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High Speed Communication Circuits
Lecture 23

***Design of Fractional-N Frequency Synthesizers and
Bandwidth Extension Techniques***

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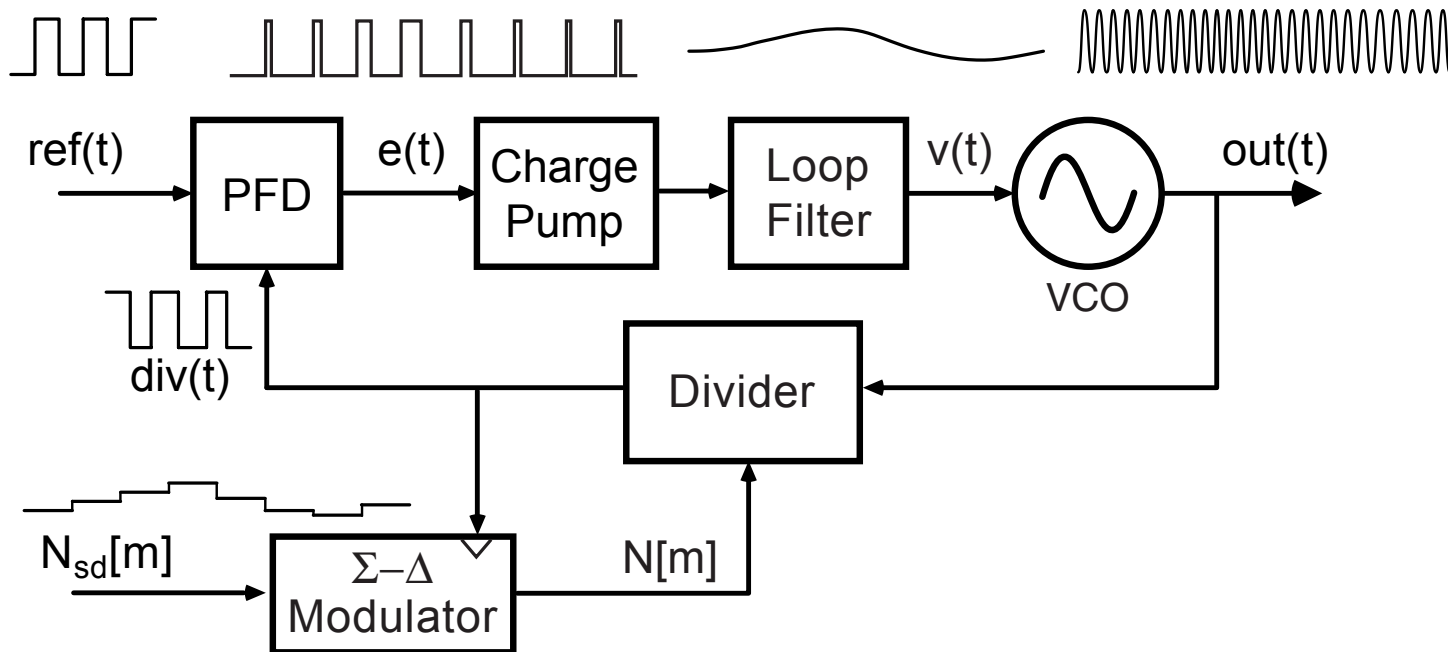
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Outline of PLL Lectures

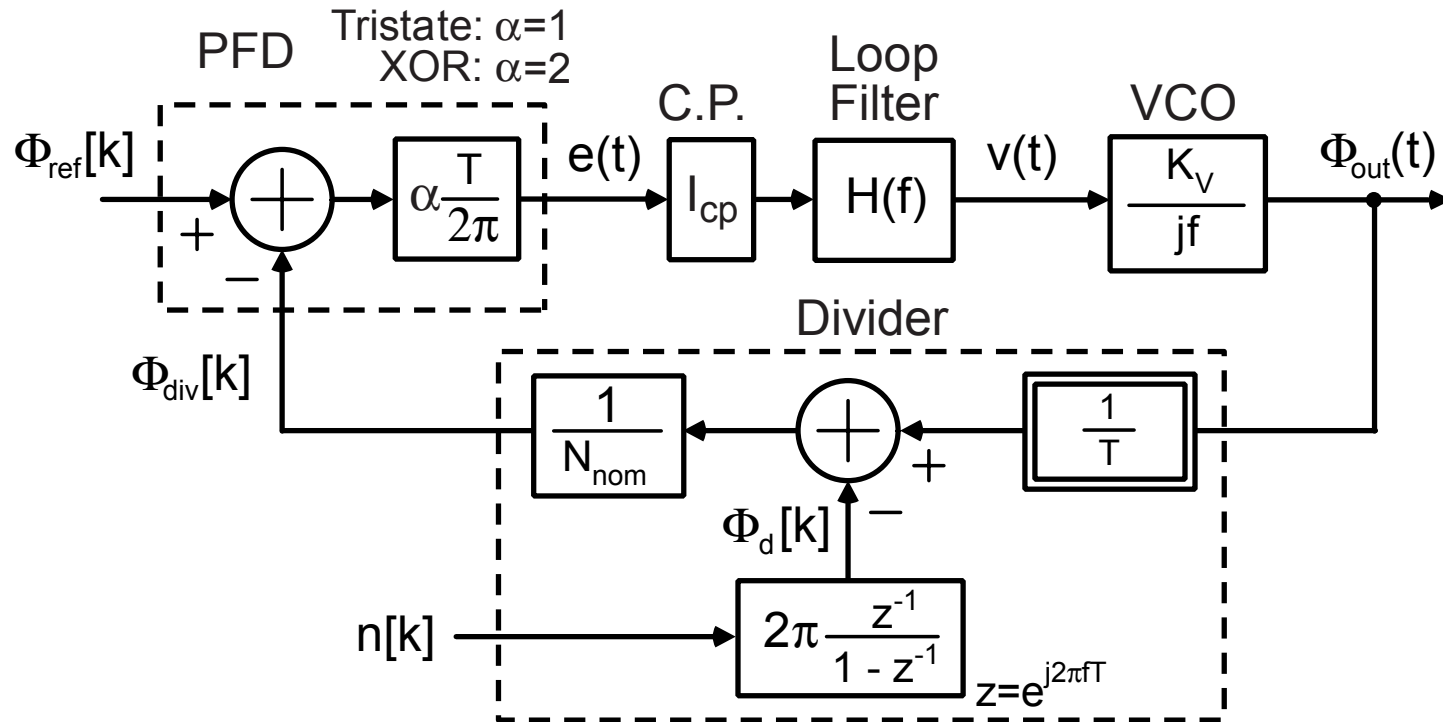
- **Integer-N Synthesizers**
 - Basic blocks, modeling, and design
 - Frequency detection, PLL Type
- **Noise in Integer-N and Fractional-N Synthesizers**
 - Noise analysis of integer-N structure
 - Sigma-Delta modulators applied to fractional-N structures
 - Noise analysis of fractional-N structure
- **Design of Fractional-N Frequency Synthesizers and Bandwidth Extension Techniques**
 - PLL Design Assistant Software
 - Quantization noise reduction for improved bandwidth and noise

Design of Frequency Synthesizers

- Focus on fractional-N architecture since it is essentially a “super set” of other PLL synthesizers
 - If we can design this structure, we can also design classical integer-N systems



Frequency-domain Model



Perrott et. al. *JSSC*, Aug. 2002

- Closed loop dynamics parameterized by

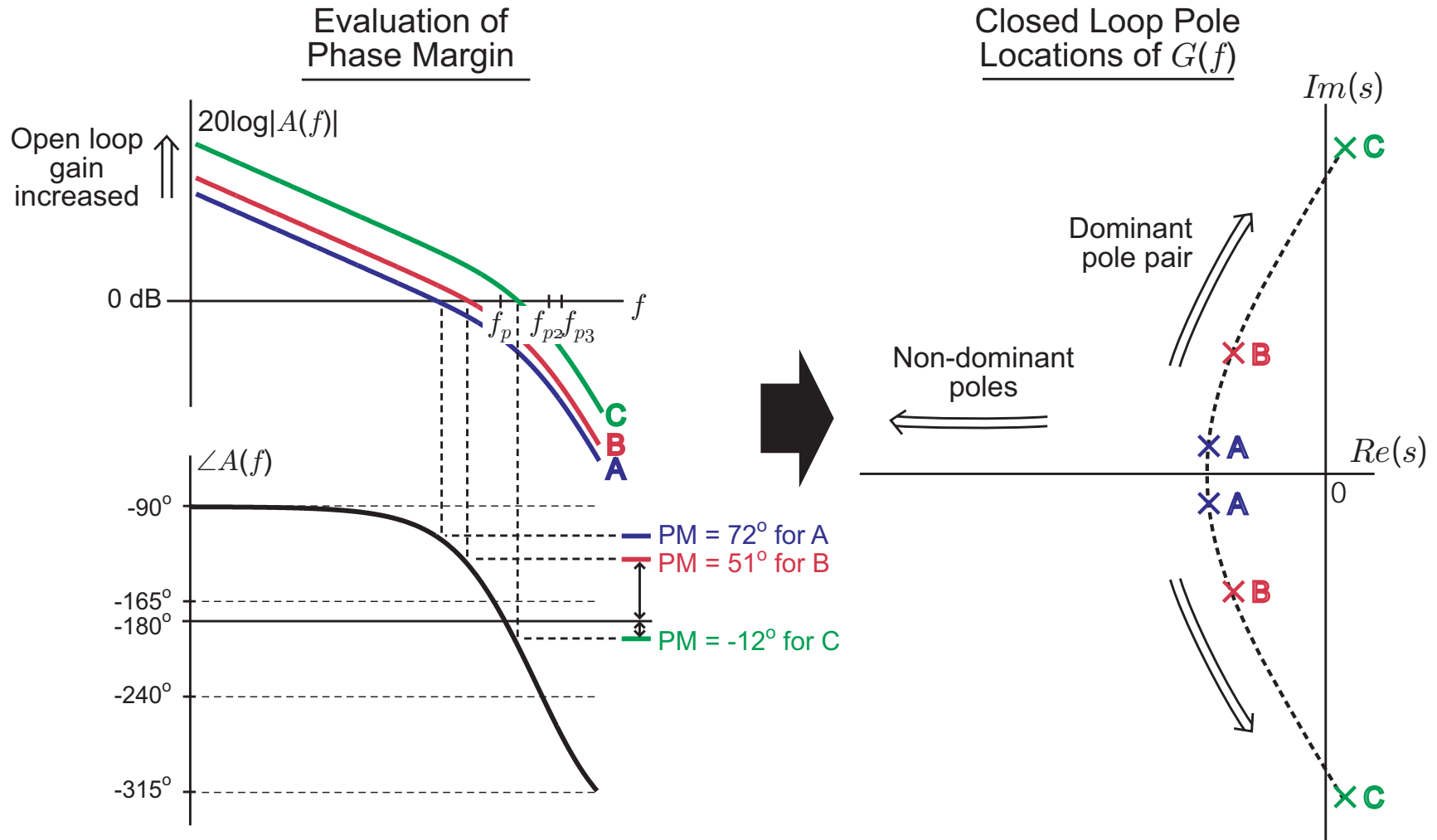
$$G(f) = \frac{A(f)}{1 + A(f)} \quad \text{where} \quad A(f) = \frac{\alpha I_{cp} H(f) K_V}{N_{nom} 2\pi j f}$$

Review of Classical Design Approach

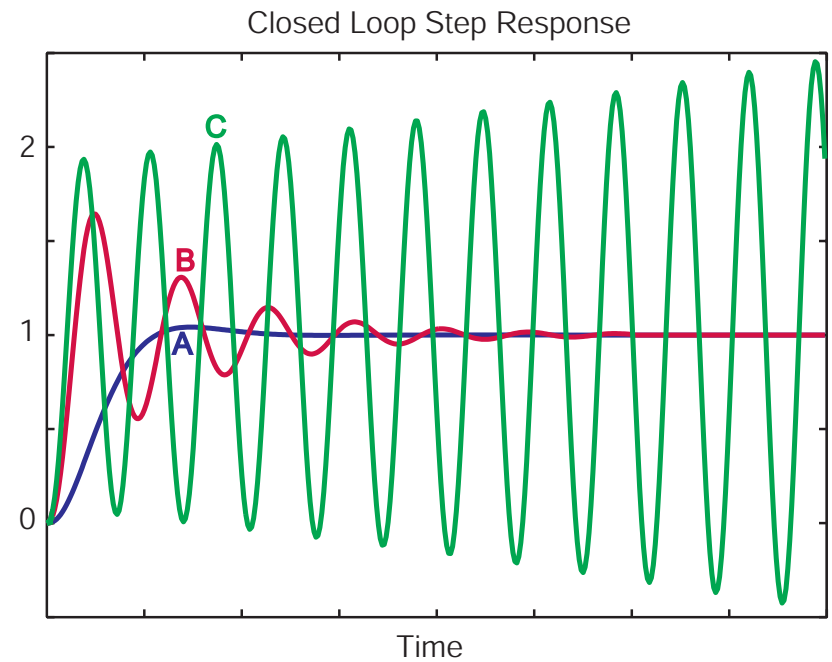
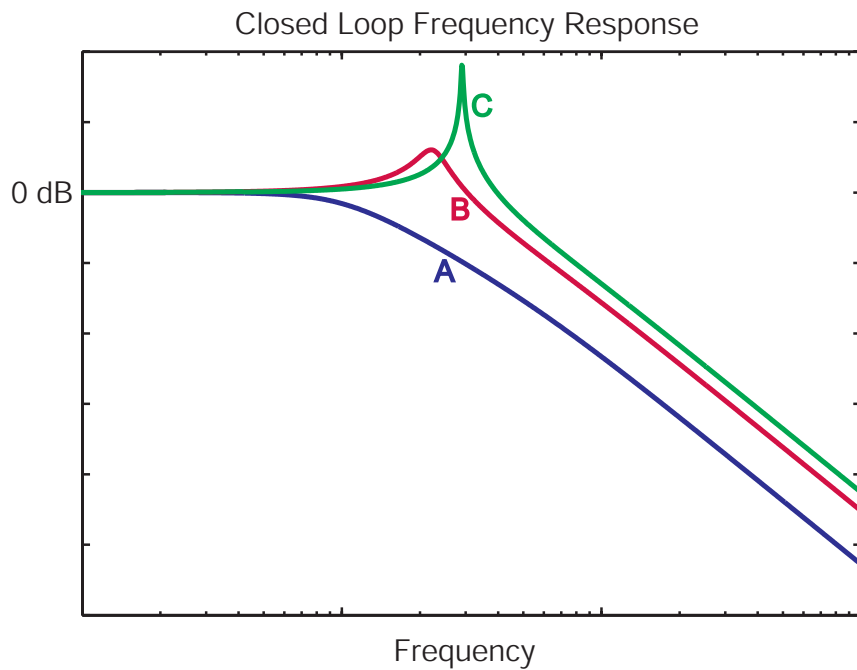
Given the desired closed-loop bandwidth, order, and system type:

- 1. Choose an appropriate topology for $H(f)$**
 - **Depends on order, type**
- 2. Choose pole/zero values for $H(f)$ as appropriate for the required bandwidth**
- 3. Adjust the open-loop gain to achieve the required bandwidth while maintaining stability**
 - **Plot gain and phase bode plots of $A(f)$**
 - **Use phase (or gain) margin criterion to infer stability**

Example: First Order, Type I with Parasitic Poles



First Order, Type I: Frequency and Step Responses



Limitations of Open Loop Design Approach

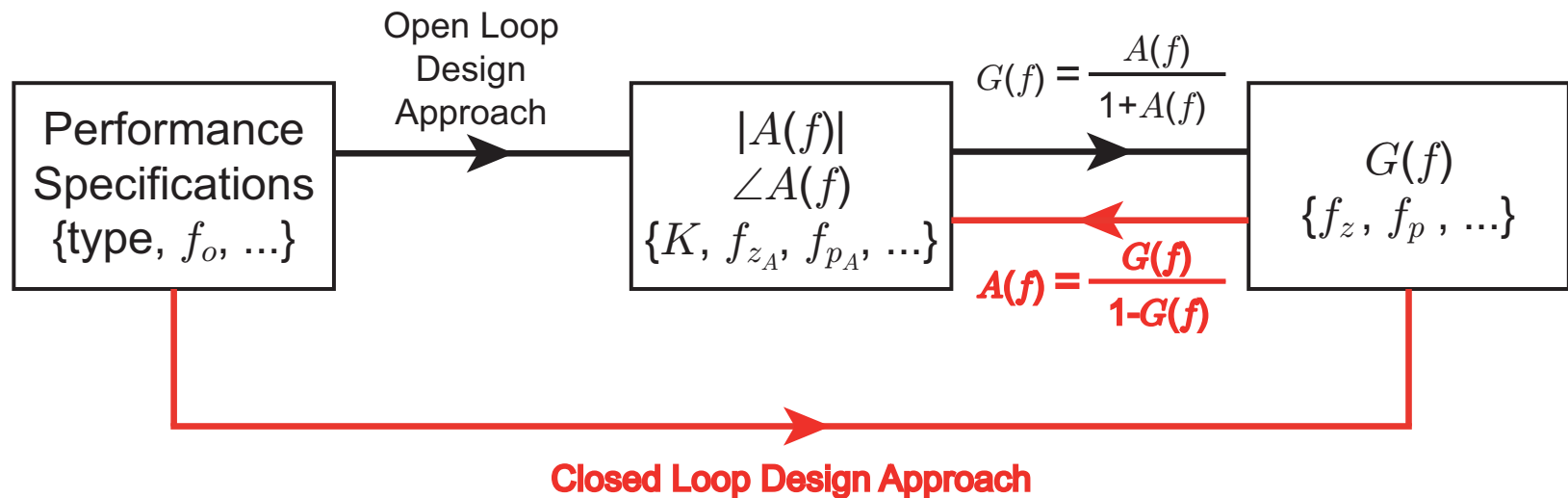
- **Constrained for applications which require precise filter response**
- **Complicated once parasitic poles are taken into account**
- **Poor control over filter shape**
- **Inadequate for systems with third order rolloff**
 - **Phase margin criterion based on second order systems**



**Closed loop design approach:
Directly design $G(f)$ by specifying dominant pole and zero locations on the s-plane (pole-zero diagram)**

Closed Loop Design Approach: Overview

- $G(f)$ completely describes the closed loop dynamics
 - Design of this function is the ultimate goal



- Instead of indirectly designing $G(f)$ using plots of $A(f)$, solve for $G(f)$ directly as a function of specification parameters
- Solve for $A(f)$ that will achieve desired $G(f)$
- Account for the impact of parasitic poles/zeros

Closed Loop Design Approach: Implementation

- **Download PLL Design Assistant Software at <http://www-mtl.mit.edu/research/perrottgroup/tools.html>**
- **Read accompanying manual**
- **Algorithm described by C.Y. Lau et. al. in “Fractional-N Frequency Synthesizer Design at the Transfer Function Level Using a Direct Closed Loop Realization Algorithm”, Design Automation Conference, 2003**

PLL Design Assistant

PLL Design Assistant

File Edit

Dynamic Parameters

fo: Hz

order: 1 2 3

shape: Butter Bessel

Cheby1 Cheby2 Elliptical

ripple: dB

type: 1 2

fz/fo: Hz

paris.pole:

paris.Q:

paris.pole:

paris.Q:

paris.pole:

paris.pole:

paris.pole:

paris.zero:

paris.zero:

Resulting Open Loop Parameters

K: alter: On

fp: Hz alter: On

fz: Hz alter: On

Qp: alter: On

Plot for PLL Design Assistant

Output Phase Noise of Synthesizer

Legend:

- S-D Noise
- Detector Noise
- VCO Noise
- Total Noise

Resulting Plots and Jitter

Apply

Pole/Zero Diagram Transfer Function

Step Response Noise Plot

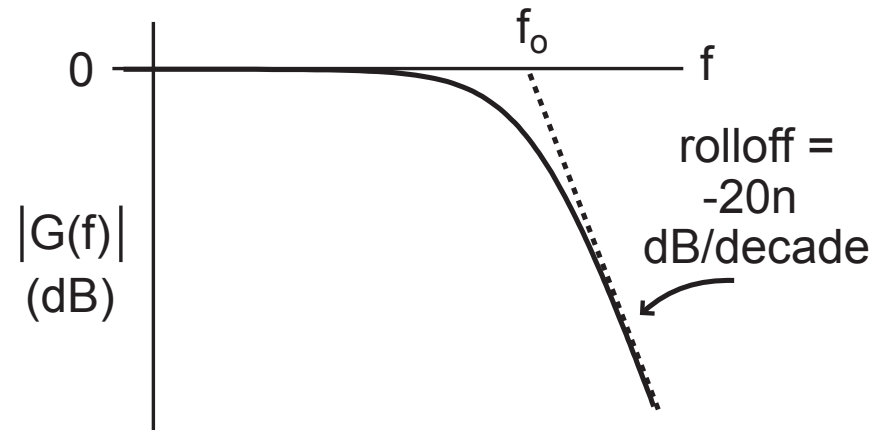
Ymin? Ymax?

rms jitter:

PLL Design Assistant

Written by Michael Perrott (<http://www-mtl.mit.edu/~perrott>)

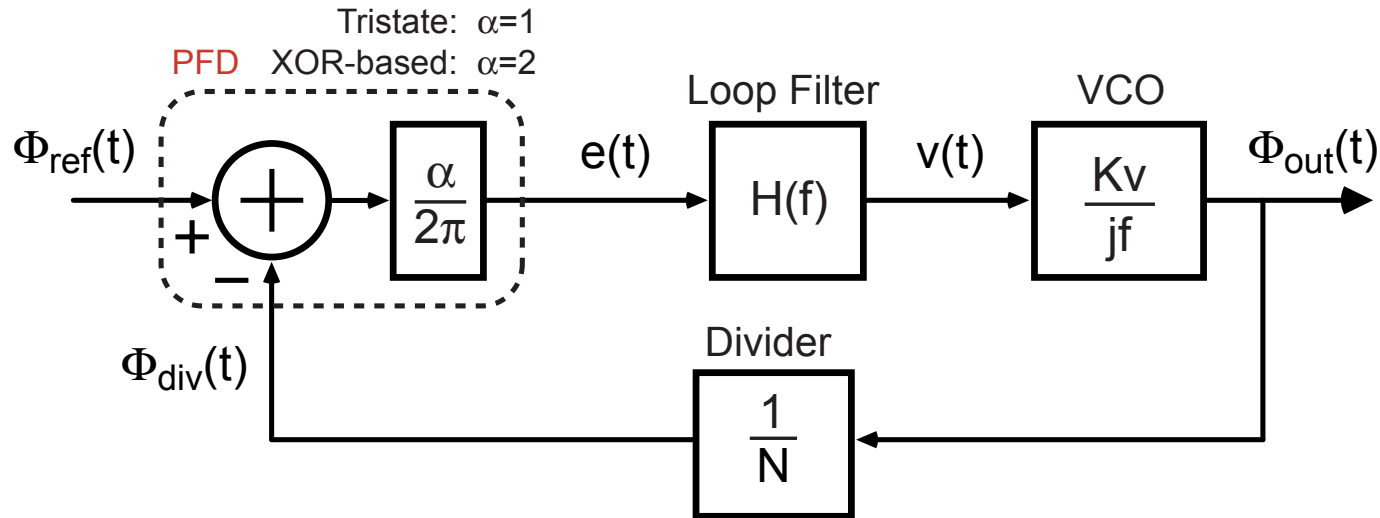
Definition of Bandwidth, Order, and Shape for $G(f)$



- **Bandwidth – f_0**
 - Defined in asymptotic manner as shown
- **Order – n**
 - Defined according to the rolloff characteristic of $G(f)$
- **Shape**
 - Defined according to standard filter design methodologies
 - Butterworth, Bessel, Chebyshev, etc.

Definition of Type

- **Type I: one integrator in PLL open loop transfer function**
 - VCO adds on integrator
 - Loop filter, $H(f)$, has no integrators
- **Type II: two integrators in PLL open loop transfer function**
 - Loop filter, $H(f)$, has one integrator



Loop Filter Transfer Function Vs Type and Order of G(f)

H(s) Topology For Different Type and Orders of G(f)

	Type I	Type II
Order 1	K_{LP}	$K_{LP} \frac{1+s/w_z}{s}$
Order 2	$\frac{K_{LP}}{1+s/w_p}$	$K_{LP} \frac{1+s/w_z}{s(1+s/w_p)}$
Order 3	$\frac{K_{LP}}{1+s/(w_p Q_p)+(s/w_p)^2}$	$\frac{K_{LP}(1+s/w_z)}{s(1+s/(w_p Q_p)+(s/w_p)^2)}$

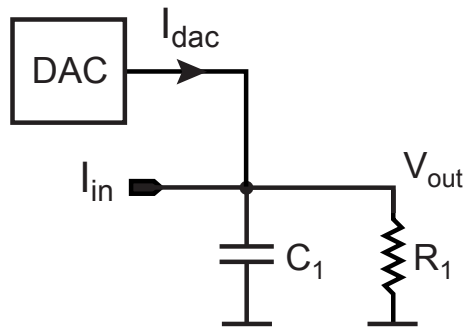
where $K_{LP} = K \frac{N_{nom}}{K_v I_{cp} \alpha}$

Calculated from software

- **Practical PLL implementations limited to above**
 - Prohibitive analog complexity for higher order, type
- **Open loop gain, K, will be calculated by algorithm**
 - Loop filter gain related to open loop gain as shown above

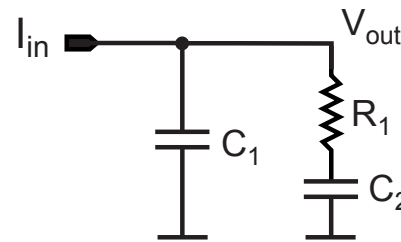
Passive Topologies to Realize a Second Order PLL

Type I, Order 2



$$\frac{V_{out}}{I_{in}} = \frac{R_1}{1+sR_1C_1}$$

Type II, Order 2

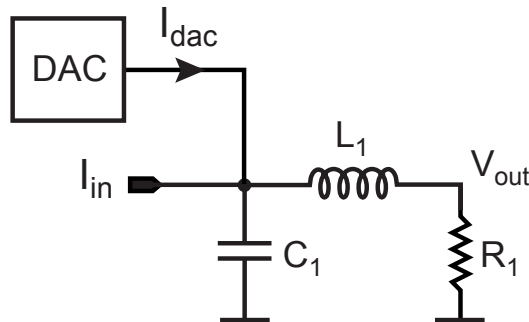


$$\frac{V_{out}}{I_{in}} = \frac{1}{s(C_1+C_2)} \frac{1+sR_1C_2}{1+sR_1C_{||}}$$

- **DAC is used for Type I implementation to coarsely tune VCO**
 - Allows full range of VCO to be achieved

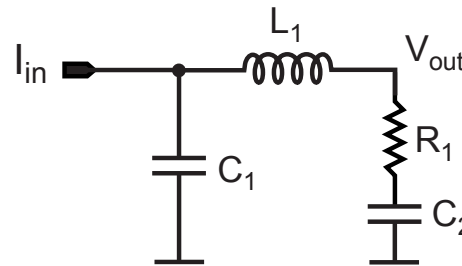
Passive Topologies to Realize a Third Order PLL

Type I, Order 3



$$\frac{V_{out}}{I_{in}} = \frac{R_1}{1+sR_1C_1+s^2L_1C_1}$$

Type II, Order 3



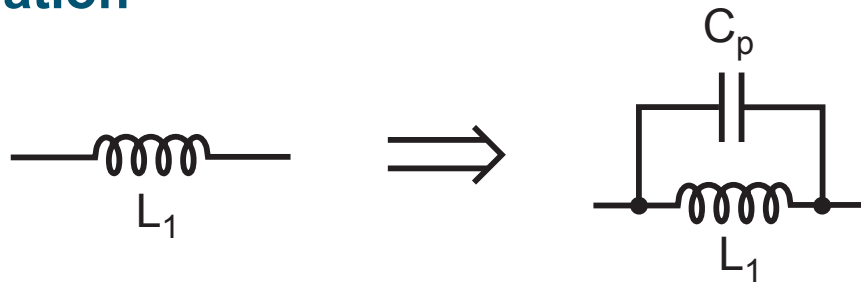
$$\frac{V_{out}}{I_{in}} = \frac{1}{s(C_1+C_2)} \frac{1+sR_1C_2}{1+sR_1C_{||}+s^2L_1C_{||}}$$

where $C_{||} = C_1C_2/(C_1+C_2)$

- Inductor is necessary to create a complex pole pair
 - Must be implemented off-chip due to its large value

Problem with Passive Loop Filter Implementations

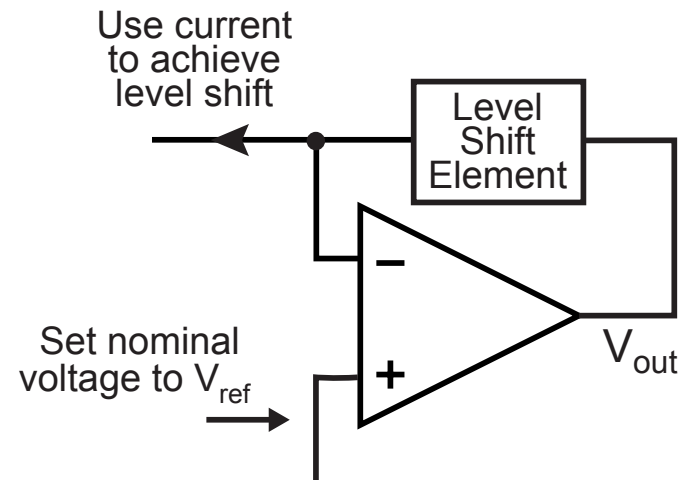
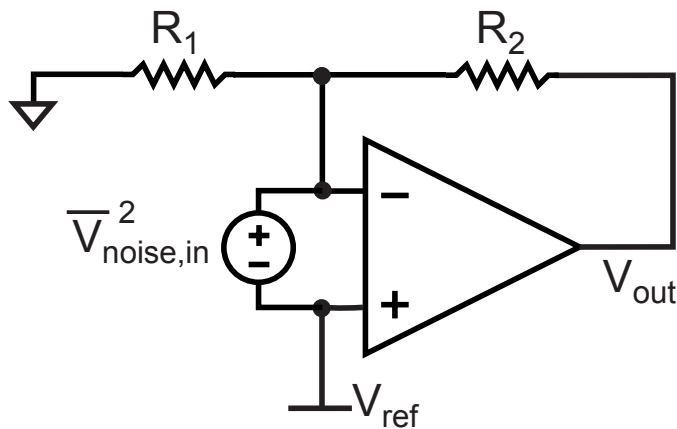
- Large voltage swing required at charge pump output
 - Must support full range of VCO input
- Non-ideal behavior of inductors (for third order $G(f)$ implementations)
 - Hard to realize large inductor values
 - Self resonance of inductor reduces high frequency attenuation



Alternative: active loop filter implementation

Guidelines for Active Loop Filter Design

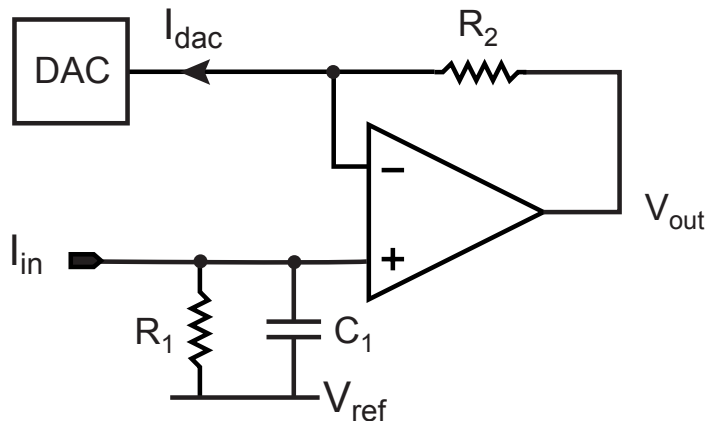
- Use topologies with unity gain feedback in the opamp
 - Minimizes influence of opamp noise
- Perform level shifting in feedback of opamp
 - Fixes voltage at charge pump output



- Prevent fast edges from directly reaching opamp inputs
 - Will otherwise cause opamp to be driven into nonlinear region of operation

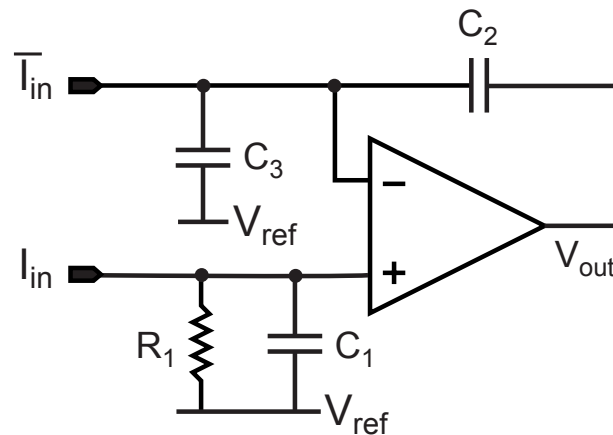
Active Topologies To Realize a Second Order PLL

Type I, Order 2



$$\frac{V_{out}}{I_{in}} = \frac{R_1}{1+sR_1C_1}$$

Type II, Order 2

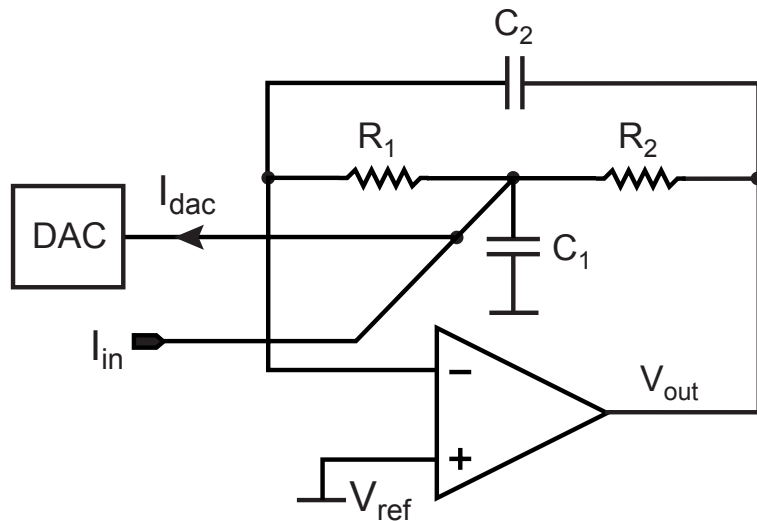


$$\frac{V_{out}}{I_{in}} = \frac{1+sR_1(C_1+C_2+C_3)}{sC_2(1+sR_1C_1)}$$

- Follows guidelines from previous slide
- Charge pump output is terminated directly with a high Q capacitor
 - Smooths fast edges from charge pump before they reach the opamp input(s)

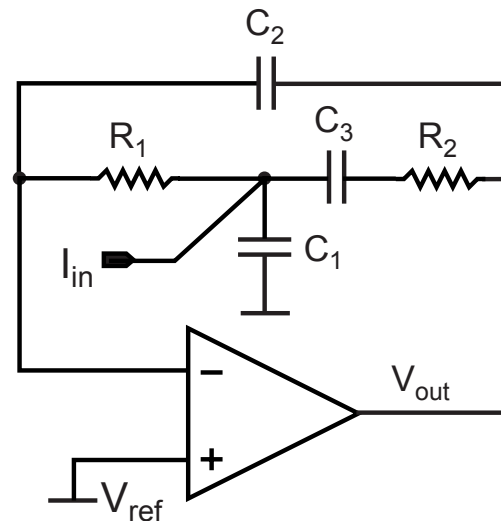
Active Topologies To Realize a Third Order PLL

Type I, Order 3



$$\frac{V_{out}}{I_{in}} = \frac{-R_2}{1+s(R_1+R_2)C_2+s^2R_1R_2C_1C_2}$$

Type II, Order 3



$$\frac{V_{out}}{I_{in}} = \frac{-1}{s(C_1+C_2)} \frac{1+sR_2C_3}{1+sC_{||}(R_1(1+C_1/C_3)+R_2)+s^2R_1R_2C_1C_{||}}$$

where $C_{||} = C_2C_3/(C_2+C_3)$

- Follows active implementation guidelines from a few slides ago

Example Design

- Type II, 3rd order, Butterworth, $f_o = 300\text{kHz}$, $f_z/f_o = 0.125$
 - No parasitic poles
- Required loop filter transfer function can be found from table:

$$\Rightarrow H(s) = \frac{K_{LF} \left(1 + \frac{s}{w_z}\right)}{s \left(1 + \frac{s}{w_p Q_p} + \left(\frac{s}{w_p}\right)^2\right)} \quad \text{where}$$

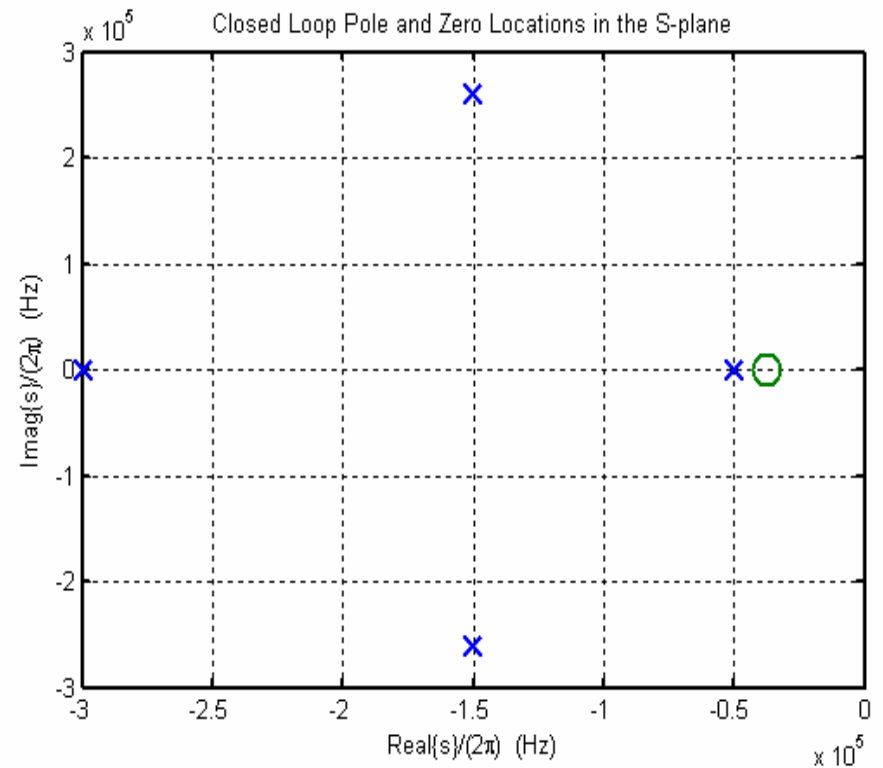
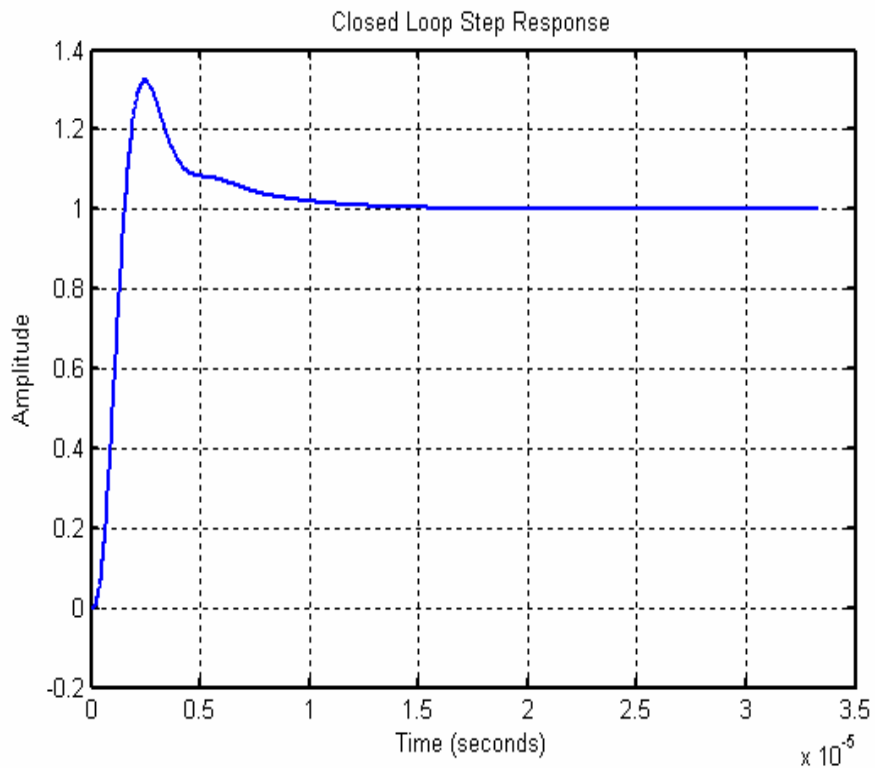
$$K_{LF} = \frac{N_{nom} K}{\alpha I_{cp} K_v}$$

Use PLL Design Assistant to Calculate Parameters

$$H(s) = \frac{K_{LF} \left(1 + \frac{s}{w_z}\right)}{s \left(1 + \frac{s}{w_p Q_p} + \left(\frac{s}{w_p}\right)^2\right)} \quad \text{where} \quad K_{LF} = \frac{N_{nom} K}{\alpha I_{cp} K_v}$$

Dynamic Parameters		Noise Parameters	
fo: <input type="text" value="300e3"/> Hz	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	ref. freq: <input type="text"/> Hz	
order: <input type="radio"/> 1 <input type="radio"/> 2 <input checked="" type="radio"/> 3	paris. Q: <input type="text"/> <input type="button" value="On"/>	out freq.: <input type="text"/> Hz	
shape: <input checked="" type="radio"/> Butter <input type="radio"/> Bessel	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	Detector: <input type="text"/> dBc/Hz <input type="button" value="On"/>	
<input type="radio"/> Cheby1 <input type="radio"/> Cheby2 <input type="radio"/> Elliptical	paris. Q: <input type="text"/> <input type="button" value="On"/>	VCO: <input type="text"/> dBc/Hz <input type="button" value="On"/>	
ripple: <input type="text"/> dB	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	freq. offset: <input type="text"/> Hz	
type: <input type="radio"/> 1 <input checked="" type="radio"/> 2	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	S-D: <input type="radio"/> 1 <input type="radio"/> 2 <input type="button" value="On"/>	<input type="text"/> <input type="button" value="On"/>
fz/fo: <input type="text" value="0.125"/> Hz	paris. zero: <input type="text"/> Hz <input type="button" value="On"/>	<input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5	
	paris. zero: <input type="text"/> Hz <input type="button" value="On"/>		
Resulting Open Loop Parameters		Resulting Plots and Jitter	
K: <input type="text" value="2.538e+011"/>	alter: <input type="text"/> <input type="button" value="On"/>	<input checked="" type="radio"/> Pole/Zero Diagram	<input type="radio"/> Transfer Function
fp: <input type="text" value="4.583e+005"/> Hz	alter: <input type="text"/> <input type="button" value="On"/>	<input type="radio"/> Step Response	<input type="radio"/> Noise Plot
fz: <input type="text" value="3.750e+004"/> Hz	alter: <input type="text"/> <input type="button" value="On"/>	Xmin? <input type="text"/>	Xmax? <input type="text"/>
Qp: <input type="text" value="7.050e-001"/>	alter: <input type="text"/> <input type="button" value="On"/>	Ymin? <input type="text"/>	Ymax? <input type="text"/>
<input type="button" value="Apply"/>		rms jitter: <input type="text"/>	
		PLL Design Assistant	
Written by Michael Perrott (http://www-mtl.mit.edu/~perrott)			

Resulting Step Response and Pole/Zero Diagram

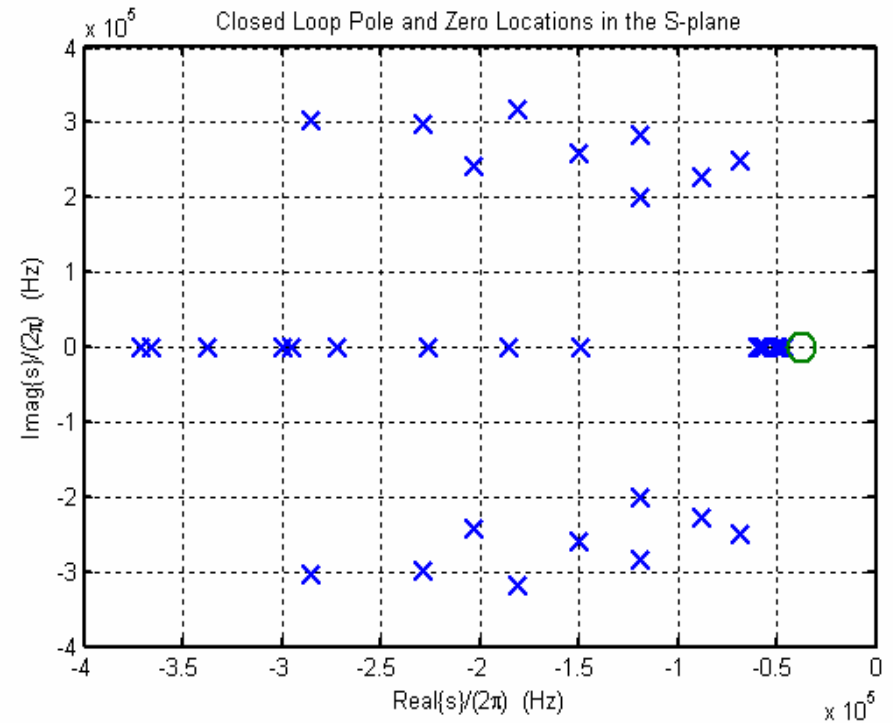
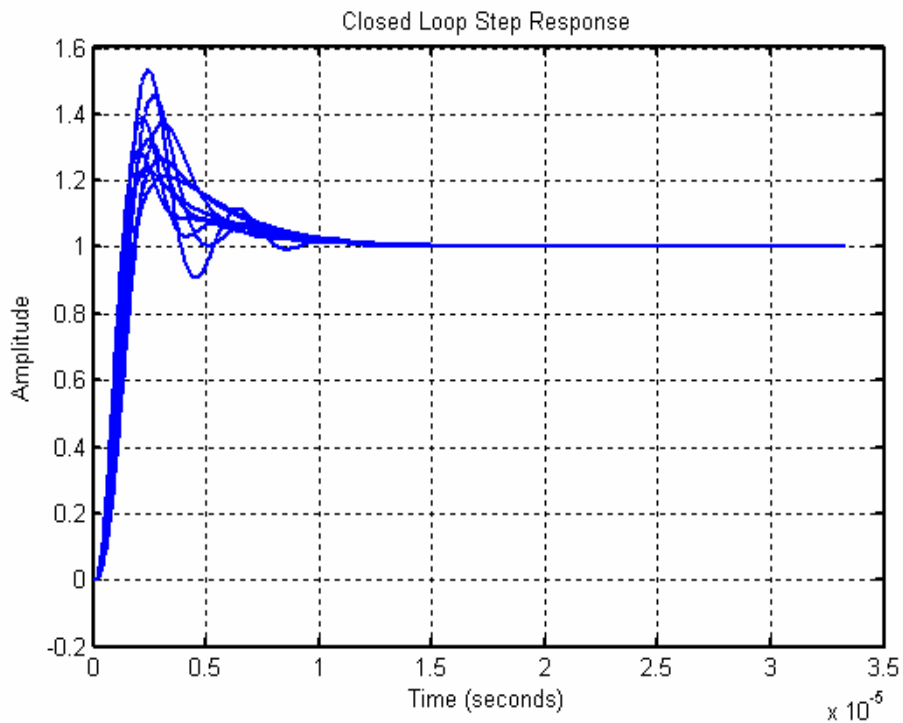


Impact of Open Loop Parameter Variations

Dynamic Parameters		Noise Parameters	
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order: <input type="radio"/> 1 <input type="radio"/> 2 <input checked="" type="radio"/> 3	paris. Q: <input type="text"/> <input type="button" value="On"/>	Detector: <input type="text"/> dBc/Hz <input type="button" value="On"/>	VCO: <input type="text"/> dBc/Hz <input type="button" value="On"/>
shape: <input checked="" type="radio"/> Butter <input type="radio"/> Bessel	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	freq. offset: <input type="text"/> Hz	S-D: <input type="radio"/> 1 <input type="radio"/> 2 <input type="button" value="On"/> <input type="text"/> <input type="button" value="On"/>
<input type="radio"/> Cheby1 <input type="radio"/> Cheby2 <input type="radio"/> Elliptical	paris. Q: <input type="text"/> <input type="button" value="On"/>		
ripple: <input type="text"/> dB	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>		
type: <input type="radio"/> 1 <input checked="" type="radio"/> 2	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>		
fz/fo: <input type="text" value="0.125"/> Hz	paris. zero: <input type="text"/> Hz <input type="button" value="On"/>		
	paris. zero: <input type="text"/> Hz <input type="button" value="On"/>		
Resulting Open Loop Parameters		Resulting Plots and Jitter	
K: <input type="text" value="2.538e+011"/>	alter: <input type="text" value="-0.2:0.2:0.2"/> <input type="button" value="On"/>	<input type="radio"/> Pole/Zero Diagram	<input type="radio"/> Transfer Function
fp: <input type="text" value="4.583e+005"/> Hz	alter: <input type="text" value="-0.2:0.2:0.2"/> <input type="button" value="On"/>	<input checked="" type="radio"/> Step Response	<input type="radio"/> Noise Plot
fz: <input type="text" value="3.750e+004"/> Hz	alter: <input type="text"/> <input type="button" value="On"/>	Xmin? <input type="text"/>	Xmax? <input type="text"/>
Qp: <input type="text" value="7.050e-001"/>	alter: <input type="text"/> <input type="button" value="On"/>	Ymin? <input type="text"/>	Ymax? <input type="text"/>
<input type="button" value="Apply"/>		rms jitter: <input type="text"/>	
PLL Design Assistant		Written by Michael Perrott (http://www-mtl.mit.edu/~perrott)	

- Open loop parameter variations can be directly entered into tool

Resulting Step Responses and Pole/Zero Diagrams



- Impact of variations on the loop dynamics can be visualized instantly and taken into account at early stage of design

Design with Parasitic Pole

- Include a parasitic pole at nominal value $f_{p1} = 1.2\text{MHz}$

$$H(s) = \frac{K_{LF} \left(1 + \frac{s}{w_z}\right)}{s \left(1 + \frac{s}{w_p Q_p} + \left(\frac{s}{w_p}\right)^2\right) \left(1 + \frac{s}{w_{p1}}\right)}$$

Dynamic Parameters		Noise Parameters	
fo: <input type="text" value="300e3"/> Hz	paris. pole: <input type="text" value="1.2e6"/> Hz <input type="checkbox"/>	ref. freq: <input type="text" value="Value?"/> Hz	
order: <input type="radio"/> 1 <input type="radio"/> 2 <input checked="" type="radio"/> 3	paris. Q: <input type="text"/> <input type="checkbox"/>	out freq: <input type="text" value="Value?"/> Hz	
shape: <input checked="" type="radio"/> Butter <input type="radio"/> Bessel	paris. pole: <input type="text"/> Hz <input type="checkbox"/>	Detector: <input type="text"/> dBc/Hz <input type="checkbox"/>	
<input type="radio"/> Cheby1 <input type="radio"/> Cheby2 <input type="radio"/> Elliptical	paris. Q: <input type="text"/> <input type="checkbox"/>	VCO: <input type="text"/> dBc/Hz <input type="checkbox"/>	
ripple: <input type="text"/> dB	paris. pole: <input type="text"/> Hz <input type="checkbox"/>	freq. offset: <input type="text"/> Hz	
type: <input type="radio"/> 1 <input checked="" type="radio"/> 2	paris. pole: <input type="text"/> Hz <input type="checkbox"/>	S-D: <input type="radio"/> 1 <input type="radio"/> 2 <input type="checkbox"/> <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="checkbox"/>	
fz/fo: <input type="text" value="1/8"/> Hz	paris. zero: <input type="text"/> Hz <input type="checkbox"/>		
	paris. zero: <input type="text"/> Hz <input type="checkbox"/>		
Resulting Open Loop Parameters		Resulting Plots and Jitter	
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fz: <input type="text" value="3.750e+004"/> Hz	alter: <input type="text"/> <input type="checkbox"/>	Xmin?: <input type="text"/>	Xmax?: <input type="text"/>
Qp: <input type="text" value="7.931e-001"/>	alter: <input type="text"/> <input type="checkbox"/>	Ymin?: <input type="text"/>	Ymax?: <input type="text"/>
PLL Design Assistant		rms jitter: <input type="text"/>	
		Written by Michael Perrott (http://www-mtl.mit.edu/~perrott)	

⇒ K , f_p and Q_p are adjusted to obtain the same dominant pole locations

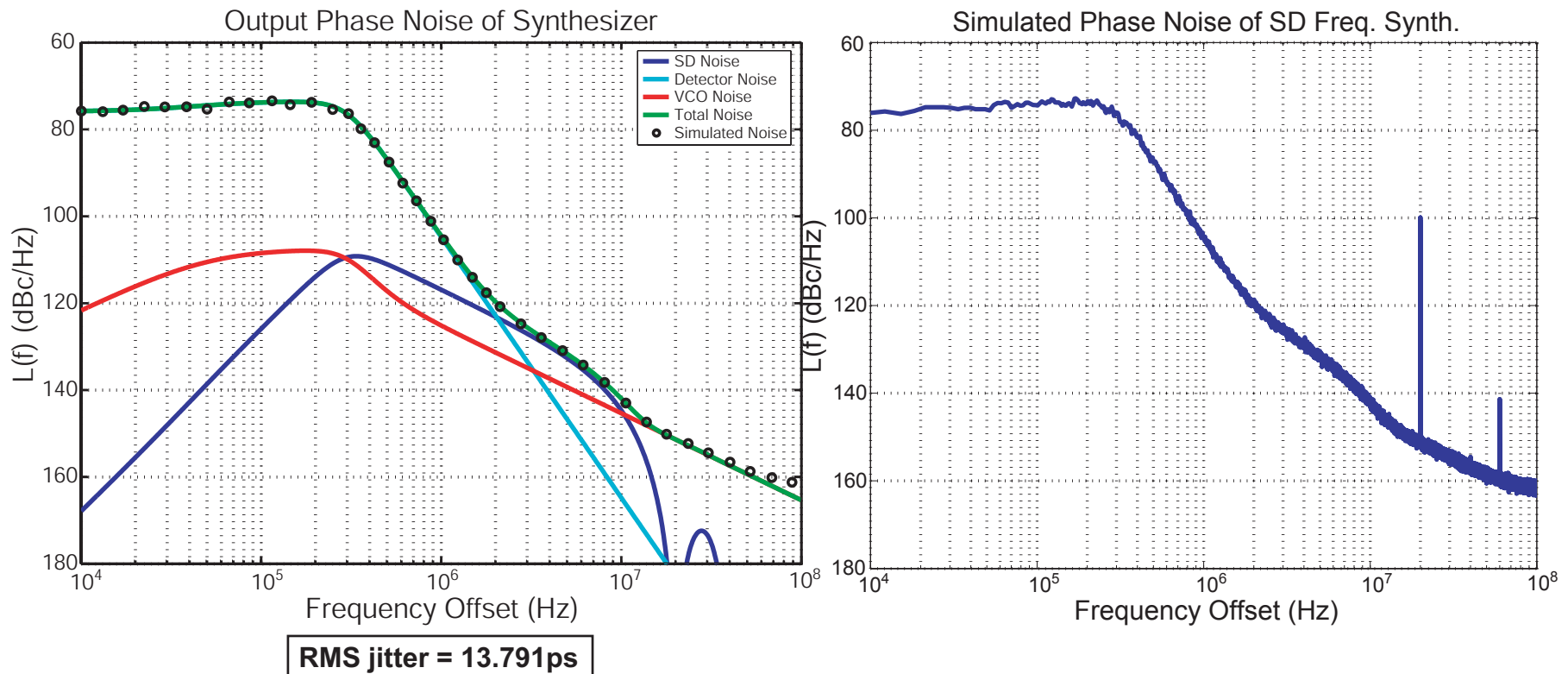
Noise Estimation

- Phase noise plots can be easily obtained
 - Jitter calculated by integrating over frequency range

Dynamic Parameters		Noise Parameters	
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order: <input type="radio"/> 1 <input type="radio"/> 2 <input checked="" type="radio"/> 3	paris. Q: <input type="text"/> <input type="button" value="On"/>	out freq.: <input type="text" value="1.84e9"/> Hz	<input type="button" value="On"/>
shape: <input checked="" type="radio"/> Butter <input type="radio"/> Bessel	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	Detector: <input type="text" value="-75.9"/> dBc/Hz <input type="button" value="On"/>	<input type="button" value="On"/>
<input type="radio"/> Cheby1 <input type="radio"/> Cheby2 <input type="radio"/> Elliptical	paris. Q: <input type="text"/> <input type="button" value="On"/>	VCO: <input type="text" value="-139.3"/> dBc/Hz <input type="button" value="On"/>	<input type="button" value="On"/>
ripple: <input type="text"/> dB	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	freq. offset: <input type="text" value="5e6"/> Hz	<input type="button" value="On"/>
type: <input type="radio"/> 1 <input checked="" type="radio"/> 2	paris. pole: <input type="text"/> Hz <input type="button" value="On"/>	S-D: <input type="radio"/> 1 <input type="radio"/> 2 <input type="button" value="On"/>	<input type="button" value="On"/>
fz/fo: <input type="text" value="0.125"/> Hz	paris. zero: <input type="text"/> Hz <input type="button" value="On"/>	<input checked="" type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5	<input type="button" value="On"/>
paris. zero: <input type="text"/> Hz <input type="button" value="On"/>			
Resulting Open Loop Parameters		Resulting Plots and Jitter	
K: <input type="text" value="2.538e+011"/> alter: <input type="text"/> <input type="button" value="On"/>	<input type="button" value="Apply"/>	<input type="radio"/> Pole/Zero Diagram	<input type="radio"/> Transfer Function
fp: <input type="text" value="4.583e+005"/> Hz alter: <input type="text"/> <input type="button" value="On"/>		<input type="radio"/> Step Response	<input checked="" type="radio"/> Noise Plot
fz: <input type="text" value="3.750e+004"/> Hz alter: <input type="text"/> <input type="button" value="On"/>		<input type="text" value="1e4"/> <input type="text" value="1e8"/> <input type="text" value="-180"/> <input type="text" value="-60"/>	
Qp: <input type="text" value="7.050e-001"/> alter: <input type="text"/> <input type="button" value="On"/>		rms jitter: <input type="text" value="13.791 ps"/>	
PLL Design Assistant		Written by Michael Perrott (http://www-ml.mit.edu/~perrott)	

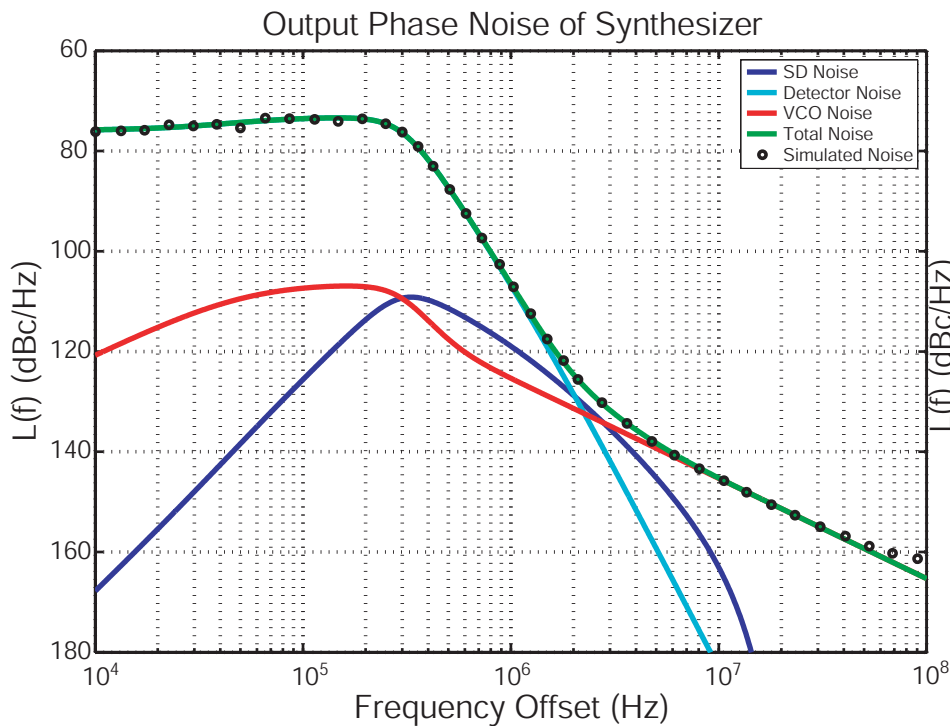
Calculated Versus Simulated Phase Noise Spectrum

Without parasitic pole:

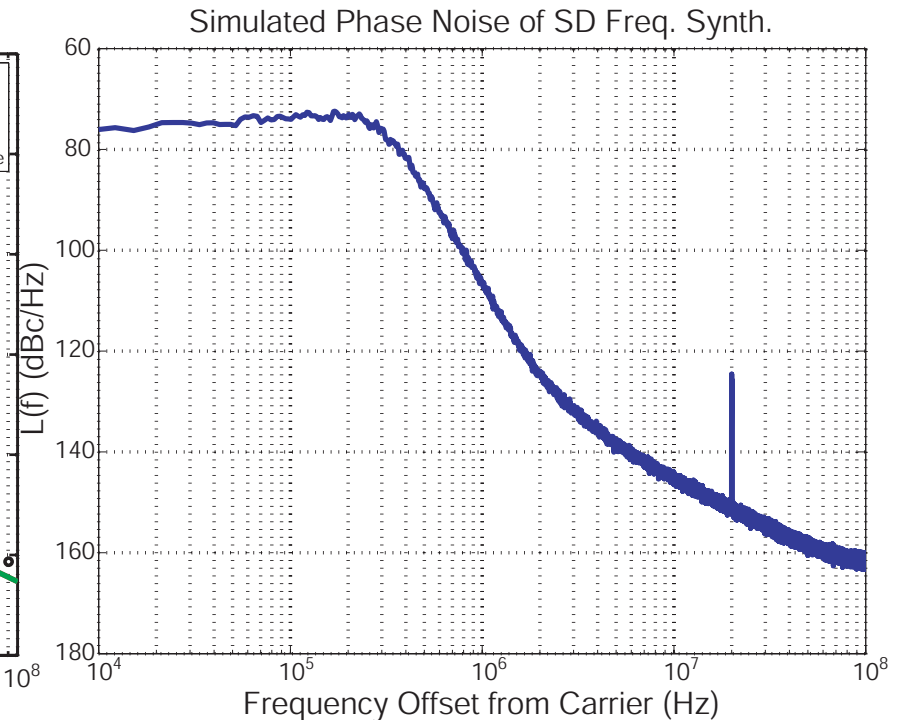


Calculated Versus Simulated Phase Noise Spectrum

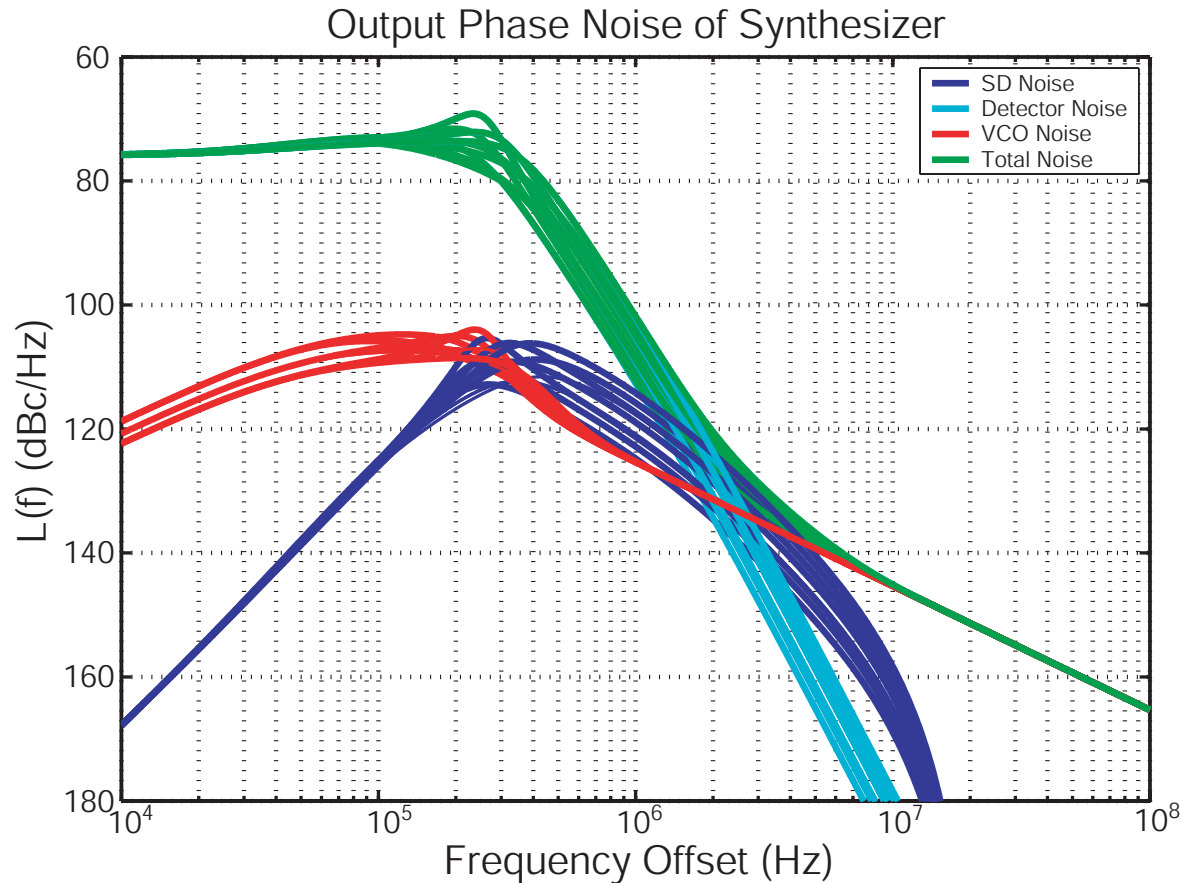
With parasitic pole at 1.2 MHz:



RMS jitter = 14.057ps



Noise under Open Loop Parameter Variations



RMS jitter = 11.678ps (min), 18.211ps (max)

- Impact of open loop parameter variations on phase noise and jitter can be visualized immediately

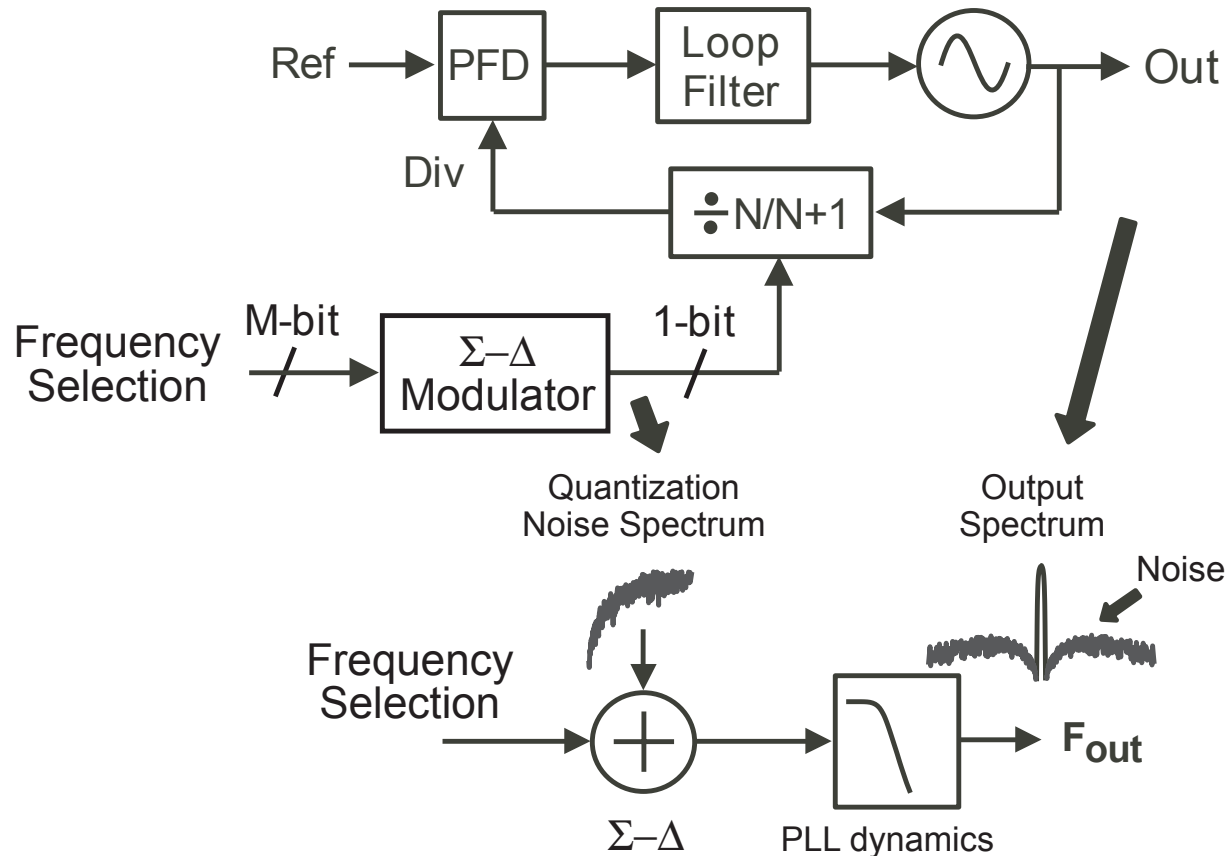
Conclusion

- **New closed loop design approach facilitates:**
 - **Accurate control of closed loop dynamics**
 - Bandwidth, Order, Shape, Type
 - **Straightforward design of higher order PLL's**
 - **Direct assessment of impact of parasitic poles/zeros**
- **Techniques implemented in a GUI-based CAD tool**

- **Beginners can quickly come up to speed in designing PLL's**
- **Experienced designers can quickly evaluate the performance of different PLL configurations**

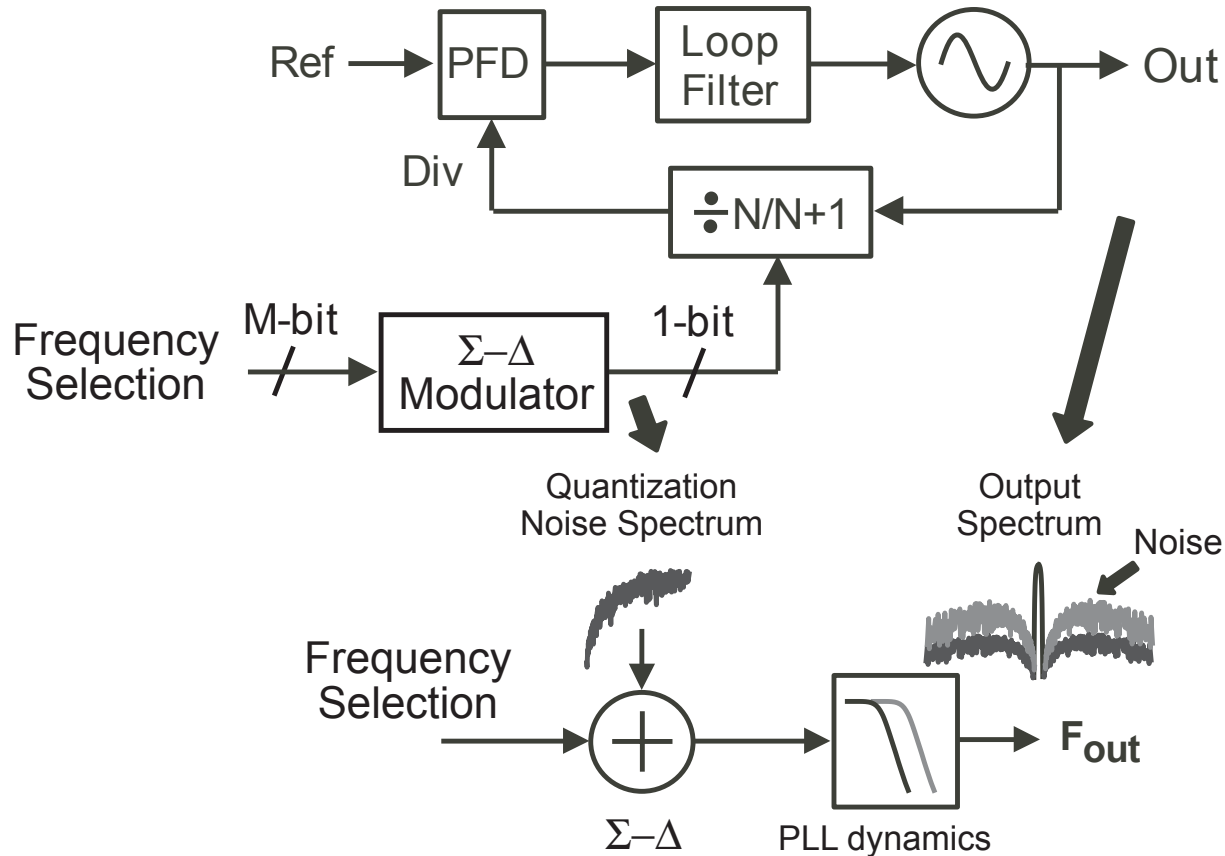
Bandwidth Extension of Fractional-N Frequency Synthesizers

Impact of $\Sigma\text{-}\Delta$ Quantization Noise on Synth. Output



- **Lowpass action of PLL dynamics suppresses the shaped $\Sigma\text{-}\Delta$ quantization noise**

Impact of Increasing the PLL Bandwidth



- Higher PLL bandwidth leads to less quantization noise suppression

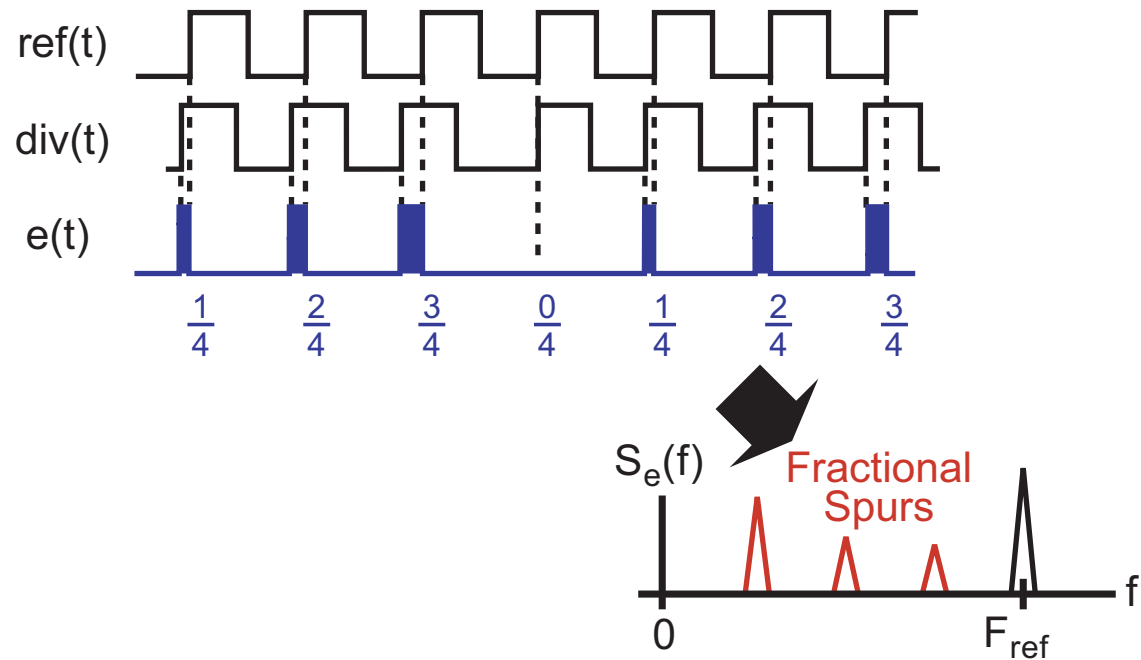
Tradeoff: Noise performance vs PLL bandwidth

Recent Approaches to Bandwidth Extension

- [1] C. Park, O. Kim, and B. Kim, “A 1.8GHz Self-Calibrated Phase-Locked Loop with Precise I/Q Matching,” IEEE JSSC, May 2001.
- [2] K. Lee, et. al., “A Single Chip 2.4GHz Direct-Conversion CMOS Receiver for Wireless Local Loop Using Multiphase Reduced Frequency Conversion Technique ,” IEEE JSSC, May 2001.
- [3] S. Pamarti, L. Jansson, and I. Galton, “A Wideband 2.4GHz Delta-Sigma Fractional-N PLL With 1Mb/s In-Loop Modulation”, IEEE JSSC, Jan 2004
- [4] E. Temporiti, et. al., “A 700kHz Bandwidth $\Sigma-\Delta$ Fractional-N Frequency Synthesizer with Spurs Compensation and Linearization Techniques for WCDMA Applications”, IEEE JSSC, Sept 2004

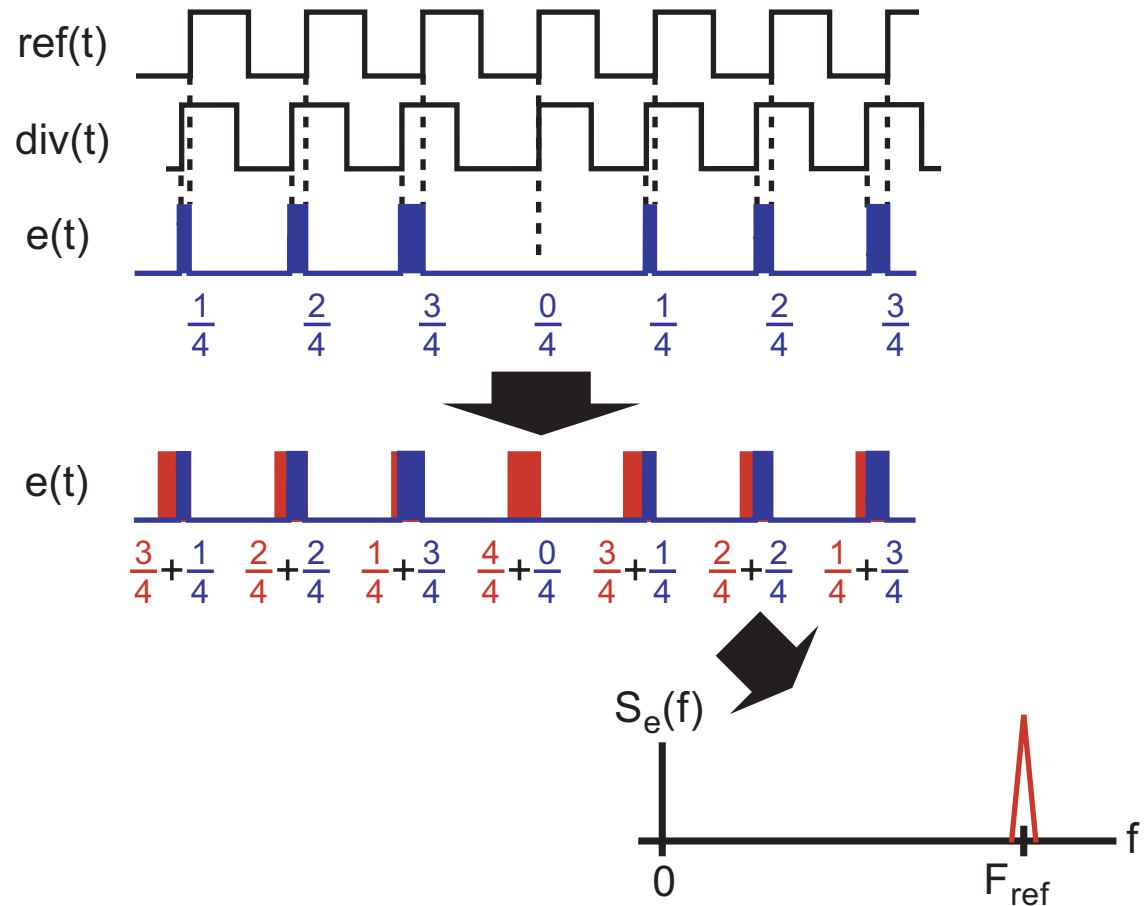
We will focus on our own approach in this talk

Examine Classical Fractional-N Signals



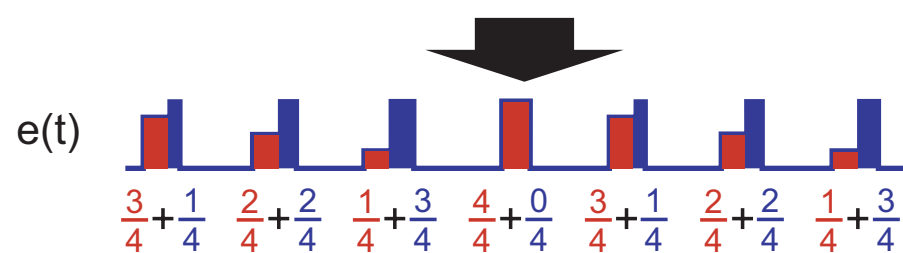
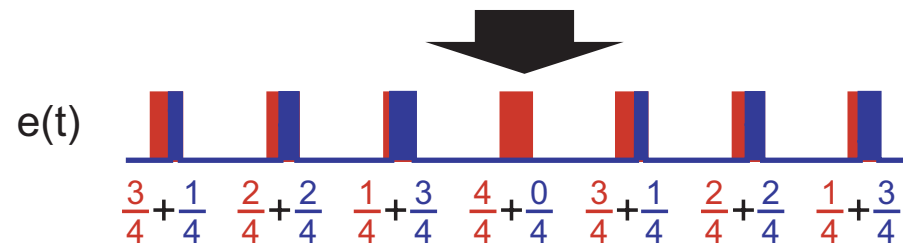
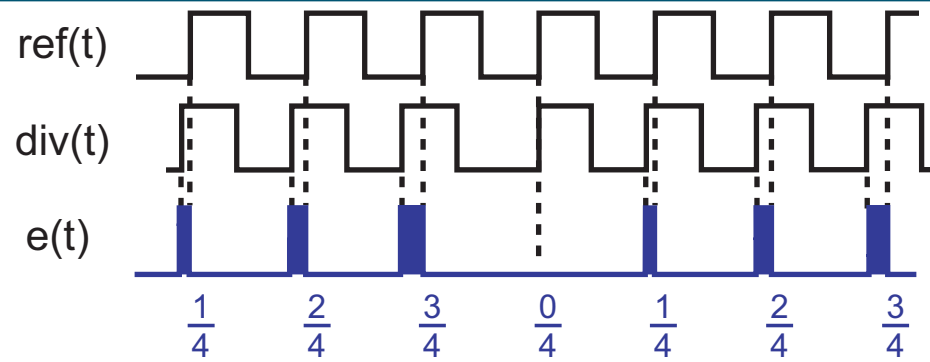
- **Goal: eliminate the fractional spurs**

Method 1: Vertical Compensation

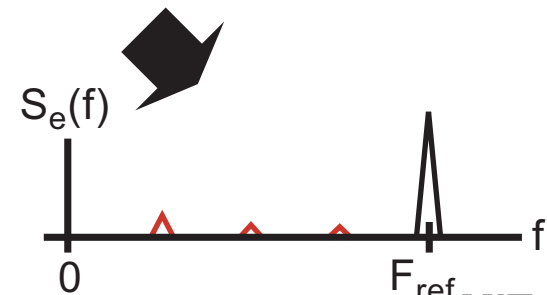


- “Fill in” pulses so that they are constant area
 - Fractional spurs are eliminated!

Method 2: Horizontal Compensation

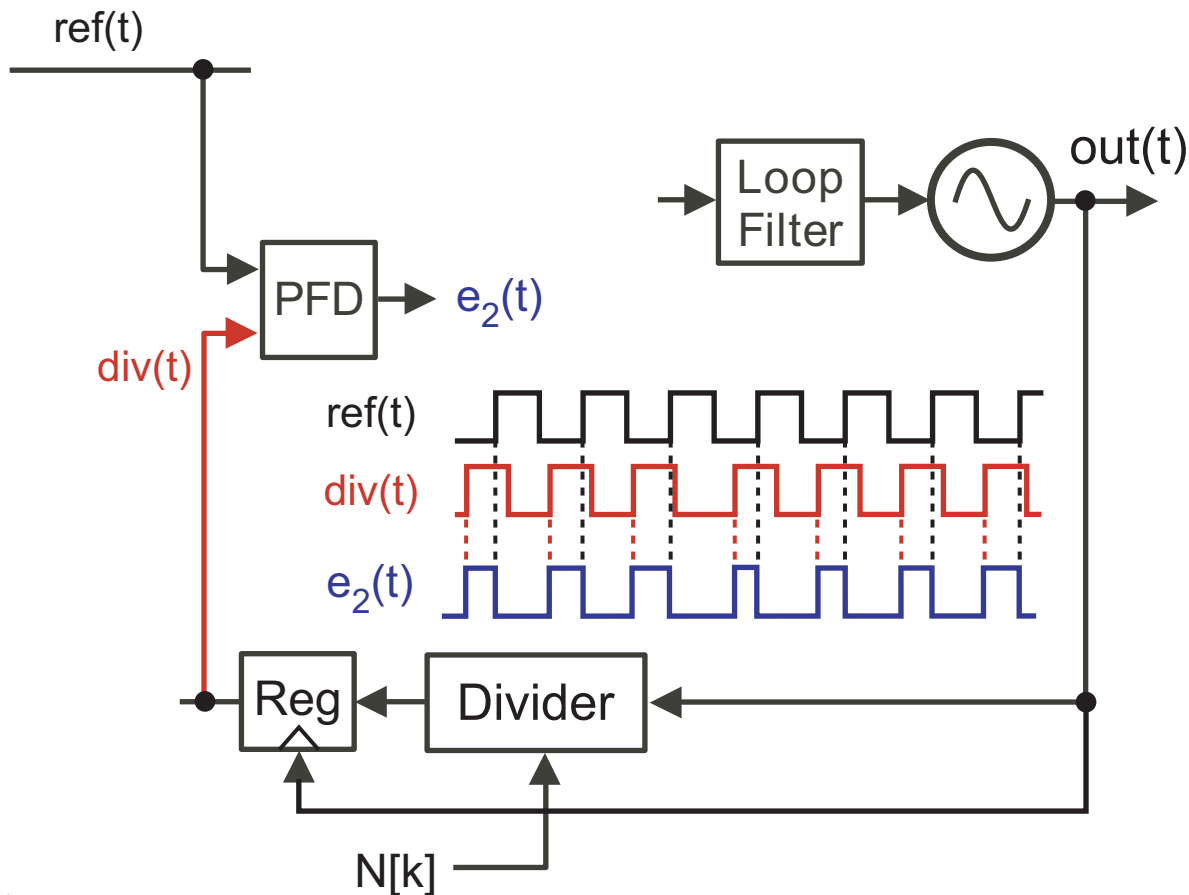


- Use constant width pulses of varying height to achieve constant area pulses
 - Largely eliminates fractional spurs

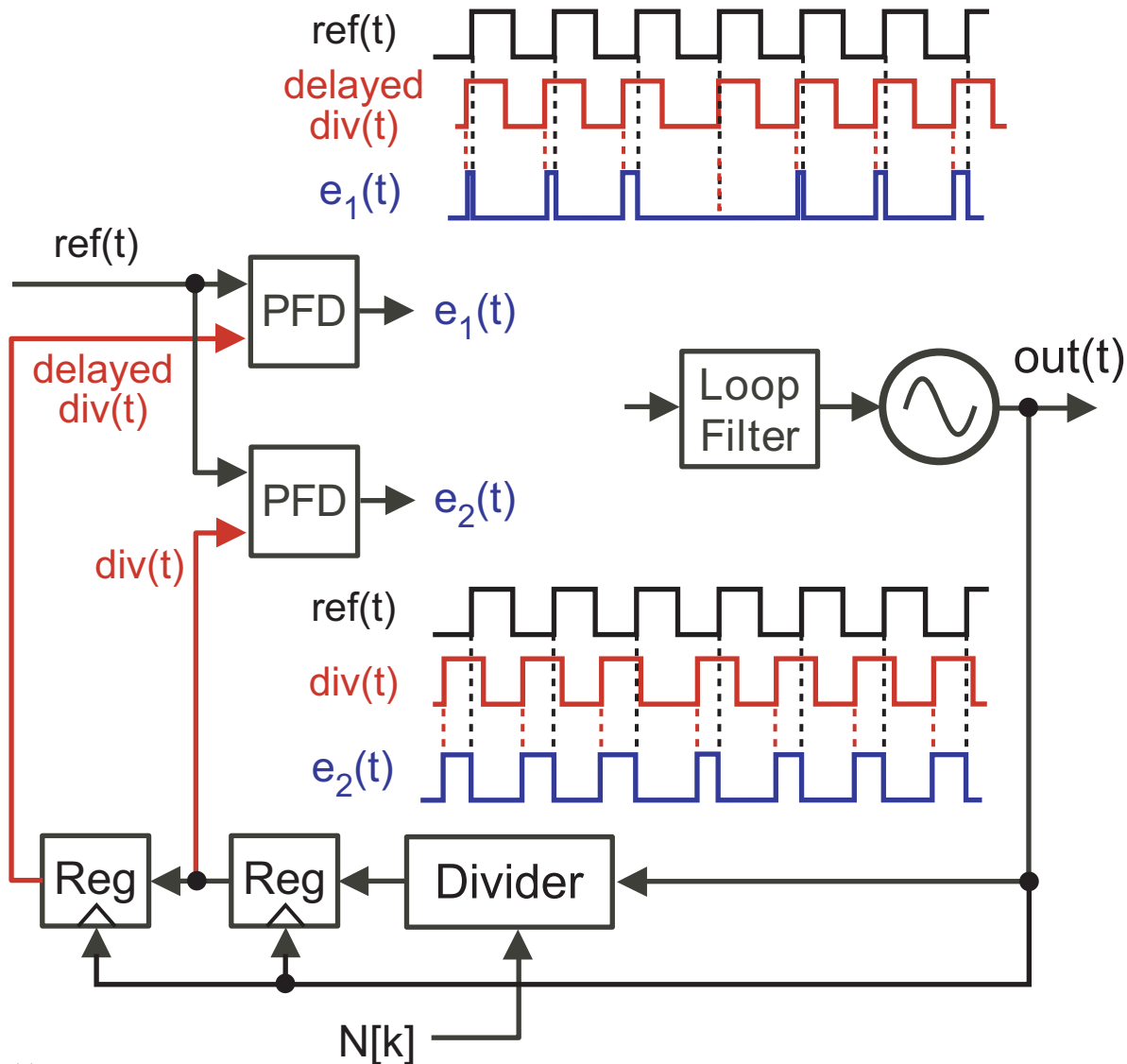


Implementation of Horizontal Cancellation

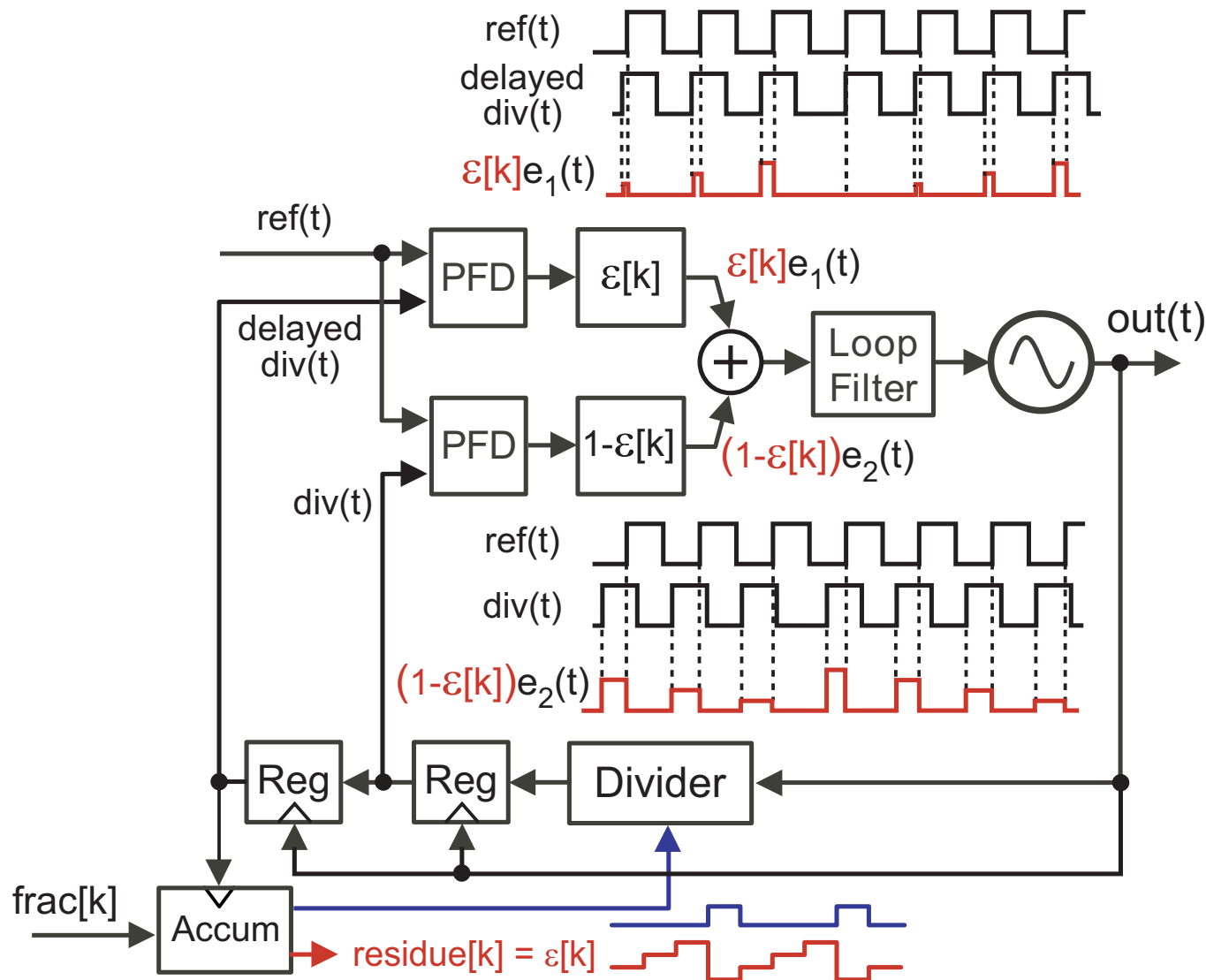
- We begin with the basic fractional-N structure



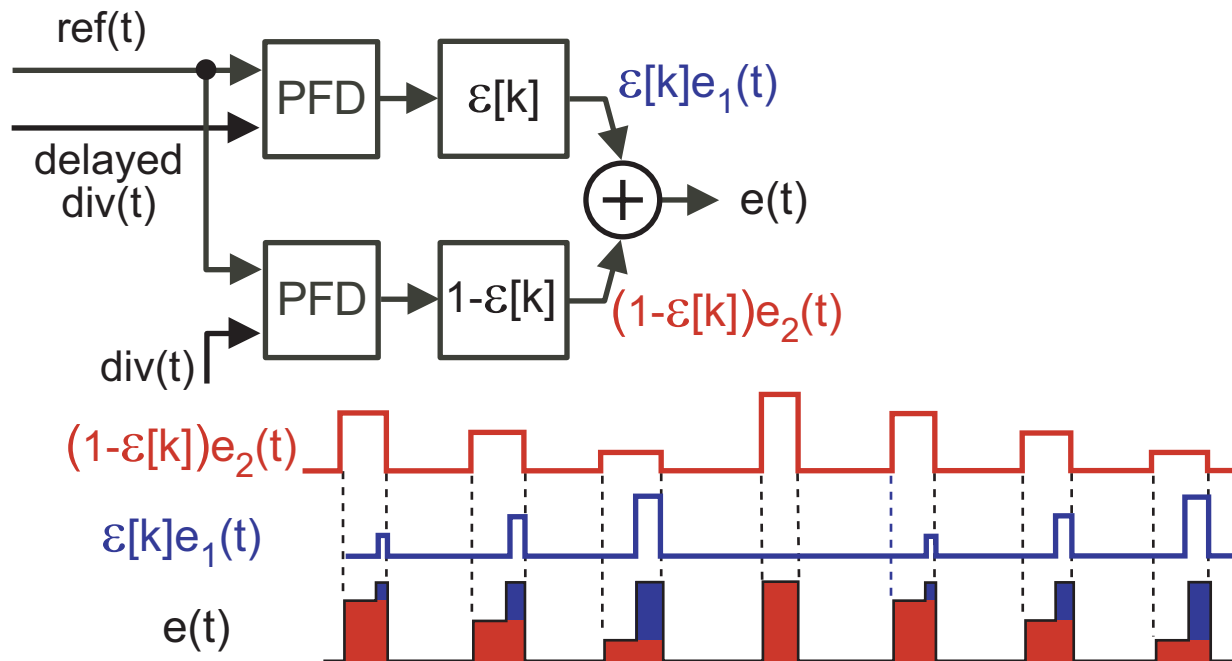
Add a Second PFD with Delayed Divider Signal



Scale Error Pulses According to Accumulator Residue



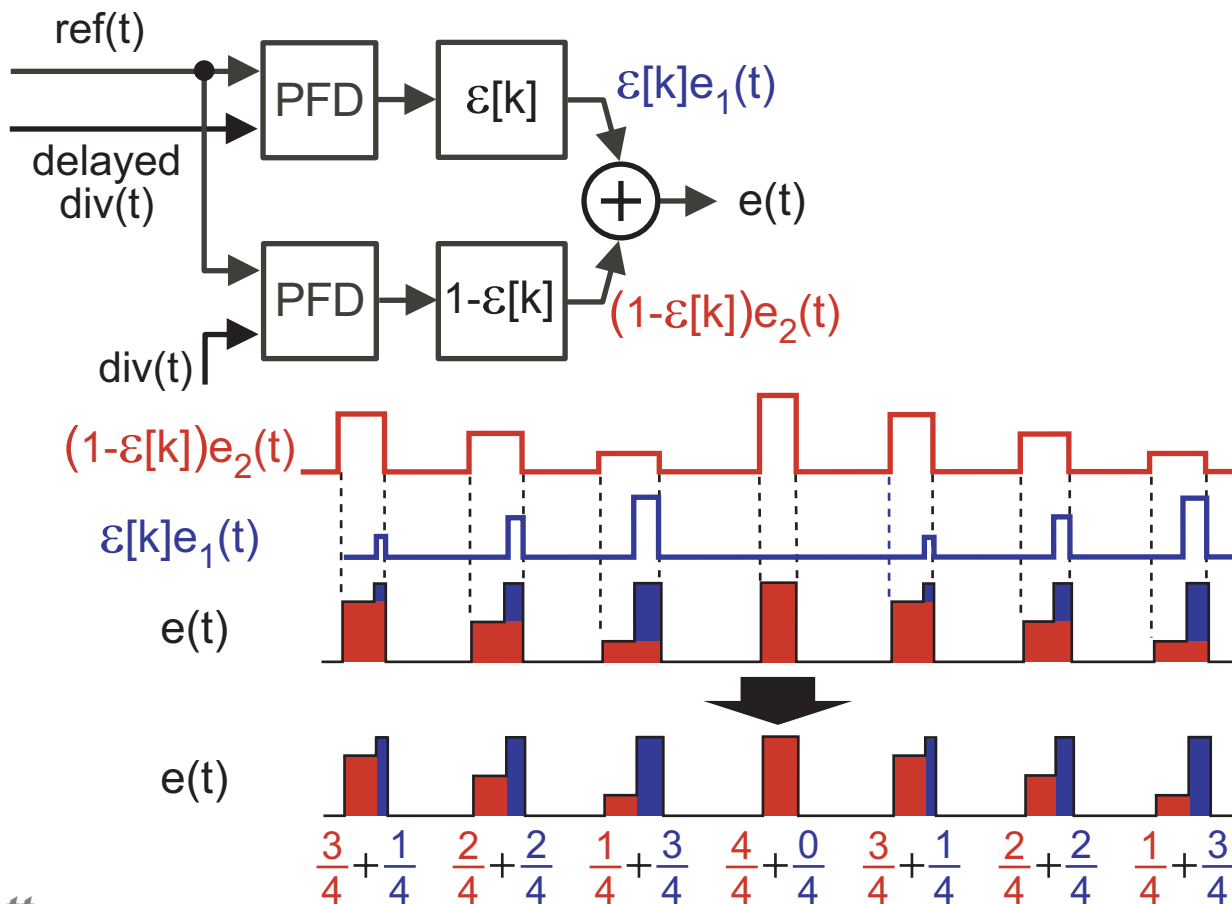
A Closer Look at Adding the Scaled Error Pulses



- **Goal – keep area constant for each pulse**
 - It's easier to see this from a slightly different viewpoint

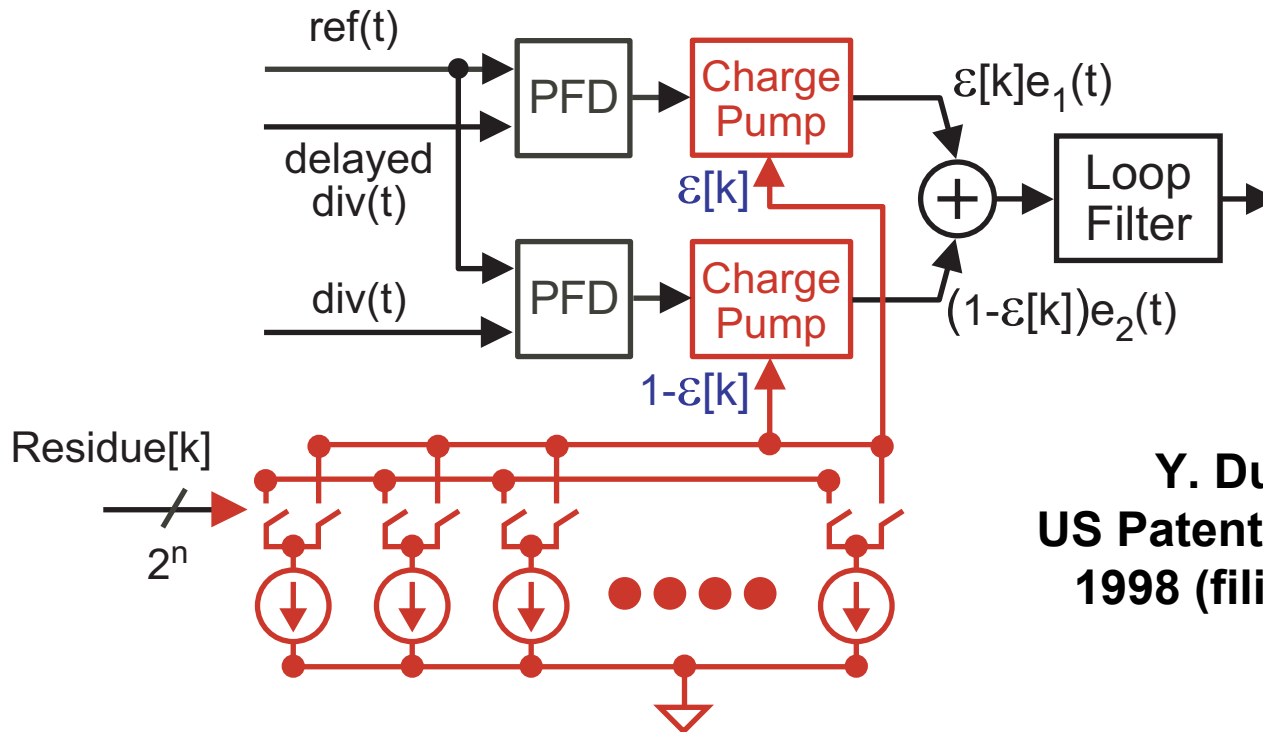
Alternate Viewpoint

- The sum of scaled pulses can now be viewed as horizontal cancellation



Implementation of Pulse Scaling Operation

- Direct output of a differential current DAC into two charge pumps



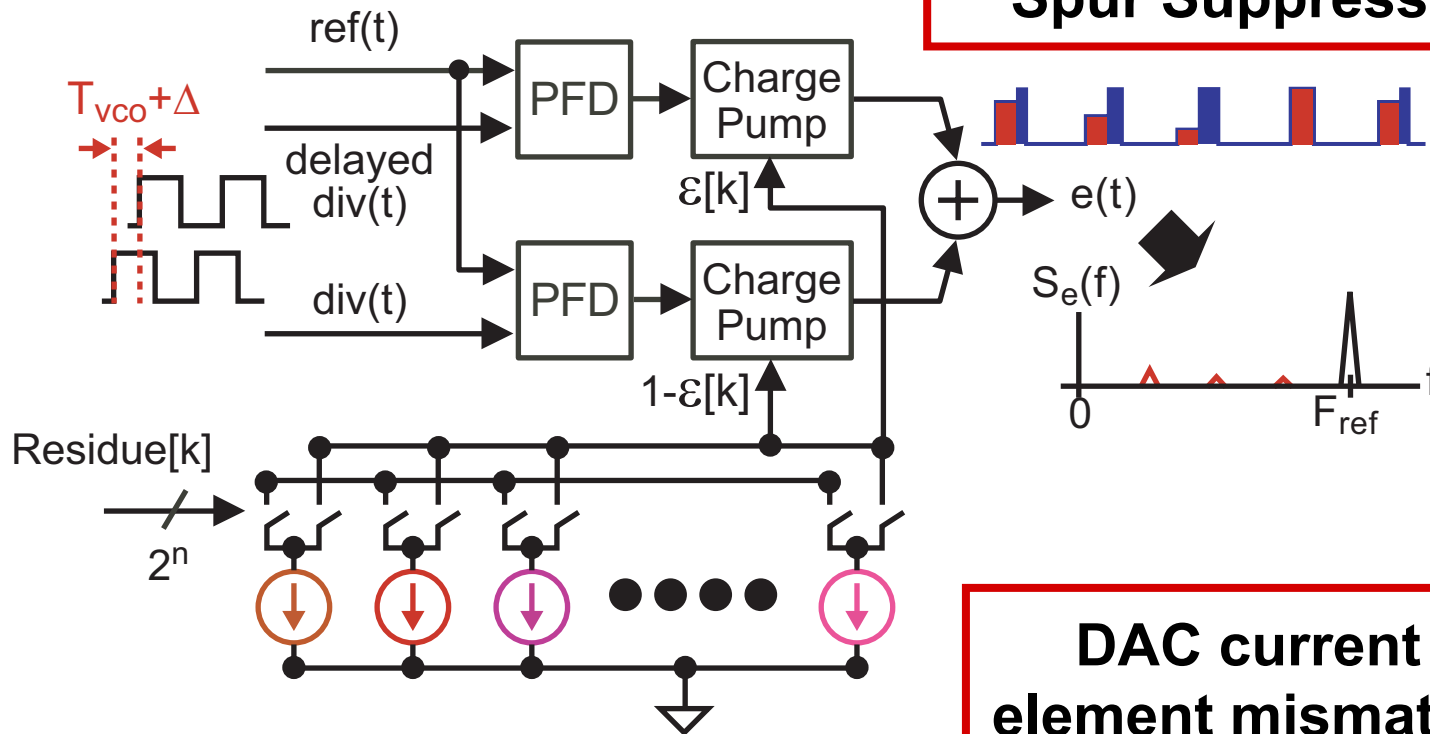
Y. Dufour
US Patent 6,130,561
1998 (filing date)

- Issue: practical non-idealities kill performance

Primary Non-idealities of Concern

Delay mismatch

Incomplete Fractional Spur Suppression

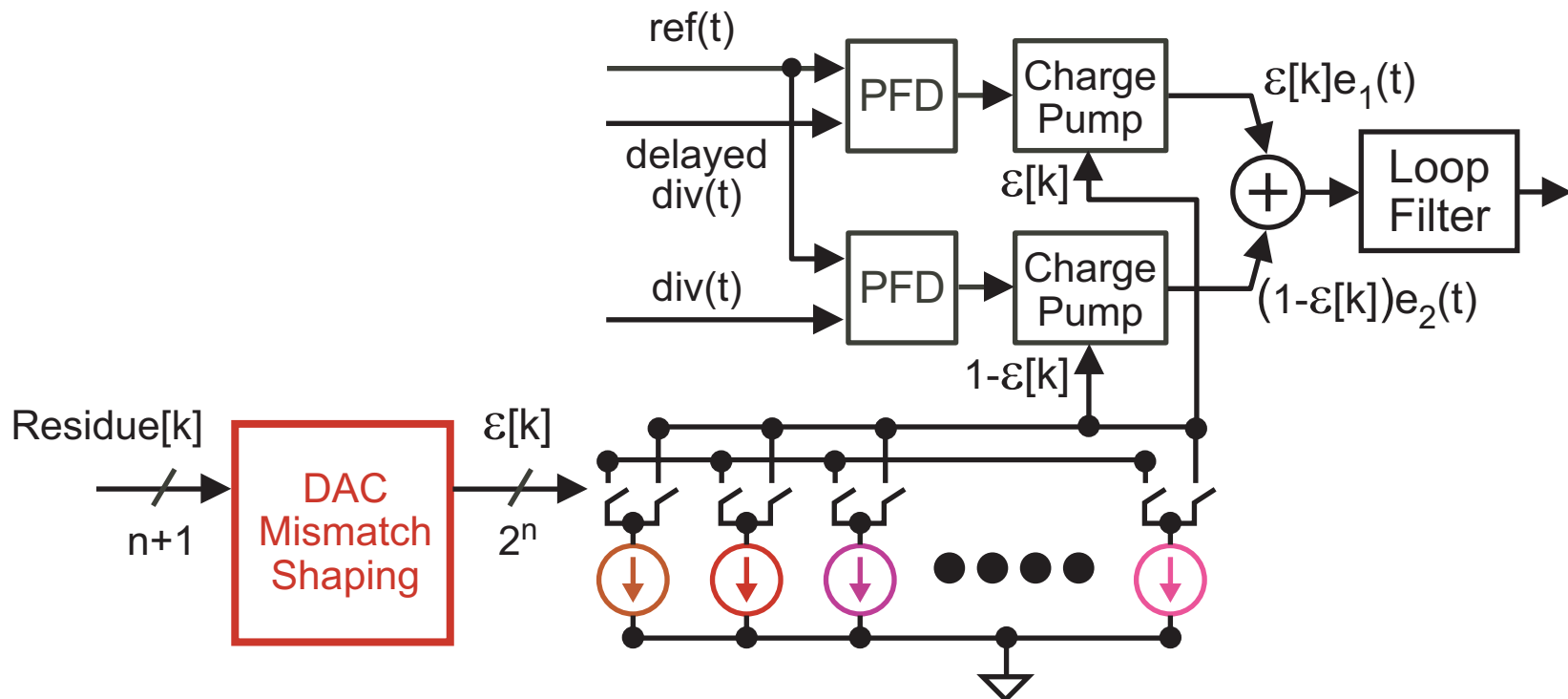


DAC current element mismatch

Proposed approach: dramatically reduce impact of these non-idealities using mixed-signal processing techniques

Eliminate Impact of DAC Current Element Mismatch

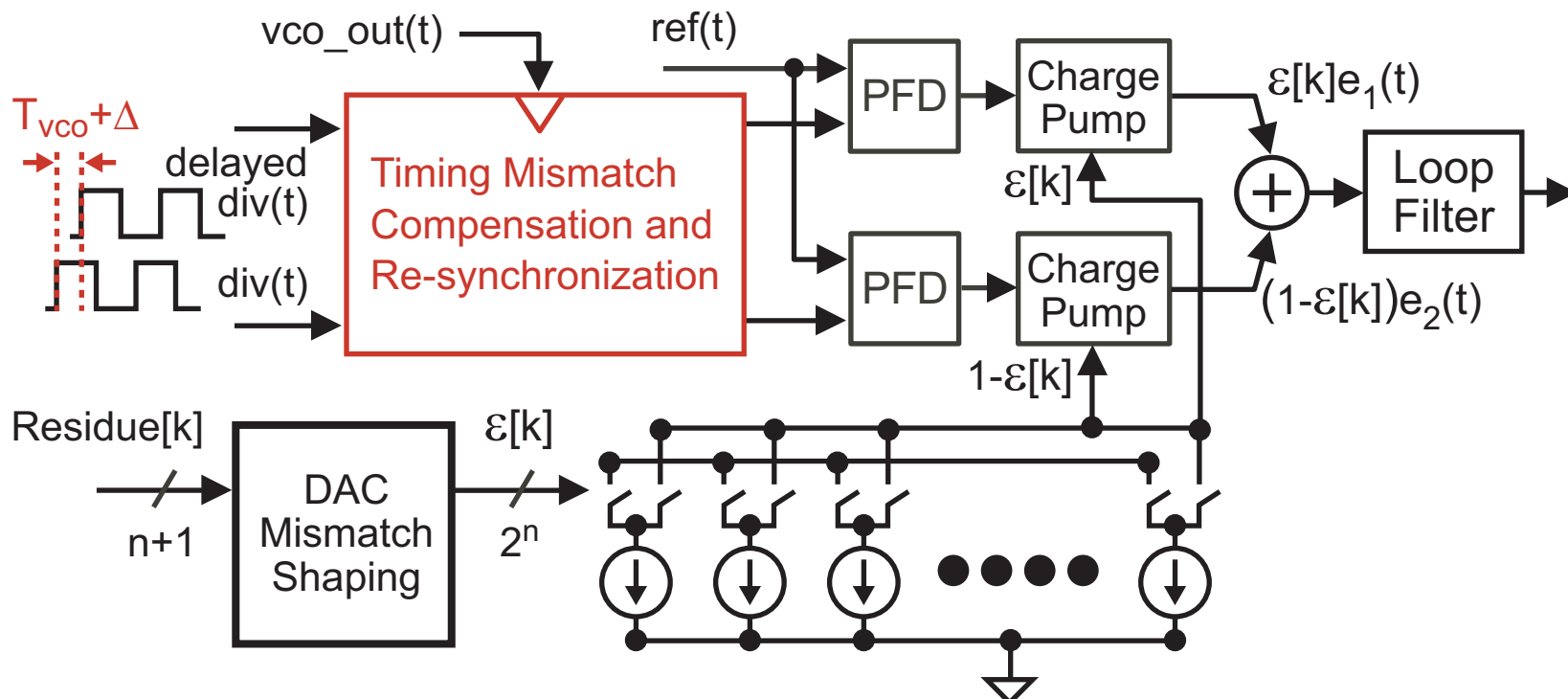
- Apply standard DAC noise shaping techniques to shape mismatch noise to high frequencies
 - See Baird and Fiez, TCAS II, Dec 1995



- Allows up to 5% mismatch between unit elements without degrading our desired performance targets

Eliminate Impact of Timing Mismatch

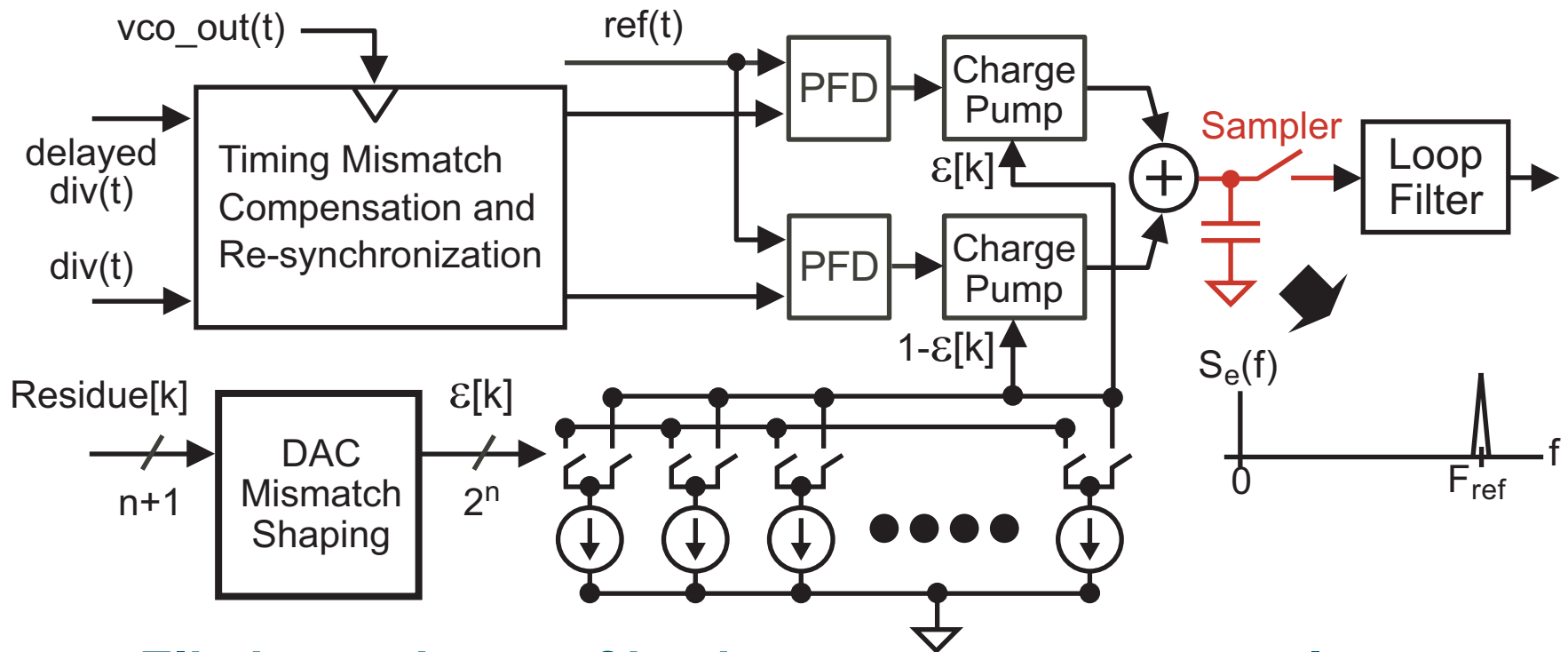
- Swap paths between divider outputs in a pseudo-random fashion
 - Need to also swap $\varepsilon[k]$ and $1-\varepsilon[k]$ sequence



- Allows up to 5 ps mismatch without degrading our desired performance targets

Improve Horizontal Cancellation Performance

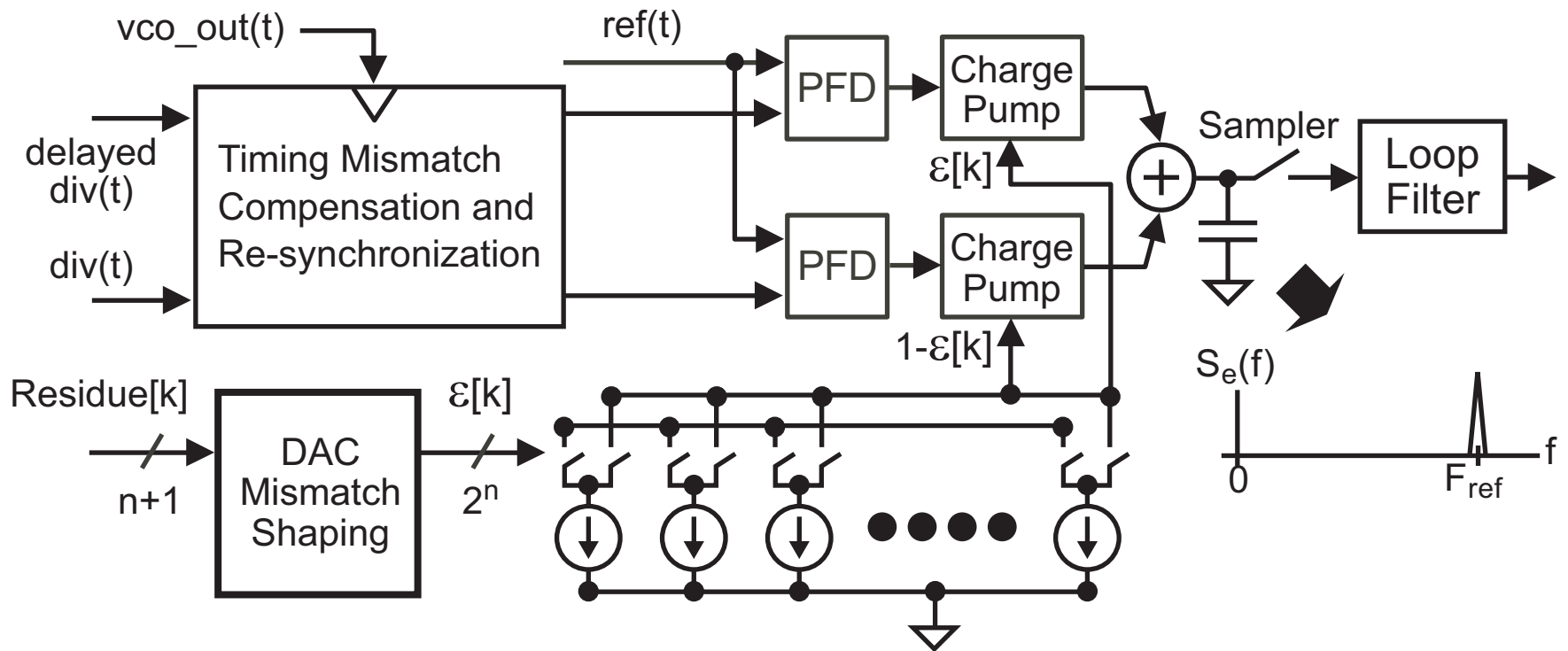
- Sampling circuit accumulates error pulses before passing their information to the loop filter
 - A common analog trick used for decades



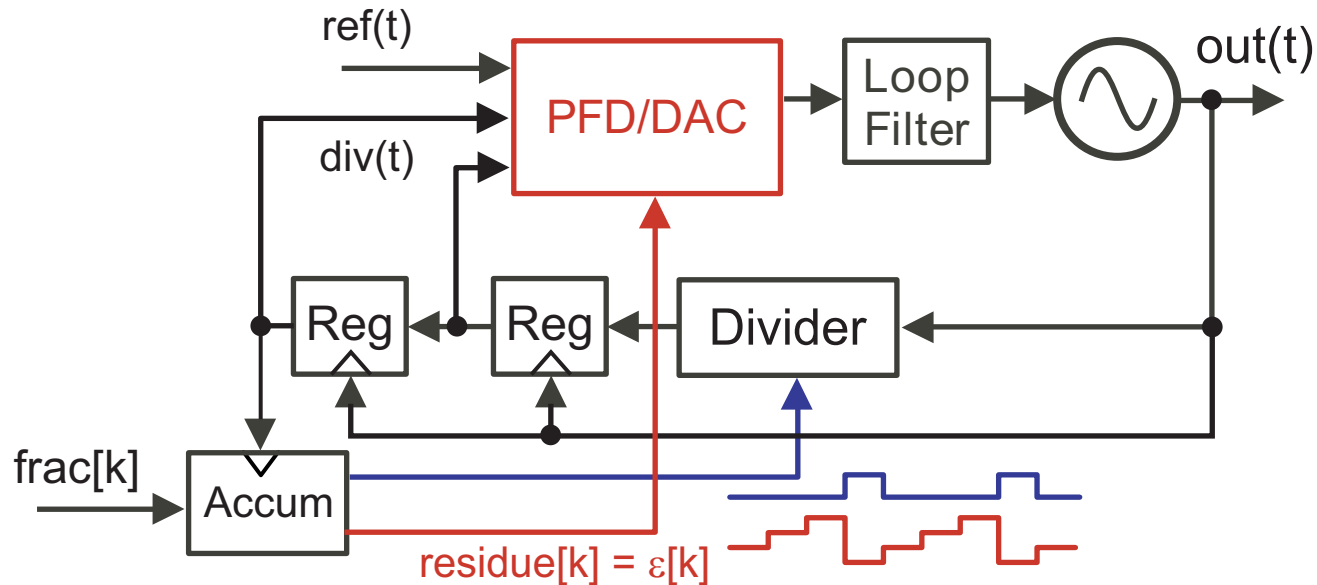
- Eliminates issue of having non-square error pulse shapes

For More Details on This Approach

- Theory and simulations presented in TCAS II paper
 - Meninger and Perrott, TCASII, Nov 2003



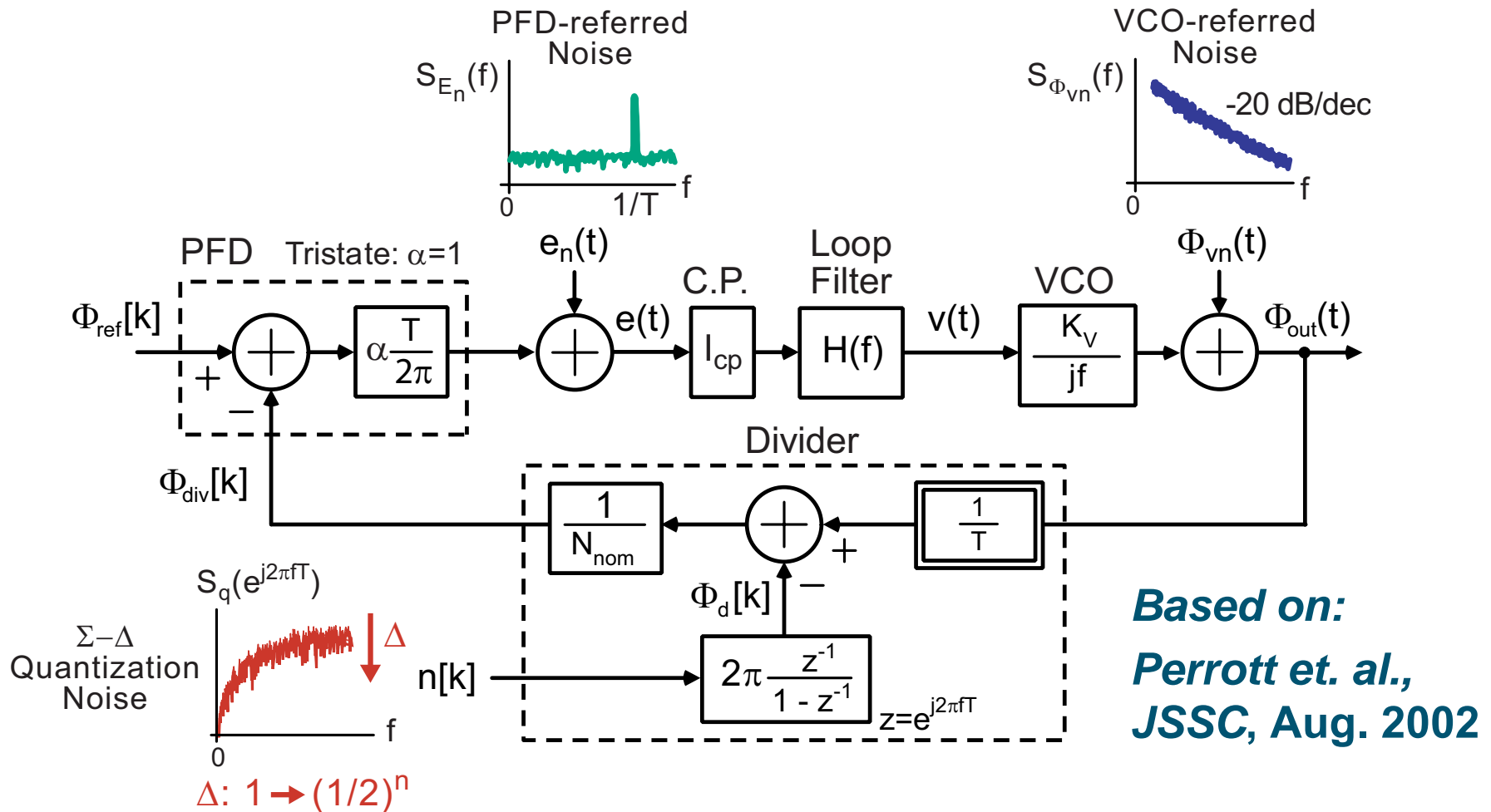
Design and Simulation of 'PFD/DAC' Synthesizer



- **Step 1: Derive analytical model**
- **Step 2: Design at system level**
- **Step 3: Simulate at system level (CppSim)**
- **Step 4: Simulate at transistor level (SPICE)**

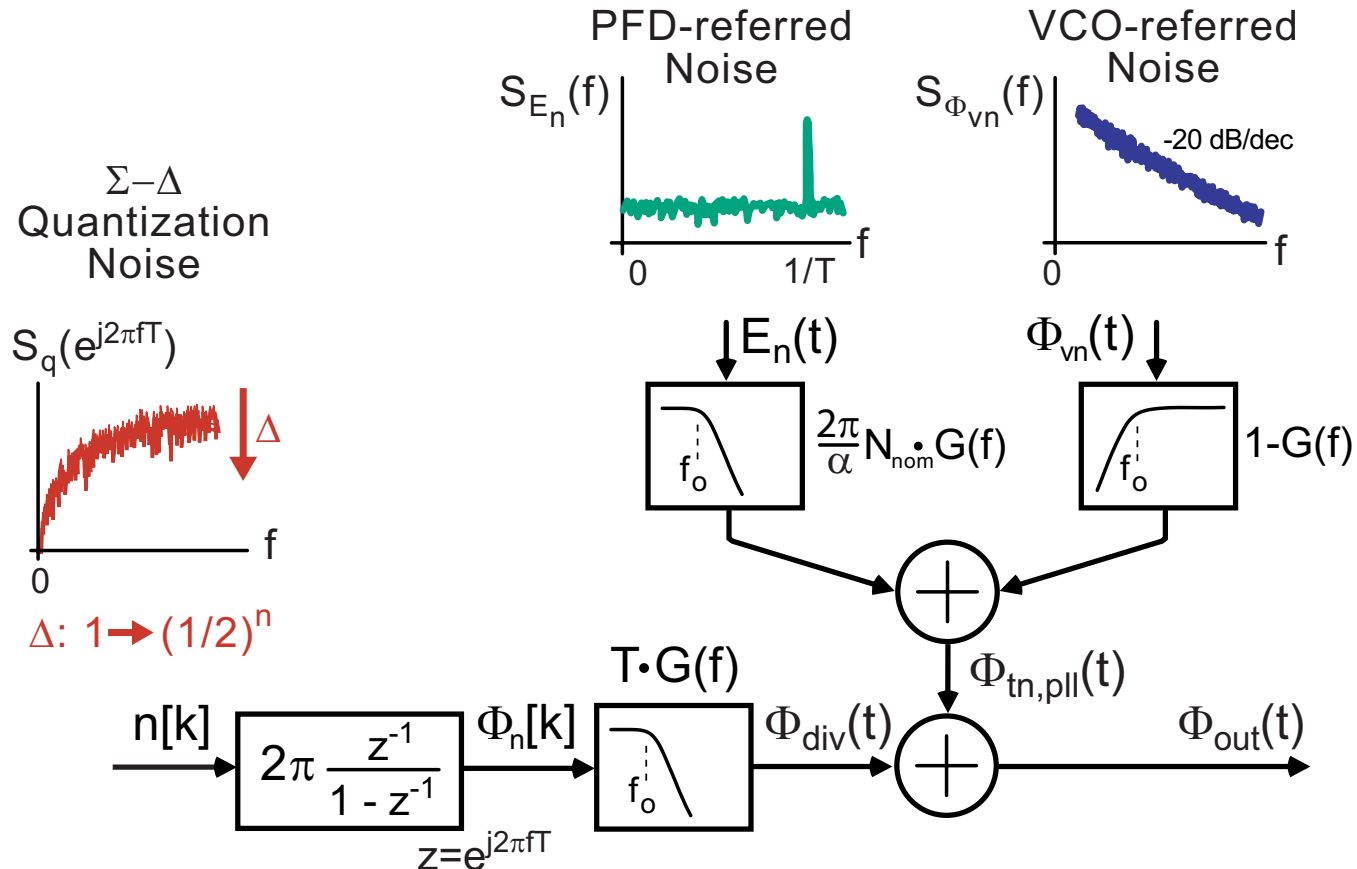
Iterate between all of these steps in practice

Analytical Model of 'PFD/DAC' Fractional-N PLL



n-bit PFD/DAC \rightarrow Δ is reduced from 1 to $(1/2)^n$

Parameterized PLL Model



$$G(f) = \frac{A(f)}{1 + A(f)}$$

where
$$A(f) = \frac{\alpha I_{cp} H(f) K_V}{N_{nom} 2\pi j f}$$

***Application:
A 1 MHz Bandwidth Fractional-N Frequency
Synthesizer Implementation***

Design Goals

- **Output frequency: 3.6 GHz**
 - Allows dual-band output (1.8 GHz and 900 MHz)
- **Reference frequency: 50 MHz**
 - Allows low cost crystal reference
- **Bandwidth: 1 MHz**
 - Allows fast settling time and ~1 Mbit/s modulation rate
- **Noise: < -150 dBc/Hz at 20 MHz offset (3.6 GHz carrier)**
 - Phase noise at the 20 MHz frequency offset is very challenging for GSM and DCS transmitters
 - GSM: -162 dBc/Hz at 20 MHz offset (900 MHz carrier)
 - DCS: -151 dBc/Hz at 20 MHz offset (1.8 GHz carrier)

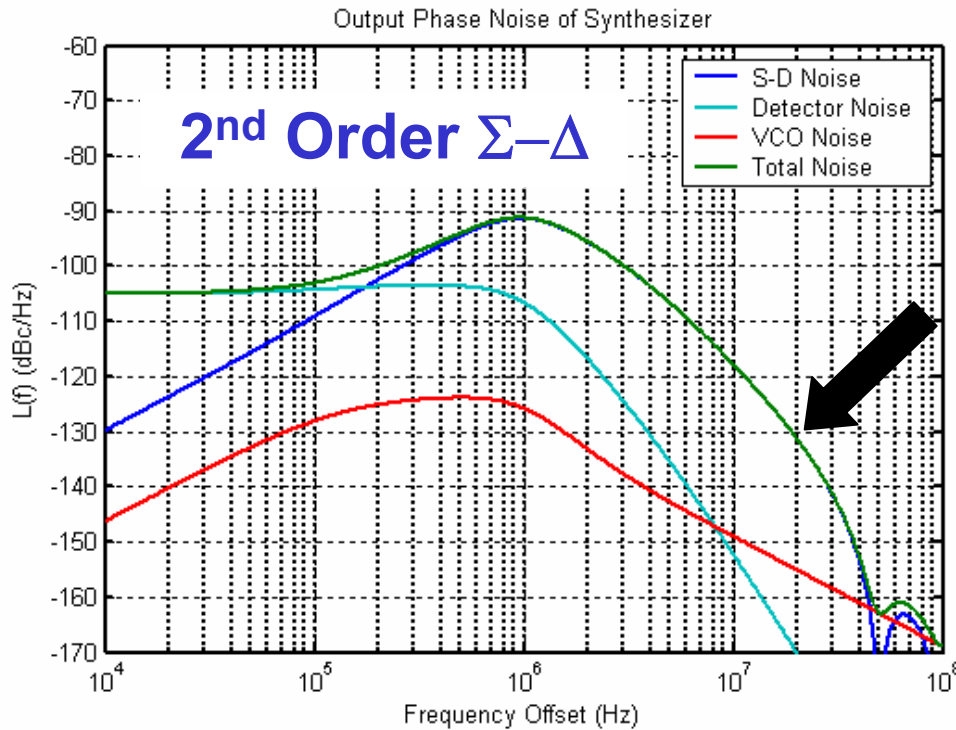
Simultaneous achievement of the above bandwidth and noise targets is very challenging

Evaluate Noise Performance with 1 MHz PLL BW

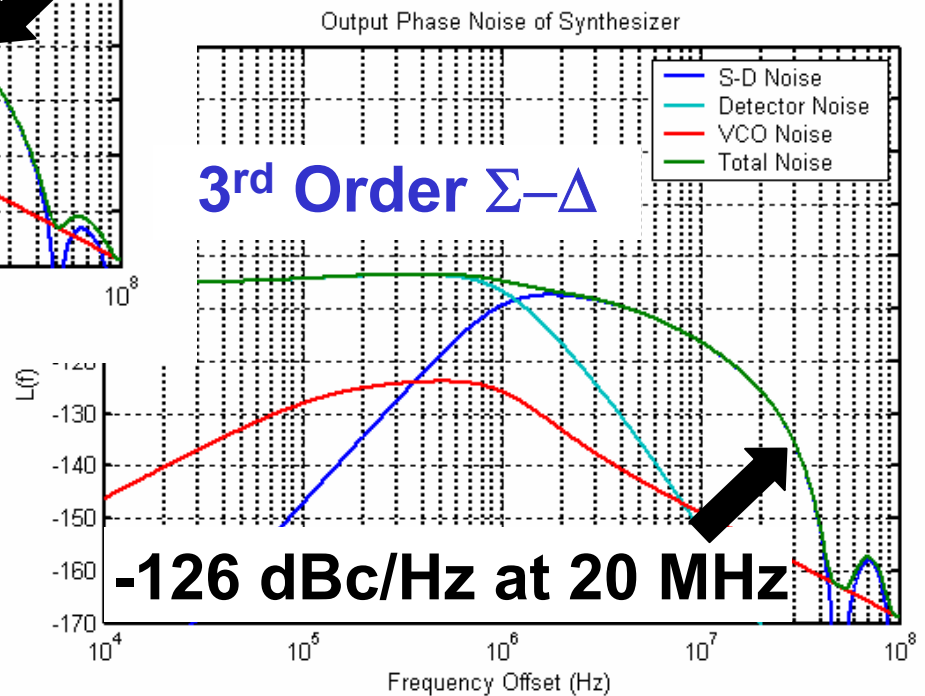
- **G(f) parameters**
 - 1 MHz BW, Type II, 2nd order rolloff, extra pole at 2.5 MHz
- **Required PLL noise parameters (with a few dB of margin)**
 - Output-referred charge pump noise: -105 dBc/Hz
 - VCO noise: -155 dBc/Hz at 20 MHz offset (3.6 GHz carrier)

Dynamic Parameters		Noise Parameters	
fo: <input type="text" value="1e6"/> Hz	paris. pole: <input type="text" value="2.5e6"/> Hz <input type="checkbox"/>	ref. freq: <input type="text" value="50e6"/> Hz	<input type="checkbox"/>
order: <input type="radio"/> 1 <input checked="" type="radio"/> 2 <input type="radio"/> 3	paris. Q: <input type="text"/>	out freq: <input type="text" value="3.6e9"/> Hz	<input type="checkbox"/>
shape: <input checked="" type="radio"/> Butter <input type="radio"/> Bessel	paris. pole: <input type="text"/> Hz <input type="checkbox"/>	Detector: <input type="text" value="-105"/> dBc/Hz <input type="checkbox"/>	<input type="checkbox"/>
<input type="radio"/> Cheby1 <input type="radio"/> Cheby2 <input type="radio"/> Elliptical	paris. Q: <input type="text"/>	VCO: <input type="text" value="-155"/> dBc/Hz <input type="checkbox"/>	<input type="checkbox"/>
ripple: <input type="text"/> dB	paris. pole: <input type="text"/> Hz <input type="checkbox"/>	freq. offset: <input type="text" value="20e6"/> Hz	<input type="checkbox"/>
type: <input type="radio"/> 1 <input checked="" type="radio"/> 2	paris. pole: <input type="text"/> Hz <input type="checkbox"/>	S-D: <input type="radio"/> 1 <input checked="" type="radio"/> 2 <input type="checkbox"/>	<input type="checkbox"/>
fz/fo: <input type="text" value="1/9"/>	paris. zero: <input type="text"/> Hz <input type="checkbox"/>	<input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5	<input type="checkbox"/>
	paris. zero: <input type="text"/> Hz <input type="checkbox"/>		
Resulting Open Loop Parameters		Resulting Plots and Jitter	
K: <input type="text" value="2.885e+012"/>	alter: <input type="text"/> <input type="checkbox"/>	<input type="radio"/> Pole/Zero Diagram	<input type="radio"/> Transfer Function
fp: <input type="text" value="2.807e+006"/> Hz	alter: <input type="text"/> <input type="checkbox"/>	<input type="radio"/> Step Response	<input checked="" type="radio"/> Noise Plot
fz: <input type="text" value="1.111e+005"/> Hz	alter: <input type="text"/> <input type="checkbox"/>	<input type="text" value="10e3"/>	<input type="text" value="100e6"/>
Qp: <input type="text"/>	alter: <input type="text"/> <input type="checkbox"/>	<input type="text" value="-170"/>	<input type="text" value="-60"/>
		rms jitter: <input type="text" value="2.197 ps"/>	
PLL Design Assistant		Written by Michael Perrott (http://www-mtl.mit.edu/~perrott)	

Calculated Phase Noise for Classical Fractional-N



-132 dBc/Hz at 20 MHz

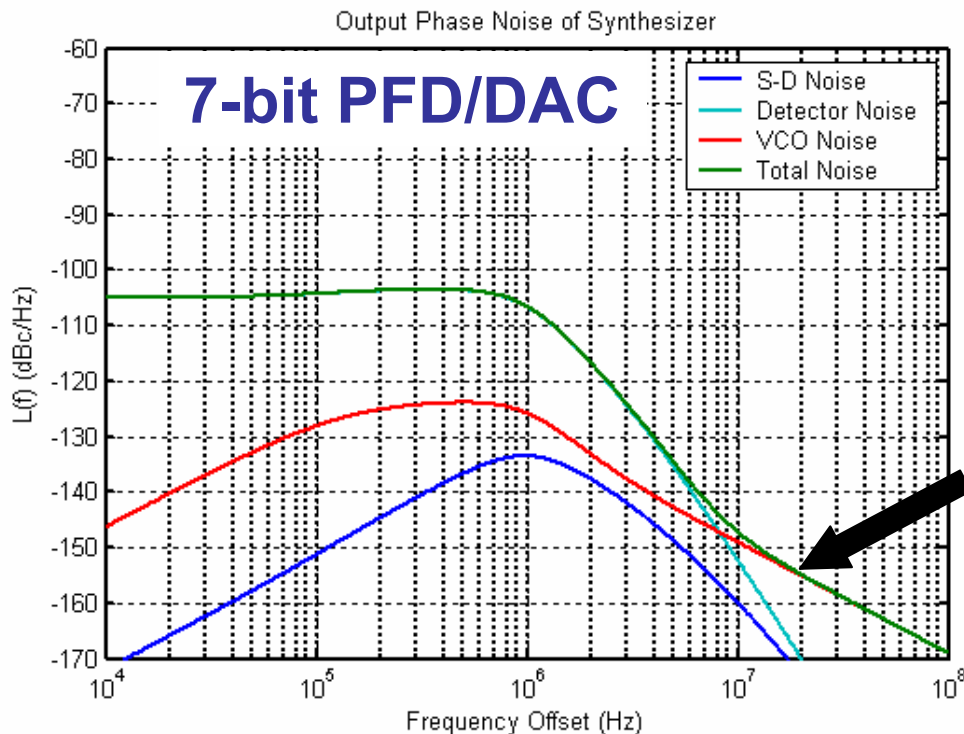


-126 dBc/Hz at 20 MHz

These do NOT meet our target of -150 dBc/Hz at 20 MHz (3.6 GHz carrier freq.)

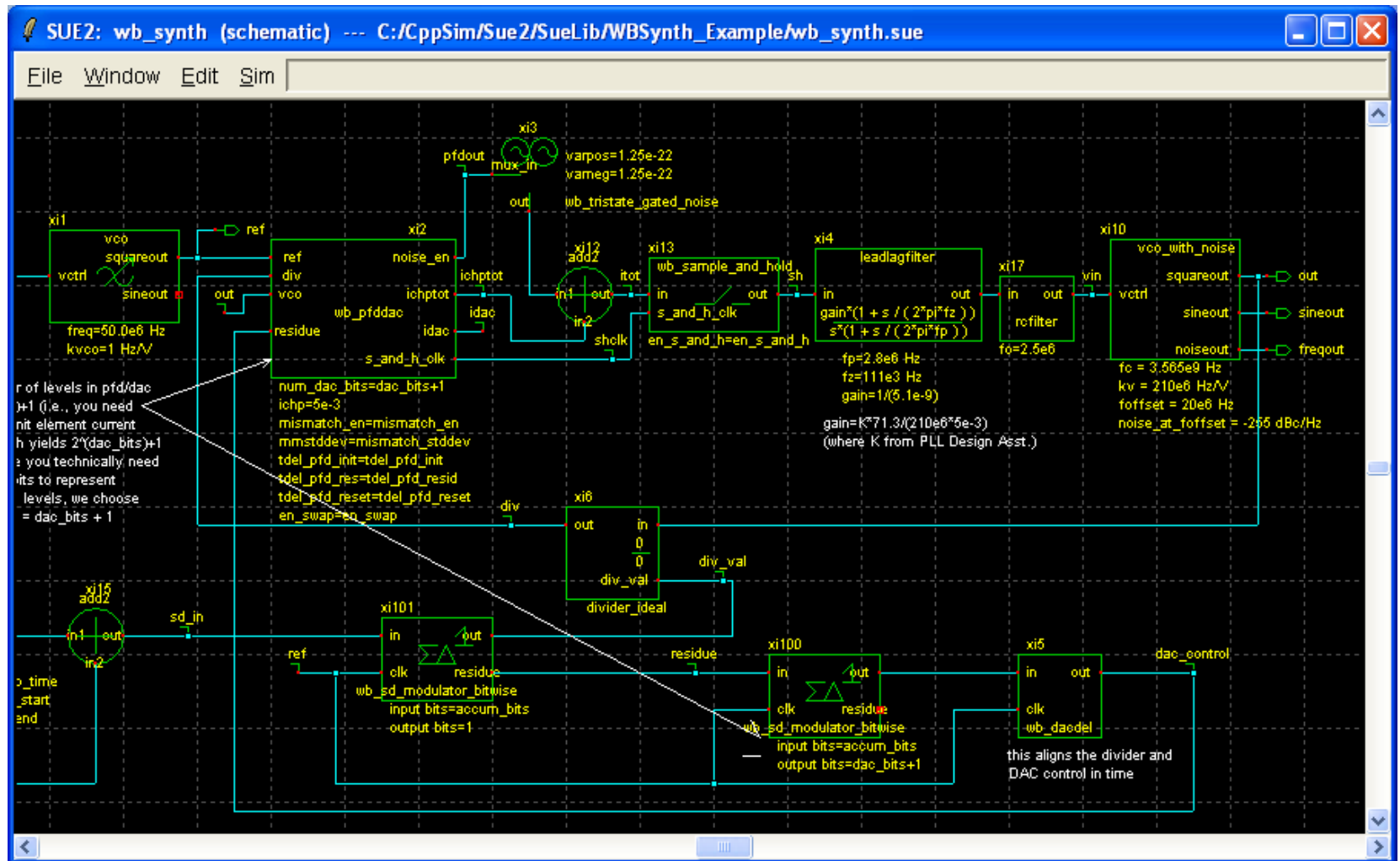
Calculated Phase Noise for 7-bit PFD/DAC Synth

Dynamic Parameters		Noise Parameters	
fo	1e6 Hz	ref. freq	50e6 Hz
order	1 2 3	out freq.	3.6e9 Hz
shape	Butter Bessel	Detector	-105 dBc/Hz
	Cheby1 Cheby2 Elliptical	VCO	-155 dBc/Hz
ripple	dB	freq. offset	20e6 Hz
type	1 2	S-D	1 2 3 4 5
fz/fo	1/9		$[1 - 2^{-1}]^{(1/2)^7}$
paris. pole	2.5e6 Hz	Resulting Plots and Jitter	
paris. Q		<input type="radio"/> Pole/Zero Diagram <input type="radio"/> Transfer Function <input type="radio"/> Step Response <input checked="" type="radio"/> Noise Plot	
paris. pole	Hz	10e3	100e6
paris. Q		-170	-60
paris. pole	Hz	rms jitter:	431.497 fs
paris. Q		by Michael Perrott (http://www-mtl.mit.edu/~perrott)	
paris. pole	Hz		
paris. zero	Hz		
paris. zero	Hz		



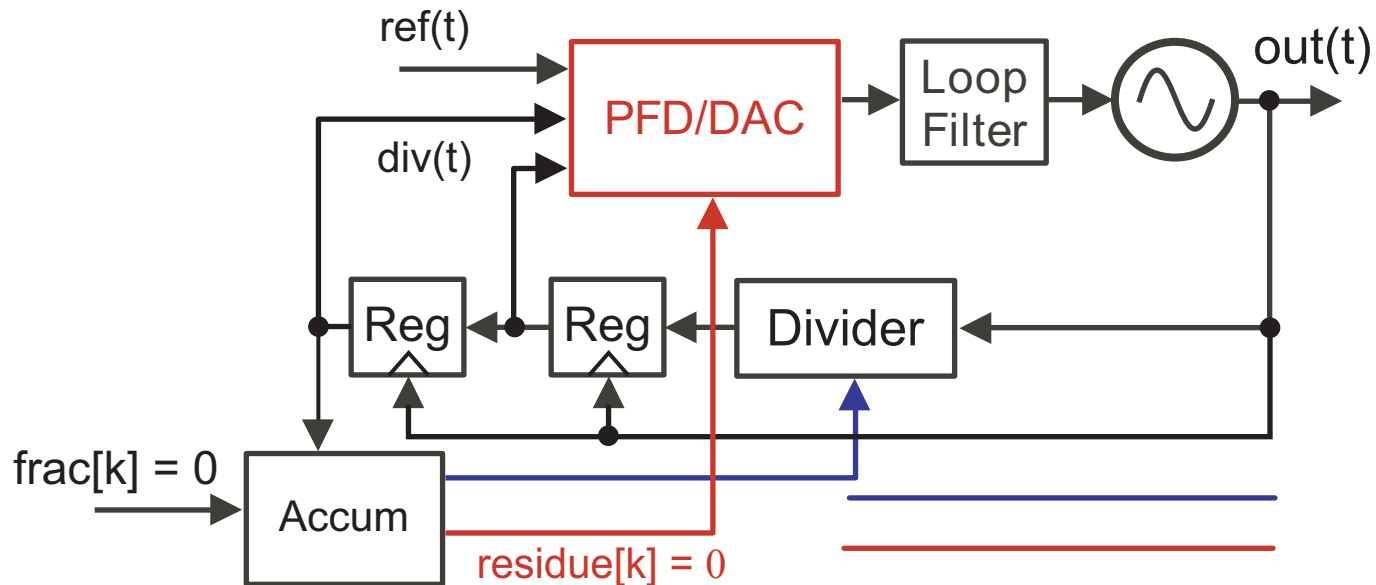
-155 dBc/Hz at 20 MHz !

Simulation of PFD/DAC Synthesizer using CppSim



- Phase noise plots to follow: 40e6 time steps in 10 min

Intrinsic PLL Noise Performance



- **Set fractional portion of divide value to zero**
 - Leads to residue variation of zero
 - No quantization noise!
- **Need to calculate and simulate impact of detector and VCO noise**

In essence, operate as integer-N synthesizer

Calculate Intrinsic PLL Noise Sources

- **Estimate detector noise (dominated by charge pump)**
 - From SPICE Simulation, $S_{I_{cp}}(f) = \text{Duty} \cdot 3e-22 \text{ A}^2/\text{Hz}$
 - Output-referred PLL noise due to above noise:

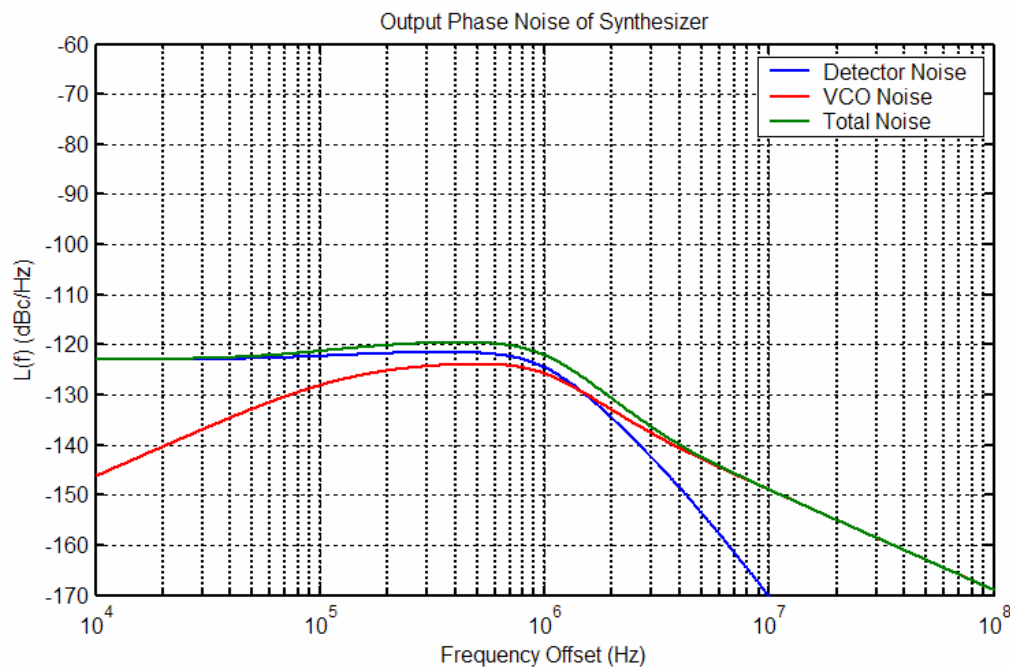
$$\begin{aligned} S_{\Phi_{out}}(f) &= \left(\frac{1}{I_{cp}}\right)^2 \left(\frac{2\pi}{\alpha} N_{nom}\right)^2 |G(f)|^2 S_{I_{cp}}(f) \\ &= \left(\frac{1}{5 \cdot 10^{-3}}\right)^2 \left(\frac{2\pi}{1} 71.3\right)^2 |G(f)|^2 0.2 \cdot 3 \cdot 10^{-22} \end{aligned}$$

$$\implies 10 \log(S_{\Phi_{out}}(f)) = -123 |G(f)|^2 \text{ dBc/Hz}$$

- **Estimate VCO noise**
 - For off-chip VCO, examine data sheet:
 - In this case, $S_{\Phi_{VCO}} = -155 \text{ dBc/Hz}$ at 20 MHz offset
 - For on-chip VCO, use Spectre RF or other CAD tool

Calculate PLL Noise Due to Intrinsic Noise Sources

Dynamic Parameters		Noise Parameters	
fo	1e6 Hz	ref. freq	50e6 Hz
order	<input type="radio"/> 1 <input checked="" type="radio"/> 2 <input type="radio"/> 3	out freq.	3.6e9 Hz
shape	<input checked="" type="radio"/> Butter <input type="radio"/> Bessel	Detector	-123 dBc/Hz <input type="checkbox"/> On
	<input type="radio"/> Cheby1 <input type="radio"/> Cheby2 <input type="radio"/> Elliptical	VCO	-155 dBc/Hz <input type="checkbox"/> On
ripple	dB	freq. offset	20e6 Hz
type	<input type="radio"/> 1 <input checked="" type="radio"/> 2	S-D	<input type="radio"/> 1 <input type="radio"/> 2 <input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5 <input type="checkbox"/> On
fz/fo	1/9		
paris. pole	2.5e6 Hz <input type="checkbox"/> On		
paris. Q	<input type="checkbox"/> On		
paris. pole	Hz <input type="checkbox"/> On		
paris. Q	<input type="checkbox"/> On		
paris. pole	Hz <input type="checkbox"/> On		
paris. pole	Hz <input type="checkbox"/> On		
paris. zero	Hz <input type="checkbox"/> On		
paris. zero	Hz <input type="checkbox"/> On		

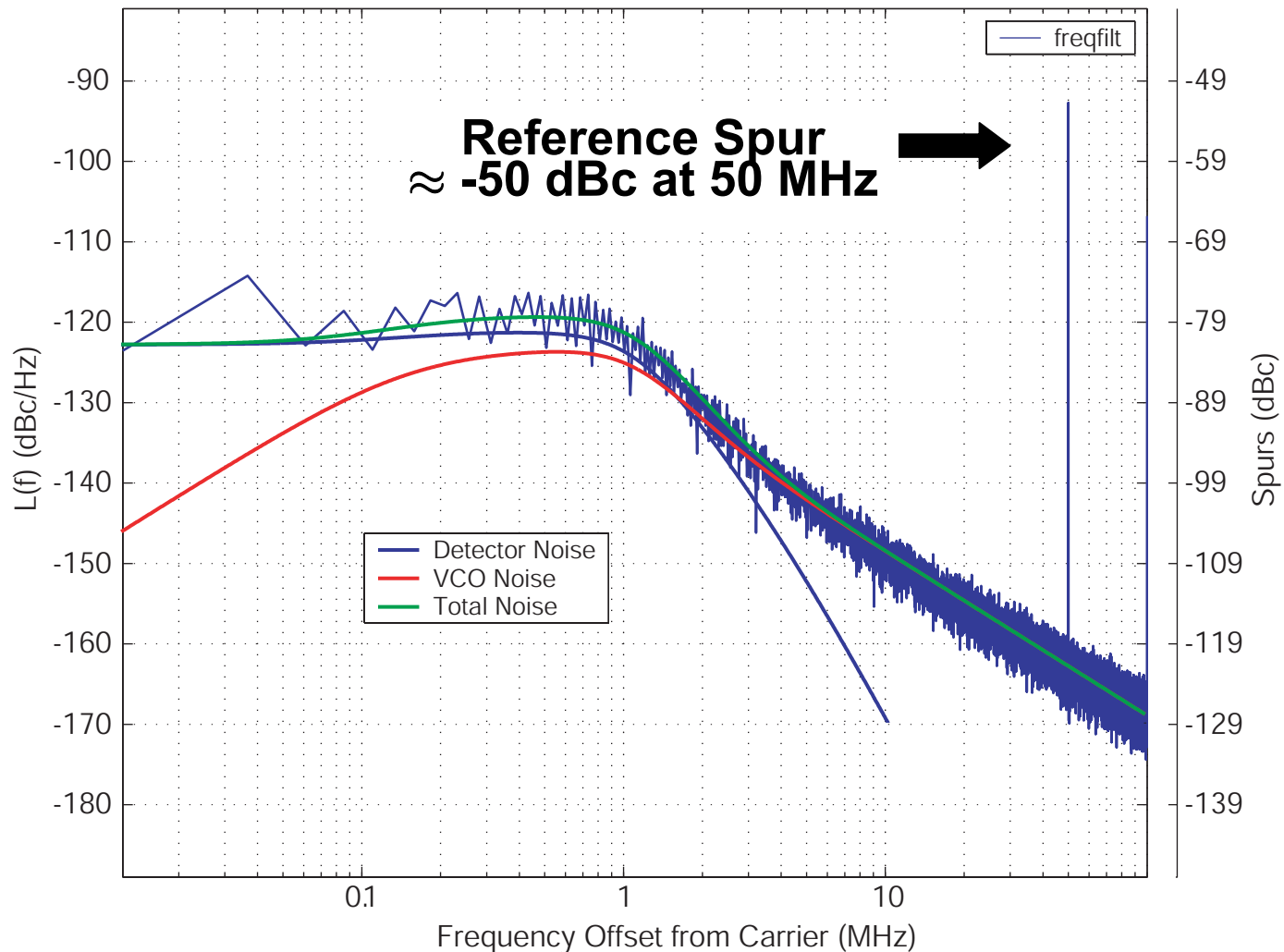


Resulting Plots and Jitter			
<input type="radio"/> Pole/Zero Diagram	<input type="radio"/> Transfer Function		
<input type="radio"/> Step Response	<input checked="" type="radio"/> Noise Plot		
10e3	100e6	-170	-60
ms jitter:	70.295 fs		
by Michael Perrott (http://www-mtl.mit.edu/~perrott)			

- We will see that we will need to include:
 - Reference noise
 - 1/f noise

Simulated Phase Noise due to Intrinsic Noise Sources

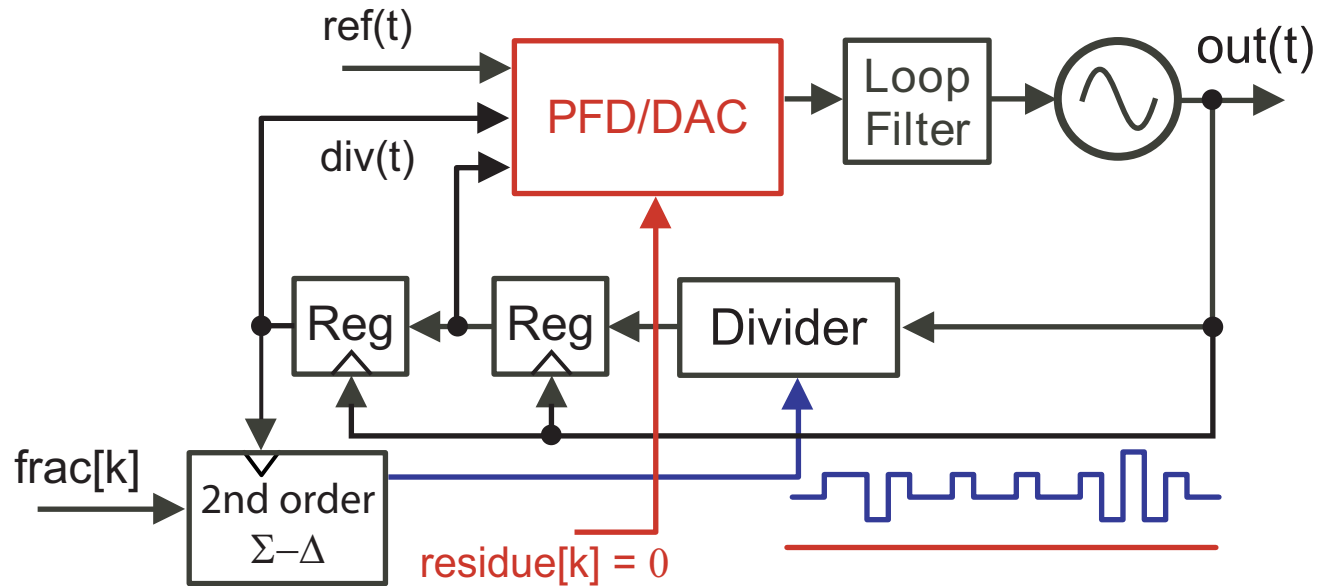
CppSim Simulated Phase Noise for Cell: wb_synth, Lib: WBSynth_Example, Sim: test_int_n.par



■ **PLL Design Assistant accurately models simulated noise!**

M.H. Perrott

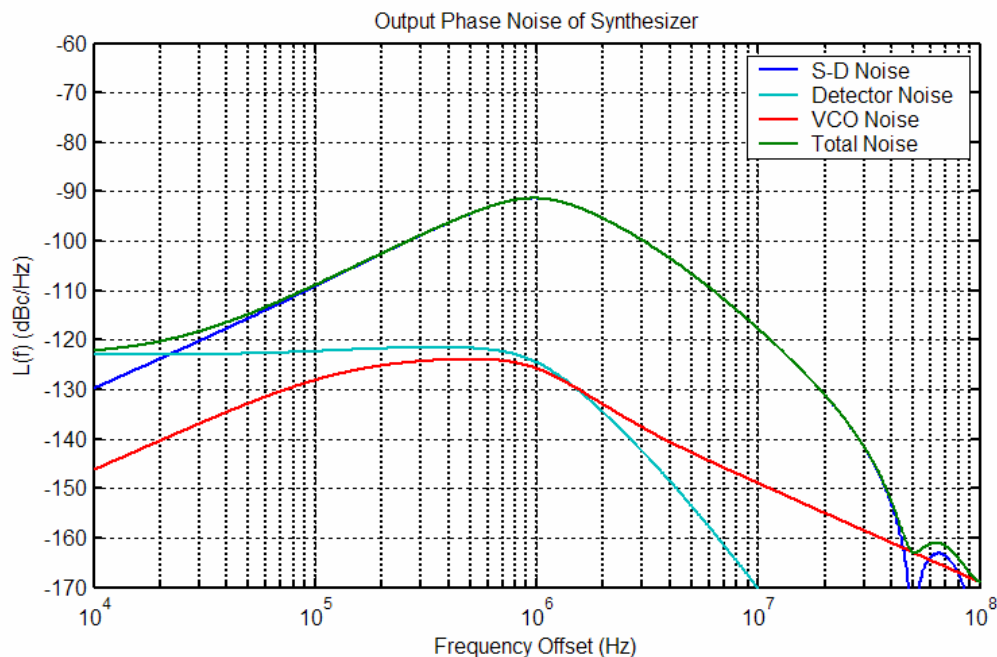
2nd Order $\Sigma\text{-}\Delta$ Fractional-N Performance



- Replace accumulator with second order $\Sigma\text{-}\Delta$ modulator
- Set residue into PFD/DAC equal to zero

Calculate PLL Noise for 2nd Order Σ - Δ Synthesizer

Dynamic Parameters		Noise Parameters	
fo	1e6 Hz	paris. pole	2.5e6 Hz
order	<input type="radio"/> 1 <input checked="" type="radio"/> 2 <input type="radio"/> 3	paris. Q	
shape	<input checked="" type="radio"/> Butter <input type="radio"/> Bessel	paris. pole	
	<input type="radio"/> Cheby1 <input type="radio"/> Cheby2 <input type="radio"/> Elliptical	paris. Q	
ripple		paris. pole	
		paris. pole	
type	<input type="radio"/> 1 <input checked="" type="radio"/> 2	paris. zero	
fz/fo	1/9	paris. zero	
		ref. freq	50e6 Hz
		out freq.	3.6e9 Hz
		Detector	-123 dBc/Hz
		VCO	-155 dBc/Hz
		freq. offset	20e6 Hz
		S-D	<input type="radio"/> 1 <input checked="" type="radio"/> 2
			<input type="radio"/> 3 <input type="radio"/> 4 <input type="radio"/> 5



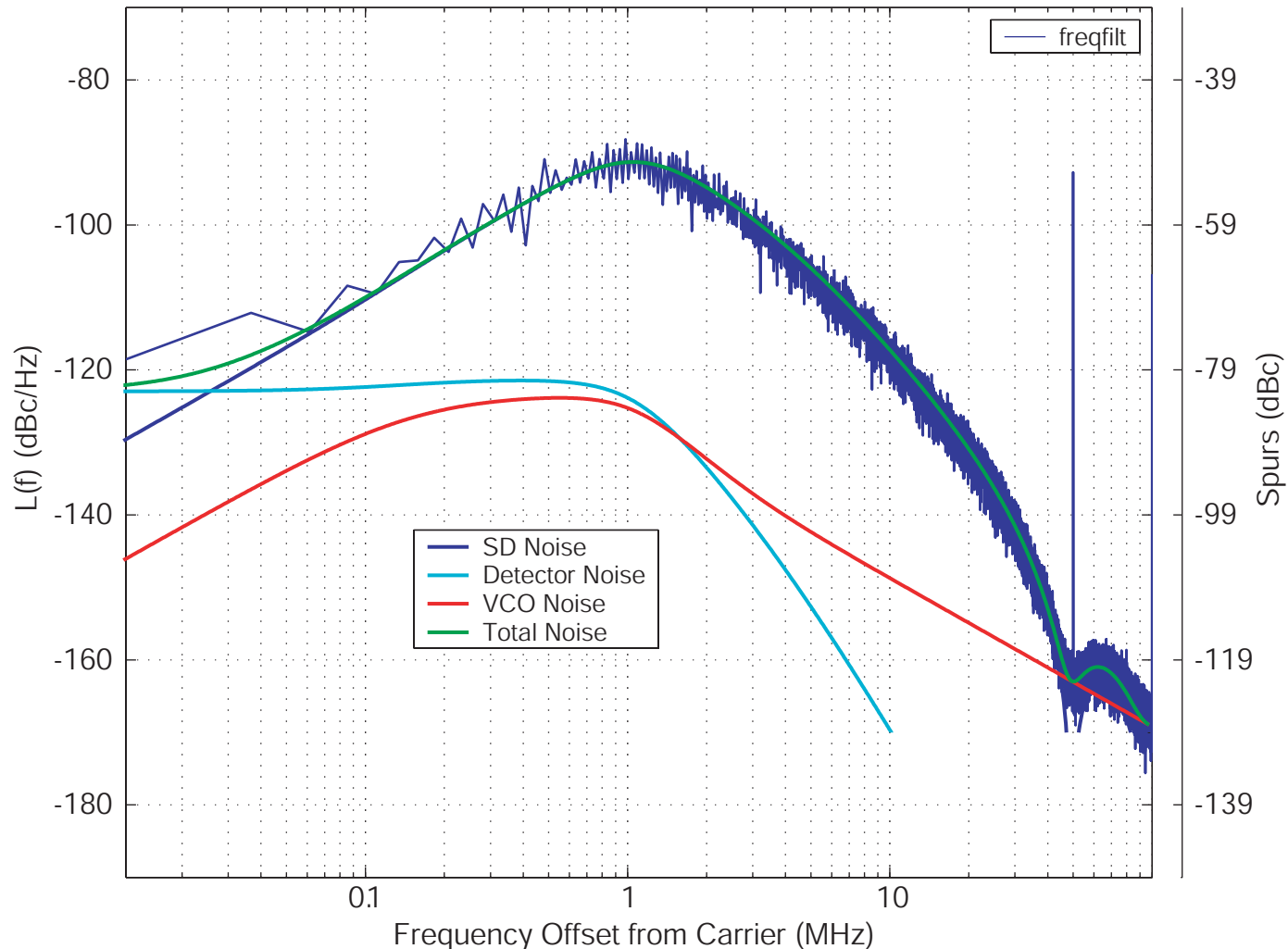
Resulting Plots and Jitter

<input type="radio"/> Pole/Zero Diagram	<input type="radio"/> Transfer Function		
<input type="radio"/> Step Response	<input checked="" type="radio"/> Noise Plot		
10e3	100e6	-170	-60
ns jitter:	2.155 ps		
by Michael Perrott (http://www-mtl.mit.edu/~perrott)			

- 2nd order Σ - Δ
- Click on 2nd order S-D quantization noise in tool

Simulated Phase Noise of 2nd Order $\Sigma\text{-}\Delta$ Synthesizer

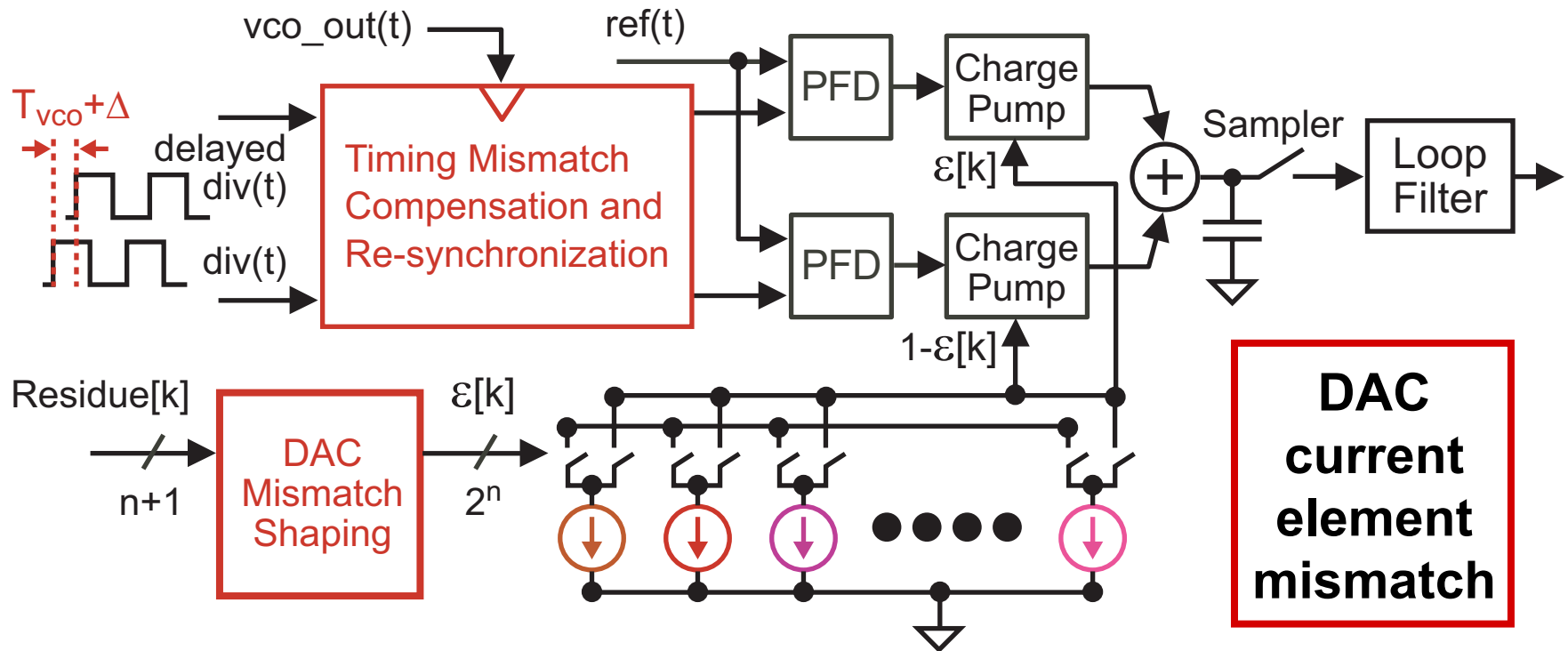
CppSim Simulated Phase Noise for Cell: wb_synth_sd2, Lib: WBSynth_Example, Sim: test.par



■ **PLL Design Assistant accurately models simulated noise!**

7-bit PFD/DAC Synthesizer Performance

Delay mismatch



Application of proposed noise scrambling/shaping techniques leads to broadband noise from delay and DAC current mismatch

Impact on PLL Noise due to Non-idealities of PFD/DAC

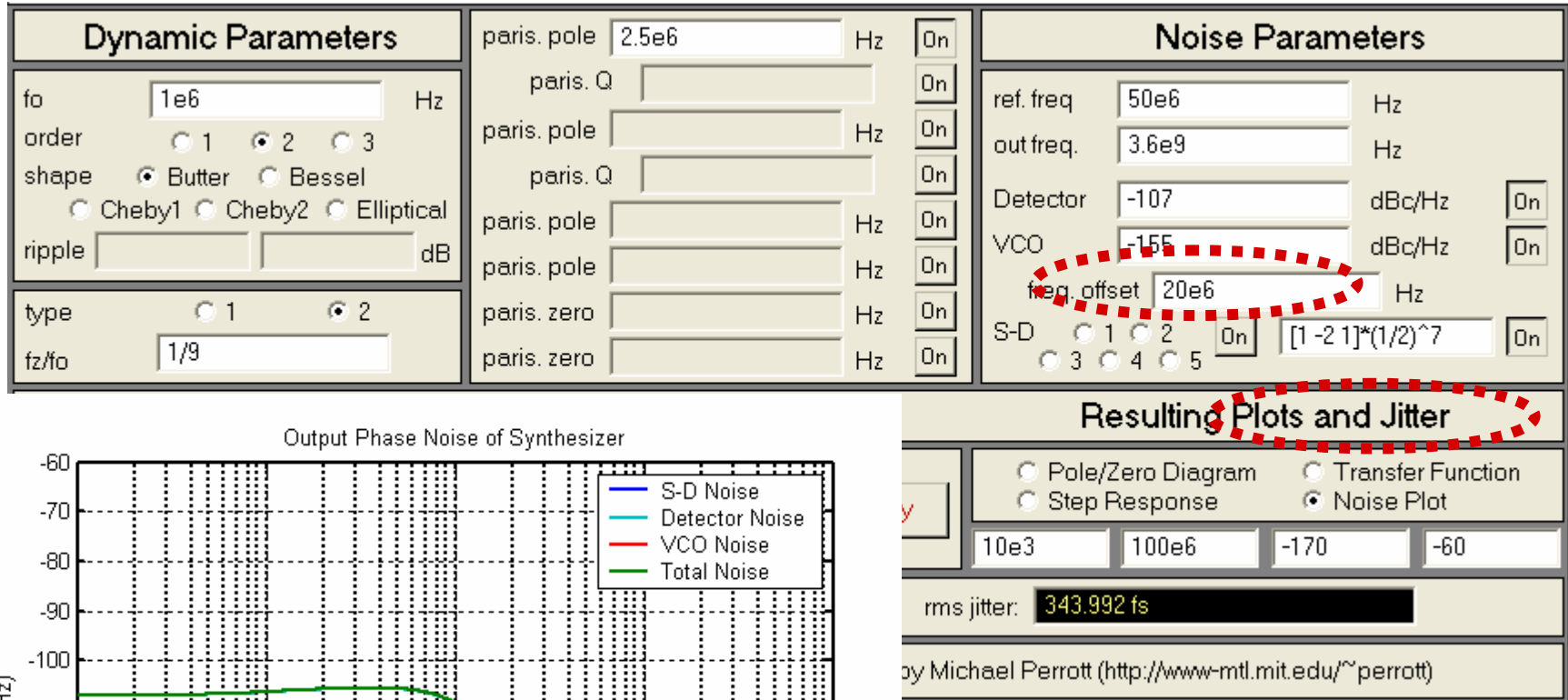
- Impact of DAC mismatch
 - Lowers achievable quantization noise suppression
 - Negligible in this case
- Impact of delay mismatch
 - Model as white reference noise uniformly distributed from 0 to Δt

- Calculation for $\Delta t = 5$ ps:

$$\begin{aligned} S_{\Phi_{out}}(f) &= \frac{1}{T} |TN_{nom}G(f)|^2 S_{\Phi_{jit}}(e^{j2\pi fT}) \\ &= \frac{1}{T} |TN_{nom}G(f)|^2 \left| \frac{2\pi}{T} \right|^2 \frac{(\Delta t)^2}{12} \\ &= (50 \text{ MHz})(71.3)^2 (2\pi)^2 \frac{(5 \text{ ps})^2}{12} \end{aligned}$$

$$\implies 10 \log(S_{\Phi_{out}}(f)) = -107 |G(f)|^2 \text{ dBc/Hz}$$

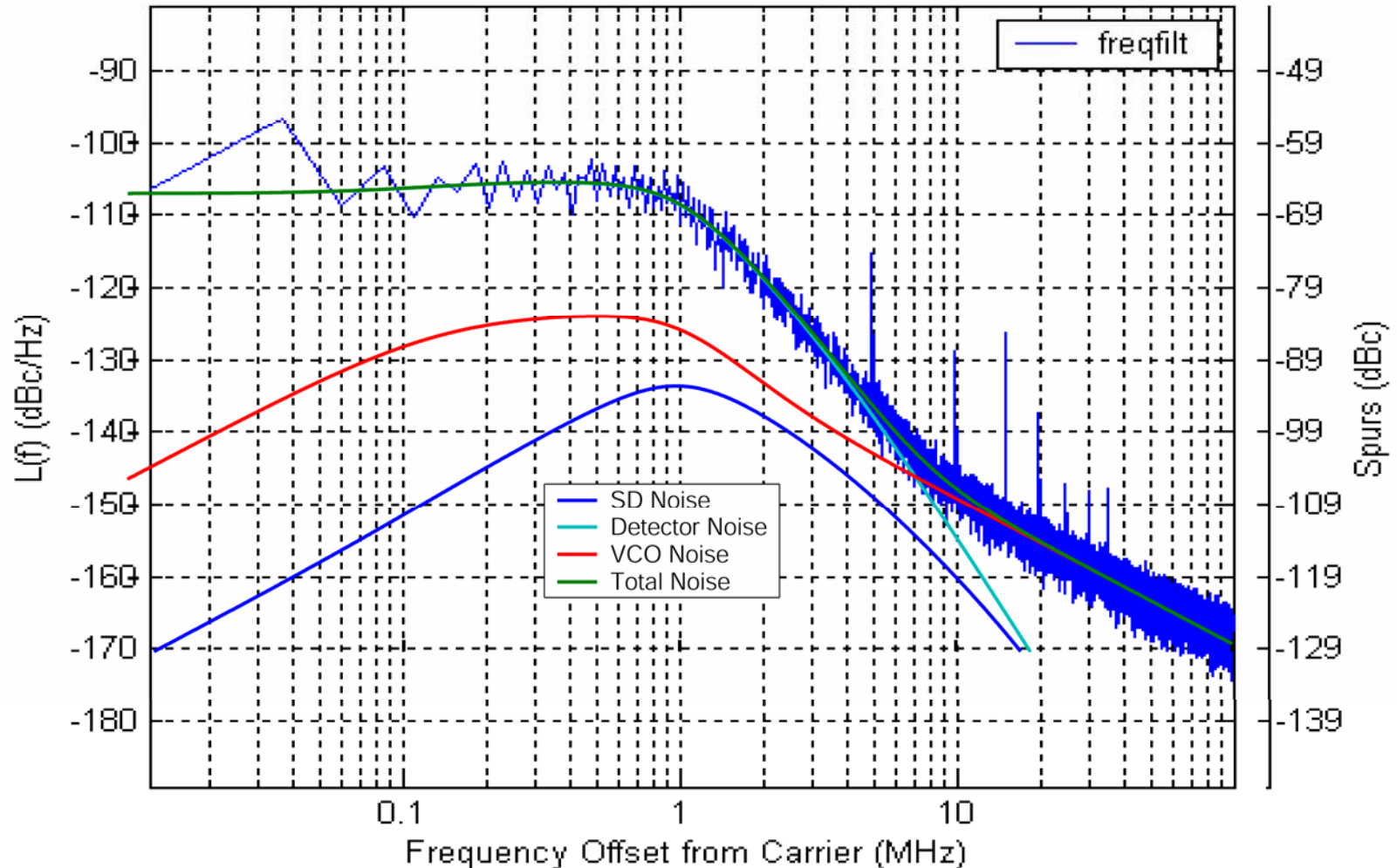
Calculate PLL Noise for 7-bit PFD/DAC Synthesizer



- **PFD/DAC**
 - Adjust S-D Quant. Noise
- **Delay mismatch**
 - Adjust Detector noise

Simulated PLL Phase Noise of 7-bit PFD/DAC

CppSim Simulated Phase Noise for Cell: wb_synth, Lib: WBSynth_Example, Sim: test.par



■ PLL Design Assistant accurately models simulated noise!

M.H. Perrott

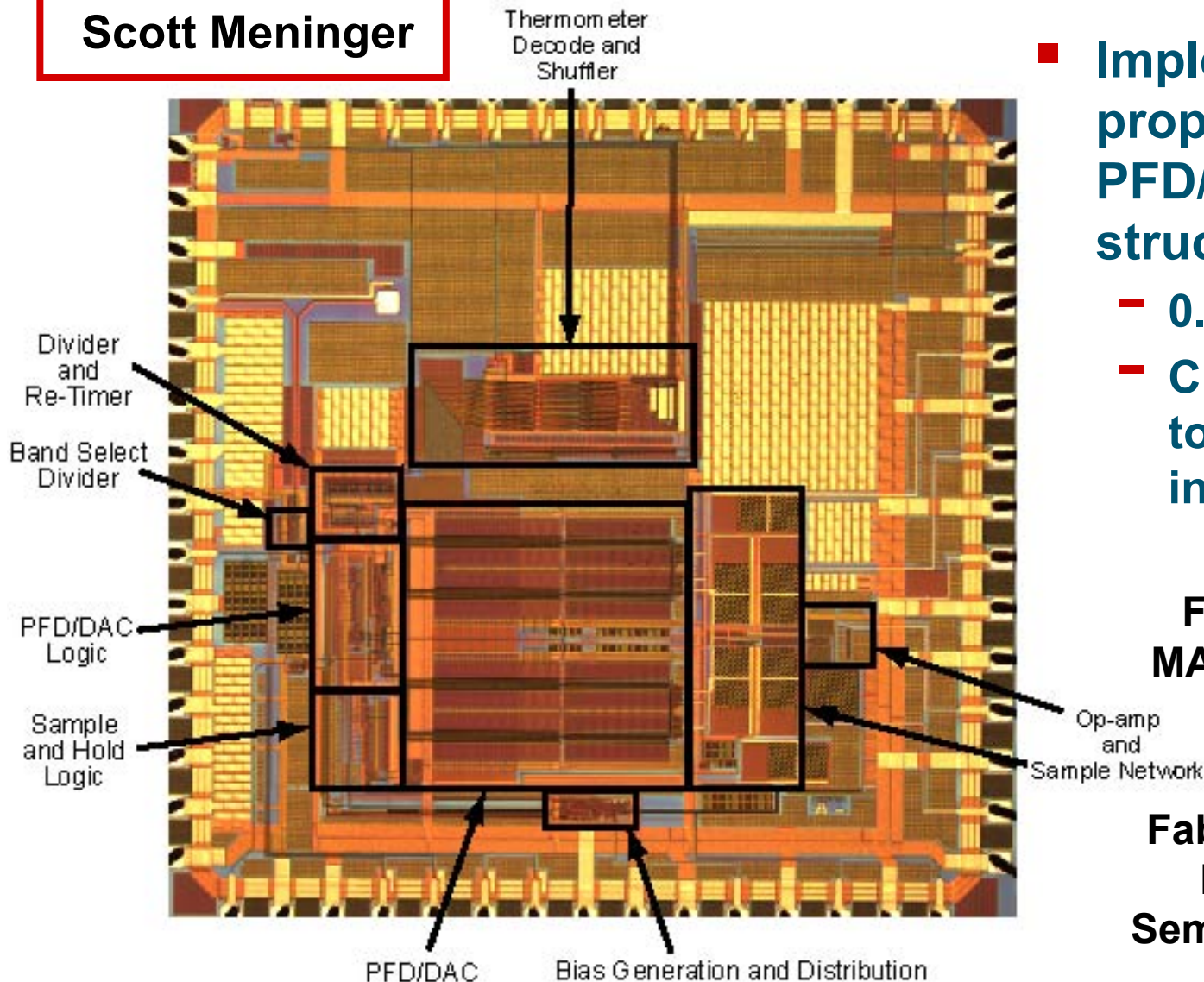
Summary of Design/Simulation Results

- The PLL Design Assistant can be used to model the impact of
 - Intrinsic PLL noise sources
 - Quantization noise due to $\Sigma-\Delta$ dithering of divide value
 - Suppression of quantization noise by n-bit PFD/DAC
 - Impact of delay and current mismatch on PLL phase noise
- CppSim simulations confirm the accuracy of the above analysis

How do PLL Design Assistant calculations compare to measured results?

A 1 MHz BW Fractional-N Frequency Synthesizer IC

Scott Meninger

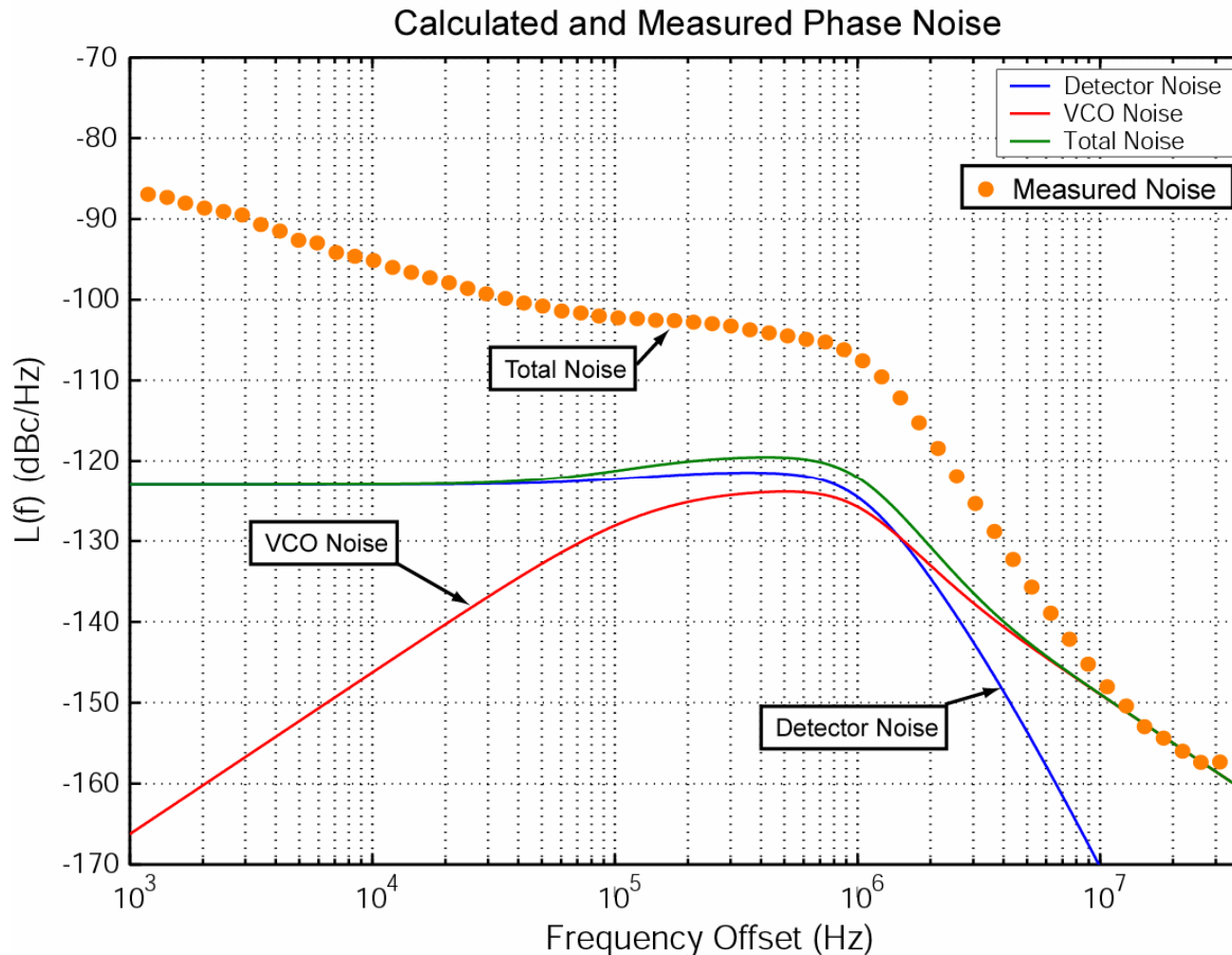


- Implements proposed 7-bit PFD/DAC structure
- 0.18u CMOS
- Circuit details to be published in the future

Funded by
MARCO C2S2

Fabricated by
National Semiconductor

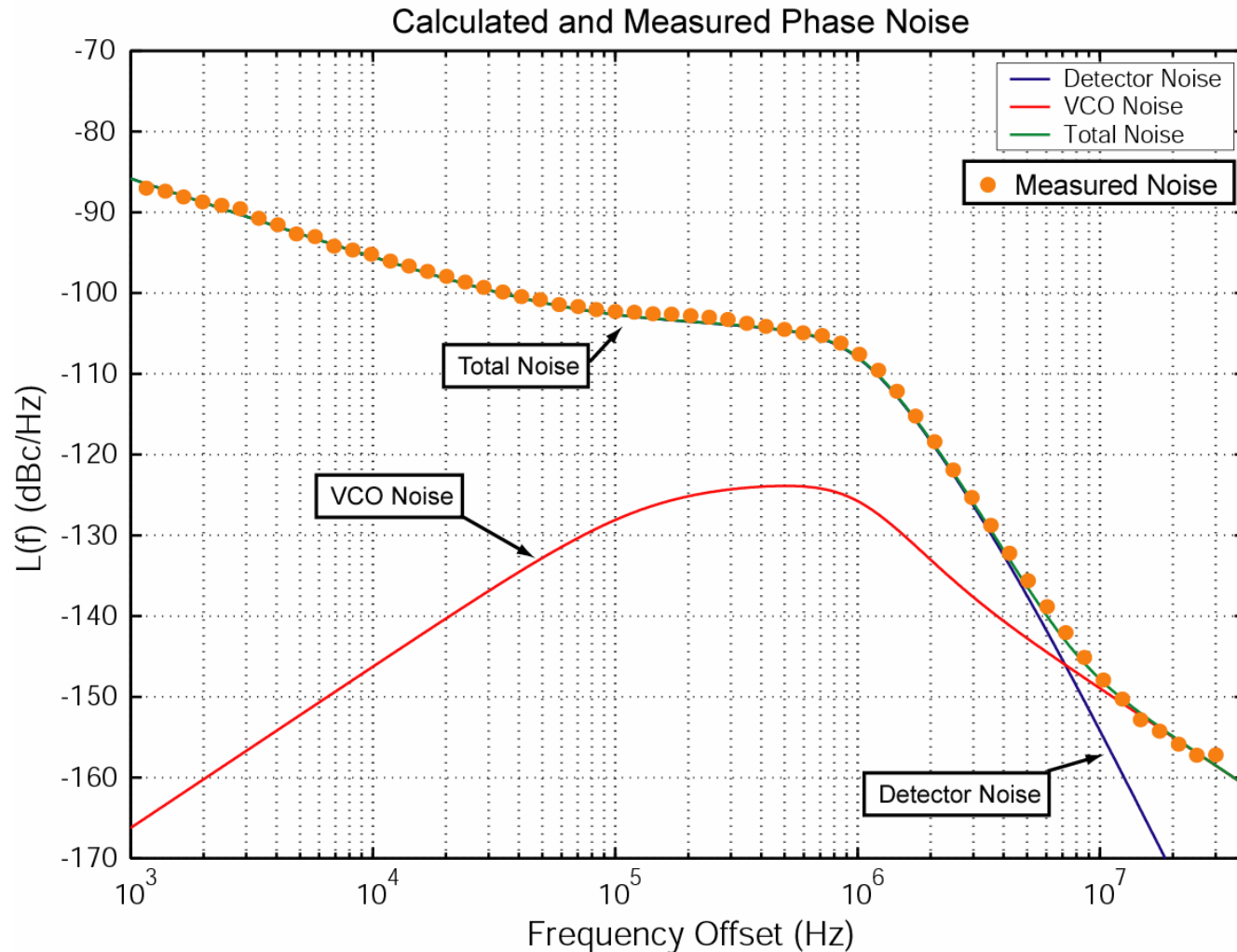
Measured vs Calculated Phase Noise (Integer-N)



- **Calculated noise is way off!**

- **Issue: we did not consider reference jitter and 1/f noise**

Adjustment of Calculations to Fit Measured Result



- **Calculated noise now assumes:**
 - **Detector noise is -107 dBc/Hz with 1/f corner of 130 kHz**

Back Extraction of Reference Jitter

$$S_{\Phi_{out}|REF}(f) = \overline{\Delta t_{jitt}^2} \cdot \frac{1}{T} \cdot \left(\frac{2\pi}{T}\right)^2 \cdot (N_{nom}T)^2 \cdot |G(f)|^2 \cdot \frac{i_{up}}{i_{down}}$$

Accounts for PFD structure
with reduced i_{up}

- Assuming $G(f) = 1$:

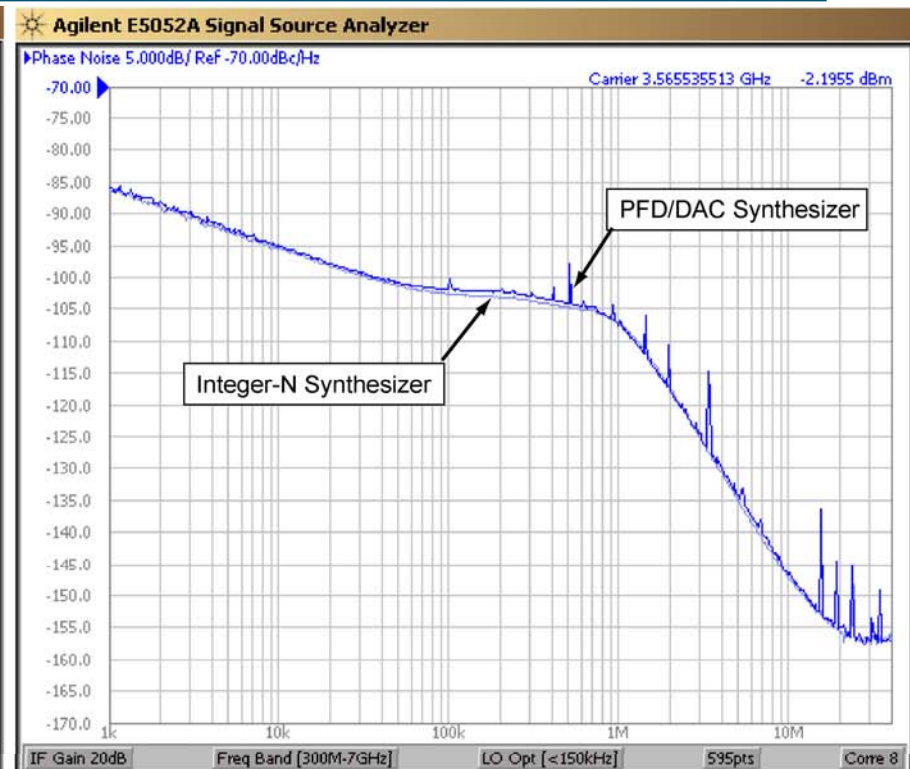
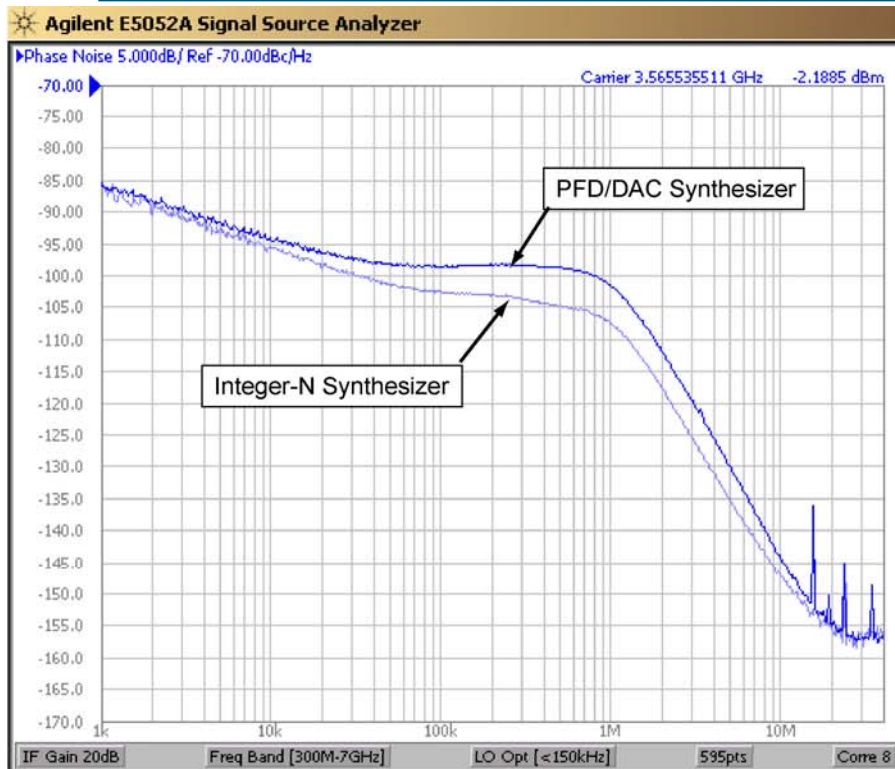
$$S_{\Phi_{out}|REF}(f) = 10^{(-107/10)}$$

$$\Rightarrow \overline{\Delta t_{jitt}^2} = S_{\Phi_{out}|REF} \cdot \frac{T}{(2\pi N_{nom})^2} \cdot \frac{i_{down}}{i_{up}}$$

- Assuming $N_{nom} = 73$, $T = 1/50\text{MHz}$, $i_{up}/i_{down} = 1/5$

$$\Rightarrow \Delta t_{jitt} = 3.08\text{ps}$$

7-bit PFD/DAC Synthesizer Vs Integer-N Configuration

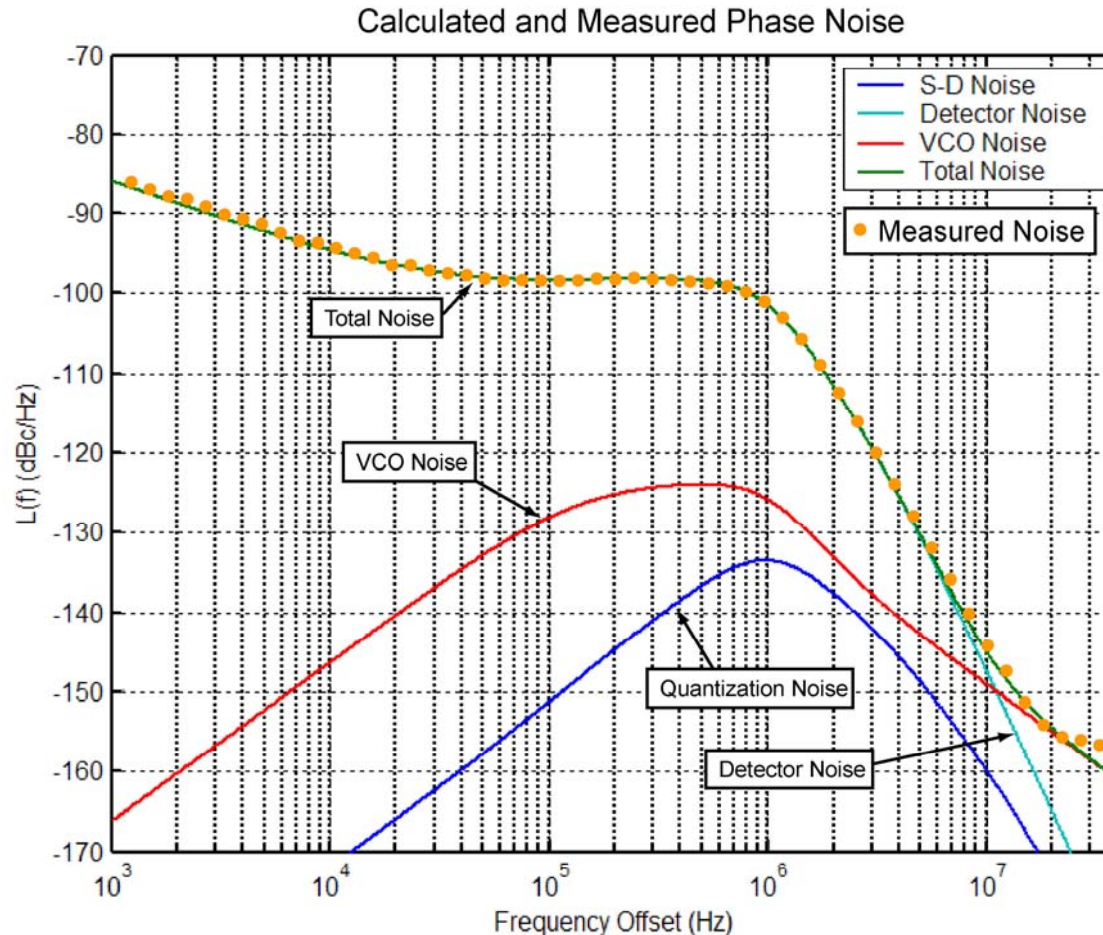


Phase Swapping Enabled

Phase Swapping Disabled

- **Left: Phase swapping enabled**
 - Timing mismatch converted into broadband noise
- **Right: Phase swapping disabled**
 - More fractional spurs, lower broadband noise

Adjustment of Calculations to fit Measured Results



- Calculated noise now assumes:
 - Detector noise is -100 dBc/Hz with 1/f corner of 20 kHz

Back Extraction of Timing Mismatch Using the Model

$$S_{\Phi_{out}|\Delta_t} = \frac{1}{T} \overline{\Delta_{t2}^2} (2\pi N_{nom})^2 \cdot |G(f)|^2$$

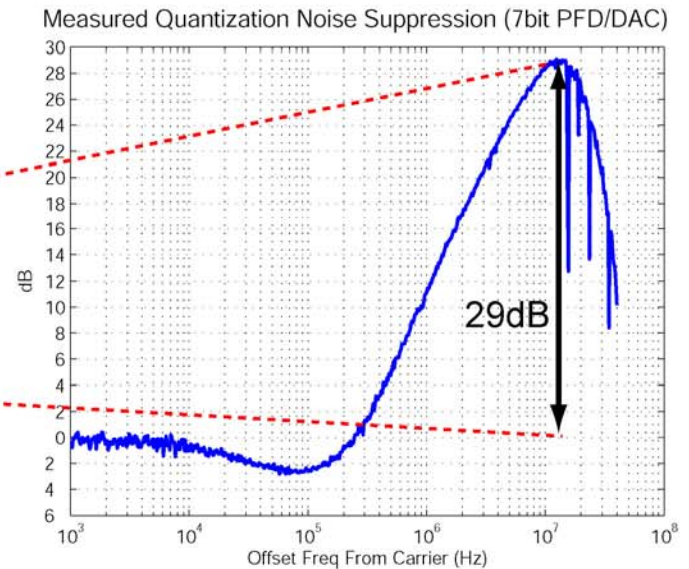
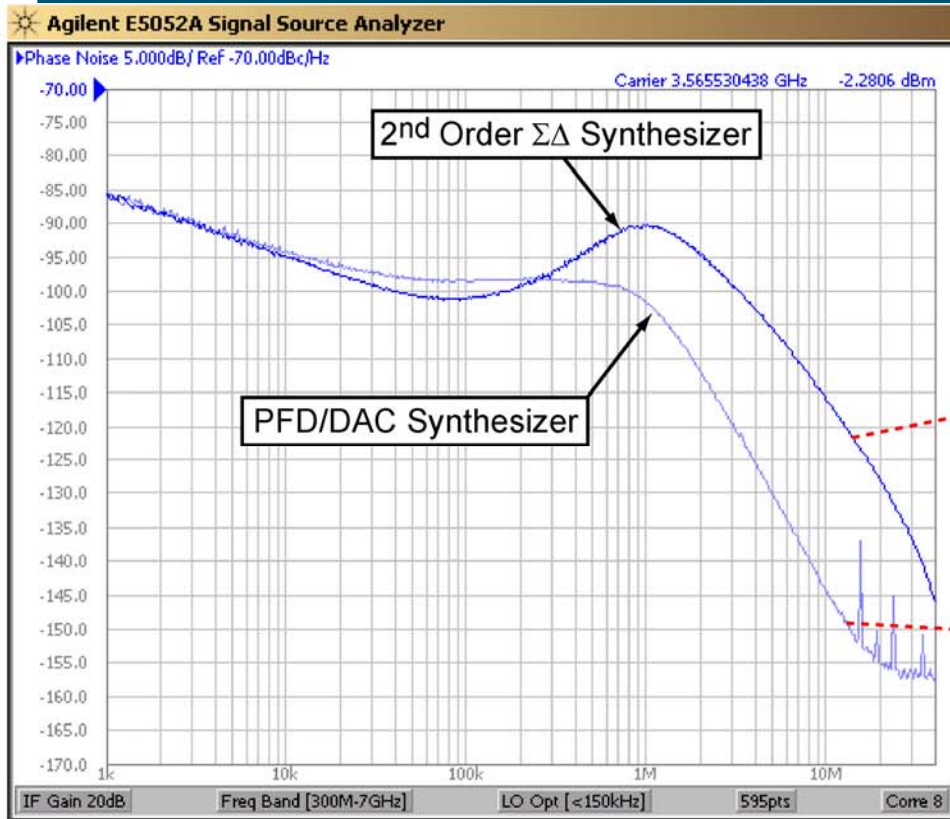
- Assuming $G(f) = 1$:

$$S_{\Phi_{out}|\Delta_t} = 10^{(-100/10)}$$

$$\Rightarrow \overline{\Delta_{t2}^2} = S_{\Phi_{out}|\Delta_t} \cdot \frac{12T}{(2\pi N_{nom})^2}$$

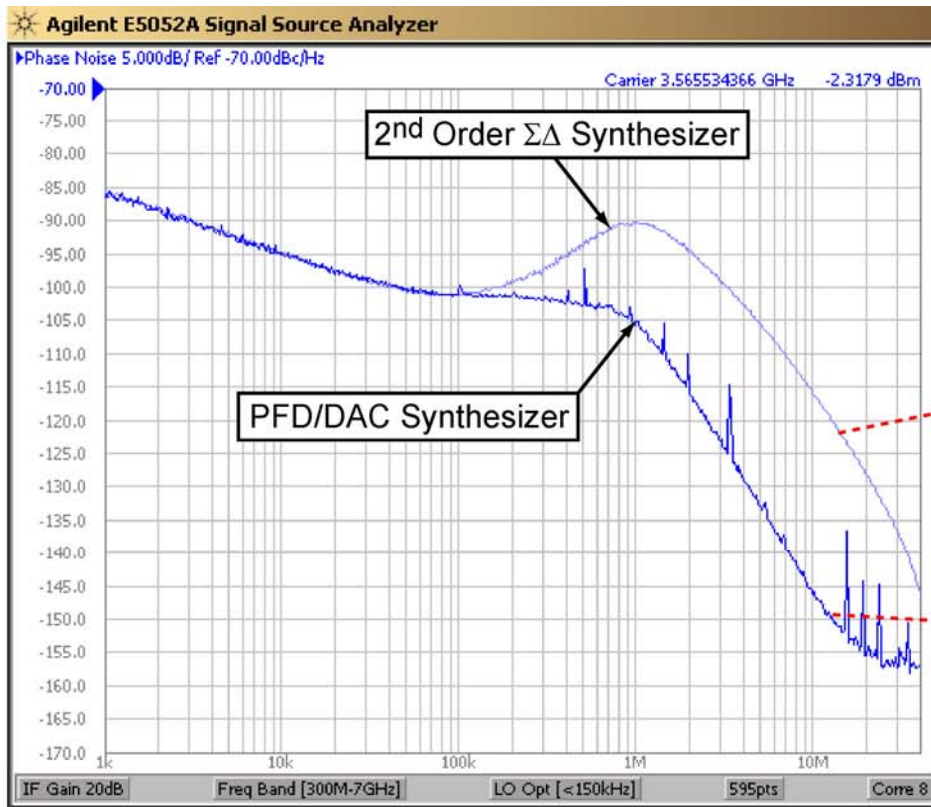
- Assuming $N_{nom} = 73$, $T = 1/50\text{MHz}$ $\Rightarrow \Delta_{t2} = 10.7 \text{ ps}$

Measured Noise Suppression

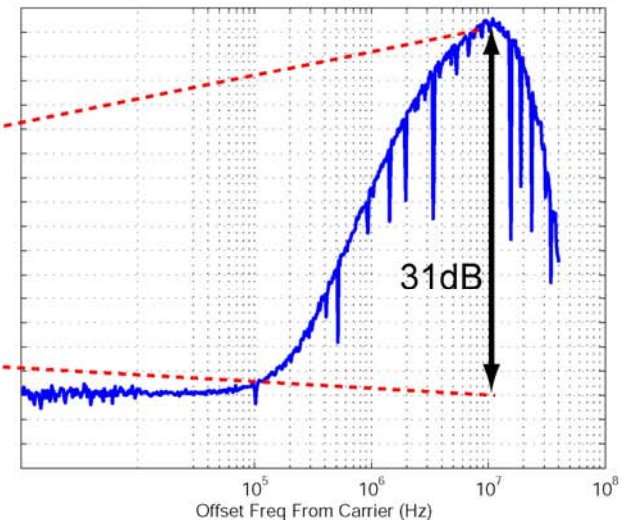


- Comparison of 7-bit PFD/DAC synthesizer with 2nd order $\Sigma\Delta$ Synthesizer
- Low freq noise ~2dB worse because of phase swapping
- **29dB quantization noise suppression measured at 10MHz !**

Measured Noise Suppression: No Swapping



Measured Quantization Noise Suppression (7bit PFD/DAC)



- Demonstrates that timing mismatch is degrading our maximum suppression by 2 dB (when swapping)
- Spurs occur due to gain error from timing mismatch

Summary of Calculation/Measured Results Comparison

- **Comparison of PLL Design Assistant results to measured data allow back extraction of key parameters:**
 - **Intrinsic noise**
 - Detector and VCO noise
 - **PFD/DAC nonidealities**
 - Delay mismatch value
- **Future work: better low frequency noise accuracy**

Conclusions

- **Fractional-N frequency synthesizers are about to undergo dramatic improvement in achieving high PLL bandwidth with excellent noise performance**
 - **The PFD/DAC approach presented here is only one of many possibilities to achieve this goal**
- **Design and simulation methodologies are starting to emerge**
 - **Analytical modeling of noise can be quite accurate**
 - **The PLL Design Assistant can be useful in this area**
 - **Behavioral simulation can be used to verify analytical models**
 - **CppSim offers a convenient and fast framework for this**

Research into High Bandwidth PLL Architectures is at an Exciting Crossroads