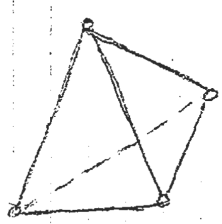
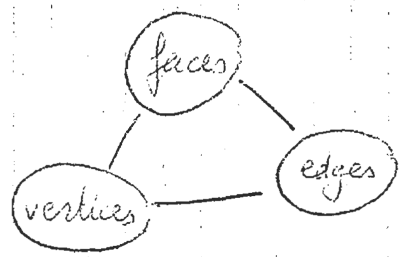
EGI (chapter 16)

Shape smooth   Representing polyhedra



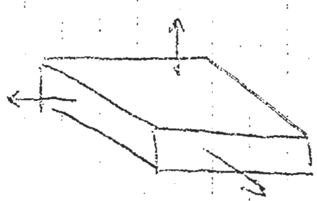


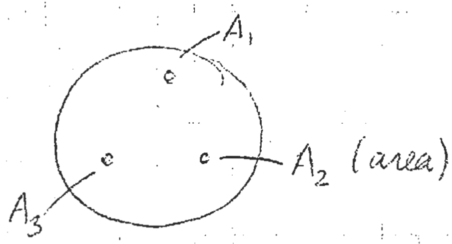
redundant

Vertex rep: $3n_v$

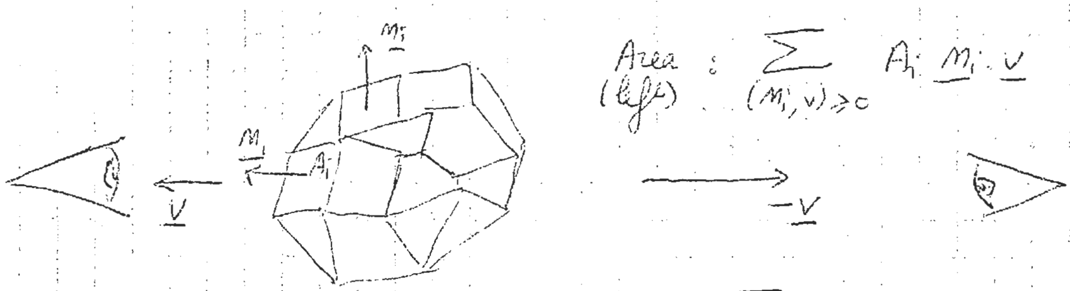
facet rep: $3n_f$ → minimum but not very convenient (edges are implicit)

Unique representation? yes: Minkowski (non-constructive)





Projection on sphere



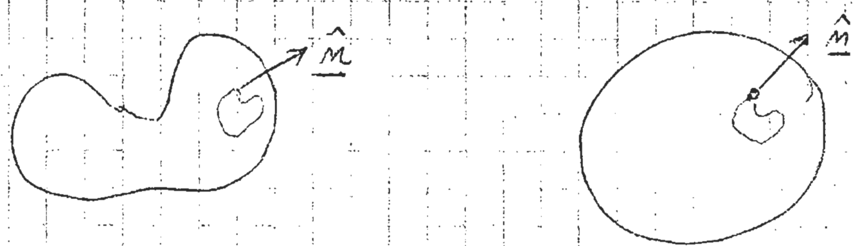
$$\text{Area (left)} : \sum_{(n_i, v) \geq 0} A_i \, \underline{n}_i \cdot \underline{v}$$

$$\text{Area (right)} = \sum_{(n_i, -\underline{v}) \geq 0} -A_i \, \underline{n}_i \cdot \underline{v}$$

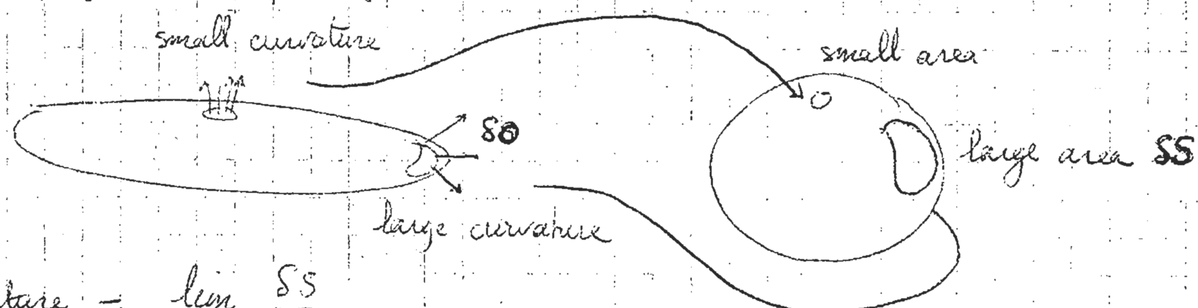
centroid of the sphere at the origin

$\boxed{\sum_i A_i \, \hat{\underline{n}}_i \cdot \hat{\underline{v}} = 0}$ for all $\underline{v}$ thus $\boxed{\sum A_i \, \hat{\underline{n}}_i = \underline{0}}$

Gaussian Image — mapping any object to a sphere



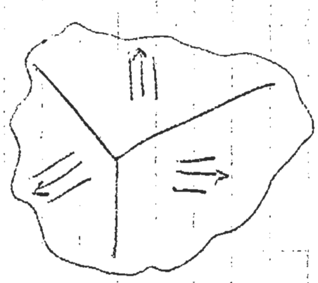
Mapping is unique if object is convex.

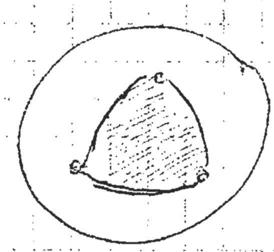


$$\text{Curvature} = \lim_{\delta O \to 0} \frac{\delta S}{\delta O}$$

$$\iint_O K \, \delta O = \iint_O \frac{dS}{dO} \, \delta O = S \longleftarrow \text{area on sphere}$$

integral of curvature



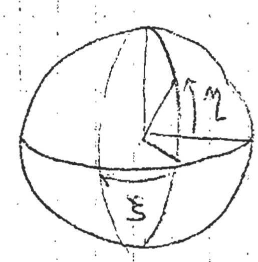


$$A = \frac{1}{8}(4\pi) = \frac{\pi}{2}.$$

$$\iint_S \frac{1}{K}\, dS = \iint_S \frac{dO}{dS}\, dS = O \leftarrow \text{area on the object}$$

$$\boxed{G(\xi,\eta) = \frac{1}{K(U,V)}}$$

EGI



Apparent area from one direction : and from the opposite direction :

$$\iint_{\underline{S}\cdot\underline{v} \geq 0} G(\hat{\underline{s}})\,(\hat{\underline{s}}\cdot\hat{\underline{v}})\, ds = \iint_{\underline{S}\cdot(-\underline{v}) \geq 0} G(\hat{\underline{s}})\,(\hat{\underline{s}}\cdot(-\hat{\underline{v}}))\, ds$$

($G(\hat{\underline{s}})\,ds$ area that has orientation $\hat{\underline{s}}$)

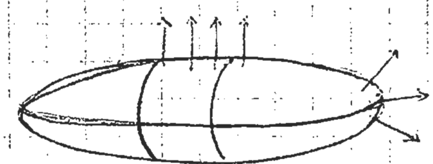
And again : $\boxed{\iint_S G(\hat{\underline{s}})\,\hat{\underline{s}} = \underline{\underline{0}}}$ (centroid at the origin)

While : $\iint_S G(\hat{\underline{s}})\,ds$ = total area of the object

<u>Example</u>: sphere, radius R

$$K = \frac{dS}{dO} = \frac{4\pi}{4\pi R^2} = \frac{1}{R^2} \qquad G = R^2 \ \text{(constant)}$$

Example: Ellipsoid



$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

$x = a.\cos\vartheta \cos\phi \qquad y = b \sin\vartheta \cos\phi \qquad z = c \sin\phi$ (explicit rep.)

$\underline{r} = (\quad " \quad , \quad " \quad , \quad " \quad)^T$

$\underline{r}_\vartheta = \dots \qquad r_\phi = \dots \qquad \underline{n} = \underline{r}_\vartheta \times \underline{r}_\phi \qquad \vartheta, \phi \rightsquigarrow \xi, \eta$

$\underline{n} = (bc \cos\vartheta \cos\phi, \; ca \sin\vartheta \cos\phi, \; ab \sin\phi)^T$

$\underline{\hat{n}} = (\cos\xi \cos\eta, \; \sin\xi \cos\eta, \; \sin\eta)^T$

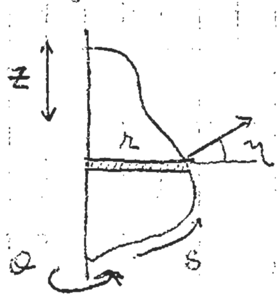
$$k = \left(\frac{abc}{(bc\cos\vartheta\cos\phi)^2 + (ca\sin\vartheta\cos\phi)^2 + (ab\sin\phi)^2} \right)^2$$

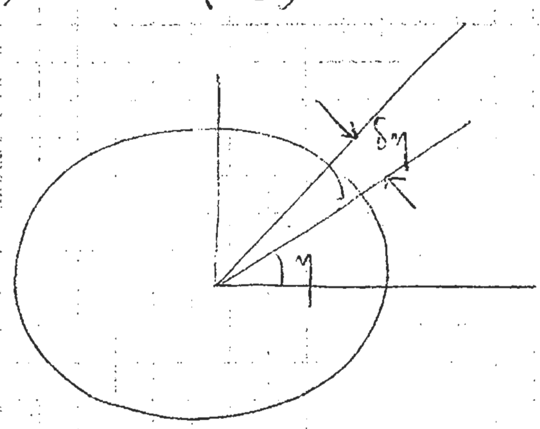
$$G(\xi, \eta) = \frac{1}{k(\vartheta,\varphi)} =$$

extrema: $(\pm 1, 0, 0) \quad (0, \pm 1, 0) \quad (0, 0, \pm 1)$

$\left(\frac{bc}{a}\right)^2 \quad \left(\frac{ca}{b}\right)^2 \quad \left(\frac{ab}{c}\right)^2$

Solids of revolution





$\delta S = 2\pi \cos\eta \, \delta\eta$

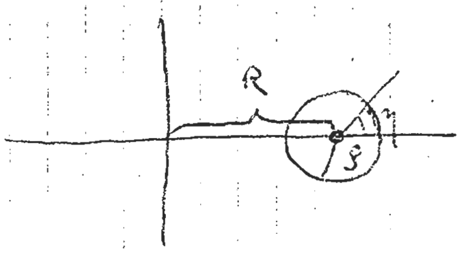
$$k = \frac{2\pi \cos\eta \, \delta\eta}{2\pi z \, \delta s}$$

$\frac{d\eta}{ds}$? measure of curvature

$\hookrightarrow K_G$ curvature of generator

$$\boxed{K = \frac{\cos\eta}{r} K_G}$$

Example: Torus



$$K_G = \frac{1}{\rho} \qquad r = R + \rho\cos\eta$$

$$K = \frac{\cos\eta}{R+\rho\cos\eta}\,\frac{1}{\rho}$$

$$G = \frac{1}{K} \qquad \left(\frac{1}{K_+} + \frac{1}{K_-}\right)$$

$$\rho\,\frac{(R+\rho\cos\eta)}{\cos\eta} + \rho\,\frac{(R-\rho\cos\eta)}{\cos\eta}$$

$$\Longrightarrow \boxed{G = 2\rho R\sec\eta}$$