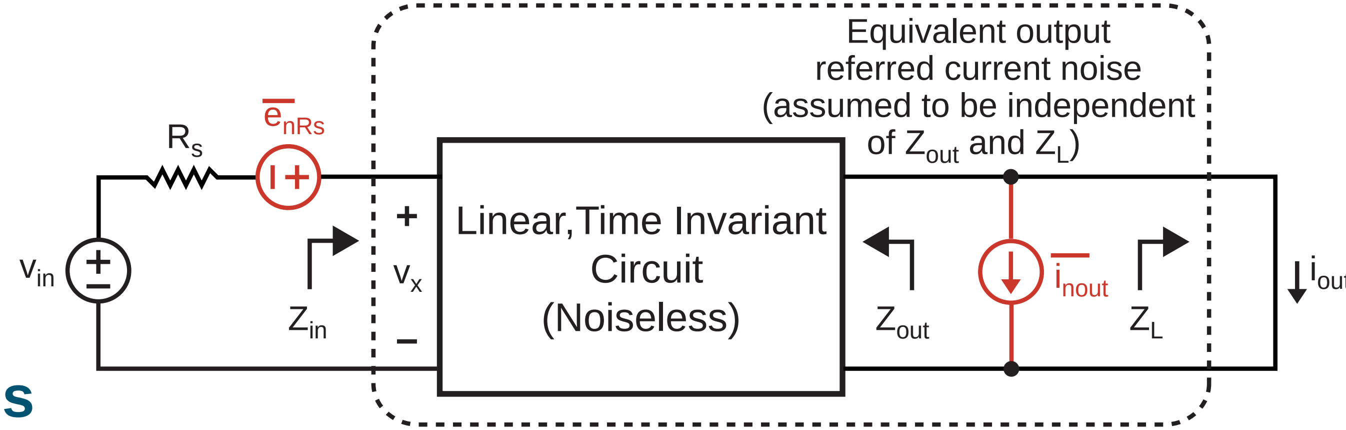


- **SNR (assume constant spectra, ignore noise from R_s):**

$$SNR_{out} = \frac{S_{iout,sig}(f)}{S_{iout,noise}(f)} \approx \frac{(g_m Q)^2 S_{in}(f)}{\overline{i_{ndg}^2}/\Delta f}$$

- **For small Q such that gate noise < drain noise**
  - **SNR_{out} improves dramatically as Q is increased**
- **For large Q such that gate noise > drain noise**
  - **SNR_{out} improves very little as Q is increased**

Noise Factor and Noise Figure



- **Definitions**

$$\text{Noise Factor} = F = \frac{SNR_{in}}{SNR_{out}}$$

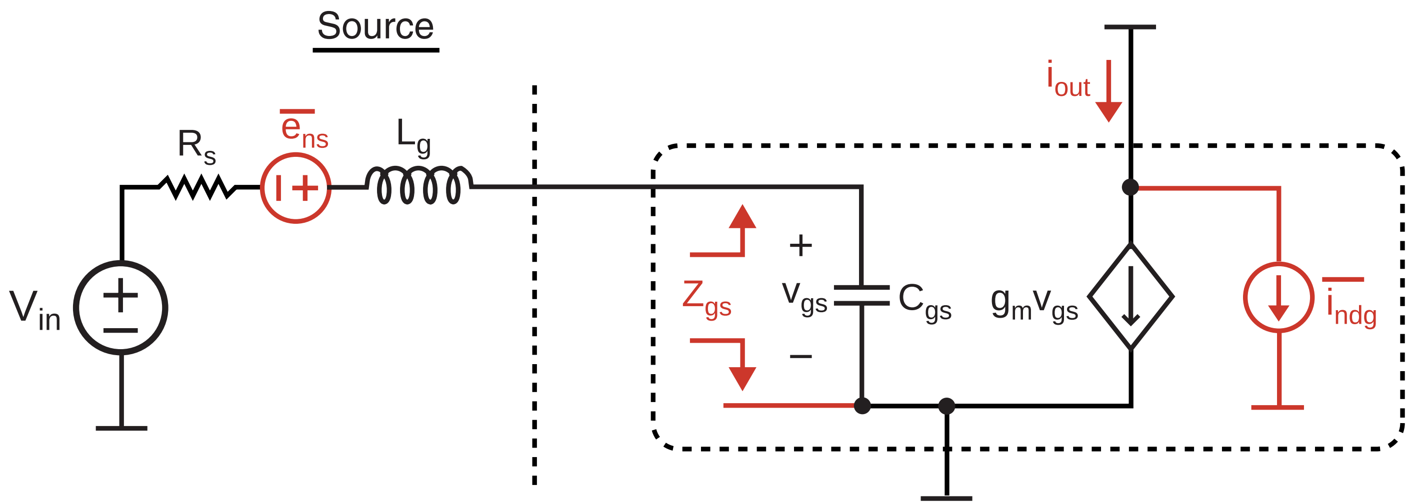
$$\text{Noise Figure} = 10\log(\text{Noise Factor})$$

- **Calculation of SNR_{in} and SNR_{out}**

$$SNR_{in} = \frac{|\alpha|^2 \overline{v_{in}^2}}{|\alpha|^2 \overline{e_{nRs}^2}} = \frac{\overline{v_{in}^2}}{\overline{e_{nRs}^2}} \quad \text{where } \alpha = \frac{Z_{in}}{R_s + Z_{in}}$$

$$SNR_{out} = \frac{|\alpha|^2 |G_m|^2 \overline{v_{in}^2}}{|\alpha|^2 |G_m|^2 \overline{e_{nRs}^2} + \overline{i_{nout}^2}} \quad \text{where } G_m = \frac{i_{out}}{v_x}$$

Calculate Noise Factor (Part 1)



- First calculate SNR_{out} (must include R_s noise for this)
  - R_s noise calculation (same as for V_{in})

$$i_{out,Rs} = g_m(-jQ)\overline{e_{ns}} \Rightarrow S_{iout,Rs}(f) = (g_m Q)^2 4kTR_s$$

  - SNR_{out}: $\Rightarrow$

$$SNR_{out} = \frac{(g_m Q)^2 S_{in}(f)}{\overline{i_{ndg}^2}/\Delta f + (g_m Q)^2 4kTR_s}$$

- Then calculate SNR_{in}:

$$SNR_{in} = \frac{S_{in}(f)}{\overline{e_{ns}^2}/\Delta f} = \frac{S_{in}(f)}{4kTR_s}$$

Calculate Noise Factor (Part 2)

$$SNR_{out} = \frac{|g_m Q|^2 S_{in}(f)}{\overline{i_{ndg}^2}/\Delta f + (g_m Q)^2 4kTR_s} \qquad SNR_{in} = \frac{S_{in}(f)}{\overline{e_{ns}^2}/\Delta f} = \frac{S_{in}(f)}{4kTR_s}$$

- **Noise Factor calculation:**

$$\text{Noise Factor} = \frac{SNR_{in}}{SNR_{out}} = \frac{\overline{i_{ndg}^2}/\Delta f + |g_m Q|^2 4kTR_s}{(g_m Q)^2 4kTR_s}$$

$$= 1 + \frac{\overline{i_{ndg}^2}/\Delta f}{(g_m Q)^2 4kTR_s}$$

- **From previous analysis**

$$\overline{i_{ndg}^2}/\Delta f = 4kT\gamma g_{do}\left(1 - 2|c|\chi_d + (Q^2+1)\chi_d^2\right)$$

$$\Rightarrow \boxed{\text{Noise Factor} = 1 + \frac{\gamma g_{do}\left(1 - 2|c|\chi_d + (Q^2+1)\chi_d^2\right)}{(g_m Q)^2 R_s}}$$

Calculate Noise Factor (Part 3)

$$\text{Noise Factor} = 1 + \frac{\gamma g_{do}\left(1 - 2|c|\chi_d + (Q^2+1)\chi_d^2\right)}{(g_m Q)^2 R_s}$$

- **Modify denominator using expressions for Q and w_t**

$$Q = \frac{1}{w_o R_s C_{gs}}, \quad w_t \approx \frac{g_m}{C_{gs}}$$

$$\Rightarrow \quad (g_m Q)^2 R_s = g_m^2 Q \frac{R_s}{w_o R_s C_{gs}} = g_m Q \frac{g_m}{C_{gs}} \frac{1}{w_o} = g_m Q \frac{w_t}{w_o}$$

- **Resulting expression for noise factor:**

$$\text{Noise Factor} = 1 + \left(\frac{w_o}{w_t}\right) \underline{\gamma \left(\frac{g_{do}}{g_m}\right) \frac{1}{Q}\left(1 - 2|c|\chi_d + (Q^2+1)\chi_d^2\right)}$$

Noise Factor scaling coefficient

  - **Noise factor primarily depends on Q, w_o/w_t, and process specs**

Minimum Noise Factor

$$\text{Noise Factor} = 1 + \left(\frac{w_o}{w_t}\right) \underline{\gamma \left(\frac{g_{do}}{g_m}\right) \frac{1}{Q} \left(1 - 2|c|\chi_d + (Q^2+1)\chi_d^2\right)}$$

Noise Factor scaling coefficient

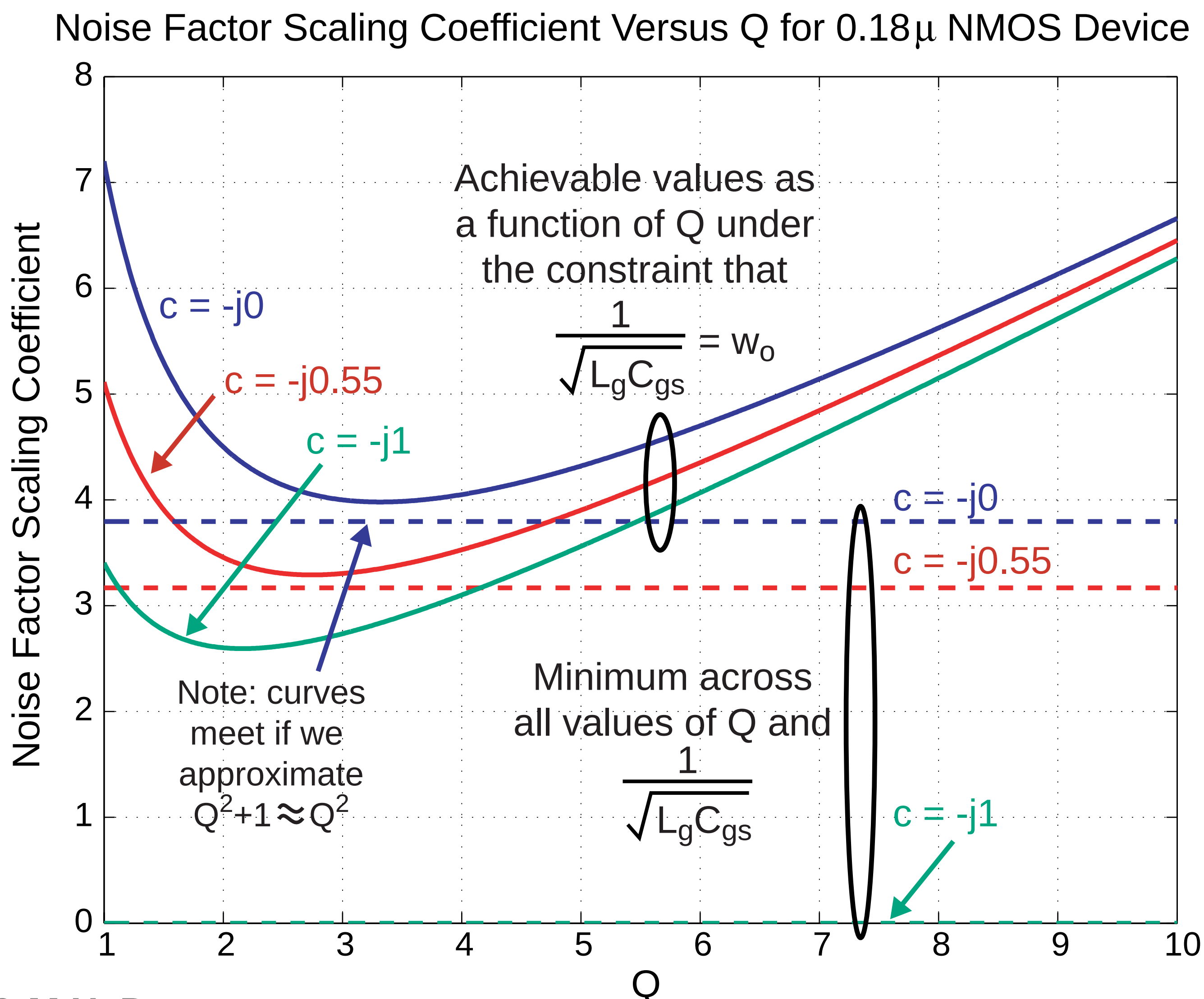
- **We see that the noise factor will be minimized for some value of Q**
  - **Could solve analytically by differentiating with respect to Q and solving for peak value (i.e. where deriv. = 0)**
- **In Tom Lee's book (pp 272-277), the minimum noise factor for the MOS common source amplifier (i.e. no degeneration) is found to be:**

$$\text{Min Noise Factor} = 1 + \left(\frac{w_o}{w_t}\right) \underline{\frac{2}{\sqrt{5}} \sqrt{\gamma\delta(1-|c|^2)}}$$

Noise Factor scaling coefficient

- **How do these compare?**

Plot of Minimum Noise Factor and Noise Factor Vs. Q



Achieving Minimum Noise Factor

- For common source amplifier without degeneration
  - Minimum noise factor can only be achieved at resonance if gate noise is uncorrelated to drain noise (i.e., if $c = 0$) – we'll see this next lecture
  - We typically must operate slightly away from resonance in practice to achieve minimum noise factor since c will be nonzero
- How do we determine the optimum source impedance to minimize noise figure in classical analysis?
  - Next lecture!