6.863J Natural Language Processing
Lecture 22: Language Learning, Part 2

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The Menu Bar

- Administrivia:
  - Project-p?
- Can we beat the Gold standard?
  - Review of the framework
  - Various stochastic extensions
- Modern learning theory & sample size
  - Gold results still hold!
- Learning by setting parameters: the triggering learning algorithm

The problem

- From finite data, induce infinite set
- How is this possible, given limited time & computation?
- Children are not told grammar rules

- Ans: put constraints on class of possible grammars (or languages)

To review: the Gold framework

- Components:
- Target language L_{gt} or L_t (with target grammar g_t), drawn from hypothesis family *H*
- Data (input) sequences *D* and texts t; t_n
- Learning algorithm (mapping) *A* ; output hypothesis after input t_n $A(t_n)$
- Distance metric d, hypotheses h
- Definition of learnability:

$$d(g_t, h_n) \rightarrow_{n\rightarrow\infty} 0$$

Framework for learning

1. Target Language $L_t \in L$ is a target language drawn from a class of possible target languages L .
2. Example sentences $s_i \in L_t$ are drawn from the target language & presented to learner.
3. Hypothesis Languages $h \in H$ drawn from a class of possible hypothesis languages that child learners construct on the basis of exposure to the example sentences in the environment
4. Learning algorithm A is a computable procedure by which languages from H are selected given the examples

Some details

- Languages/grammars – alphabet Σ^*
- Example sentences
  - Independent of order
  - Or: Assume drawn from probability distribution μ (relative frequency of various kinds of sentences) – eg, hear shorter sentences more often
  - If $\mu \in L_t$, then the presentation consists of positive examples, o.w.,
  - examples in both L_t & $\Sigma^* - L_t$ (negative examples), I.e., all of Σ^* ("informant presentation")

Learning algorithms & texts

- A is mapping from set of all finite data streams to hypotheses in H
- Finite data stream of k examples $(s_1, s_2, \ldots, s_k)$
- Set of all data streams of length k,

$$D^k = \{(s_1, s_2, \ldots, s_k) \mid s_i \in \Sigma^*\} = (\Sigma^*)^k$$

- Set of all finite data sequences $D = \cup_{k>0} D^k$ (enumerable), so:

  $A : D \rightarrow H$

  - Can consider A to flip coins if need be

If learning by enumeration: The sequence of hypotheses after each sentence is $h1, h2, \ldots,$

Hypothesis after n sentences is h_n

ID in the limit - dfns

- Text t of language L is an infinite sequence of sentences of L with each sentence of L occurring at least once ("fair presentation")
- Text t_n is the first n sentences of t
- Learnability: Language L is learnable by algorithm A if for each t of L if there exists a number m s.t. for all $n>m$, $A(t_n) = L$
- More formally, fix distance metric d, a target grammar g_t and a text t for the target language. Learning algorithm A identifies (learns) g_t in the limit if

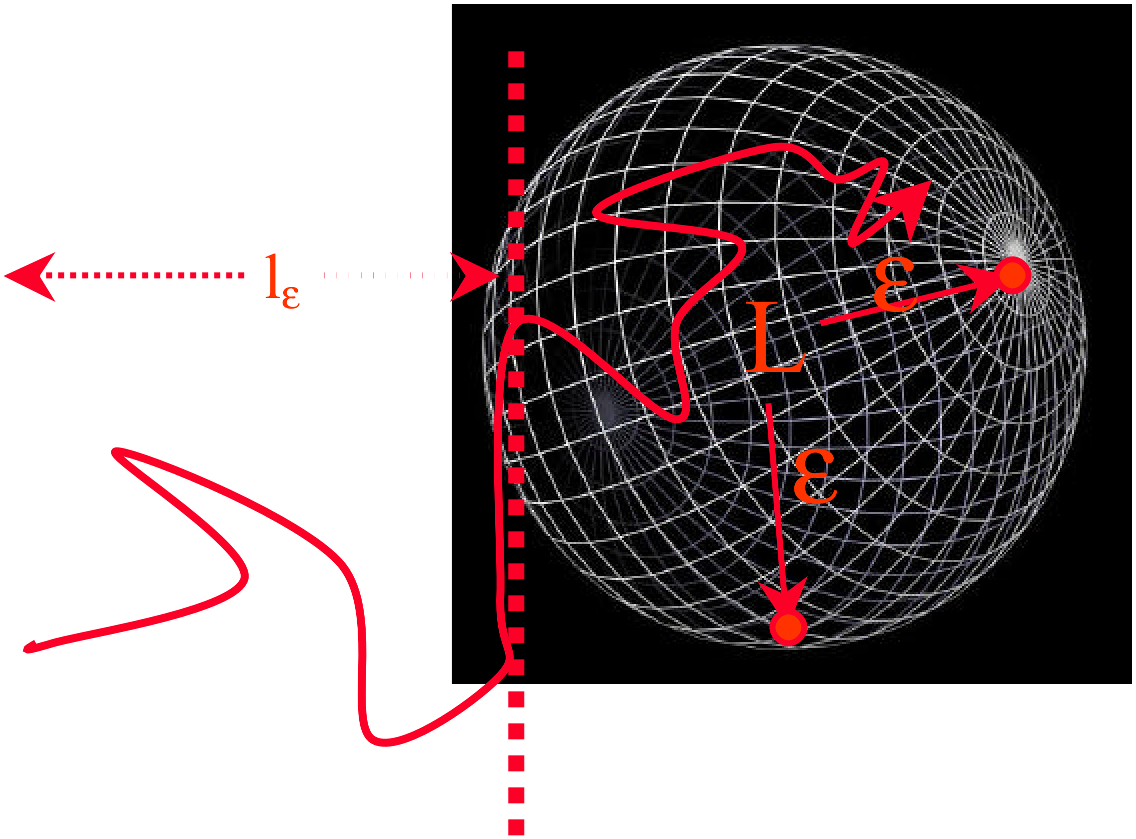
$$d(A(t_k), g_t) \to 0 \quad k \to \infty$$

Convergence in the limit

$$d(g_t\ ,\ h_n\ ) \rightarrow_{n\rightarrow\infty} 0$$

- This quantity is called generalization error
- Generalization error goes to 0 as # of examples goes to infinity
- In statistical setting, this error is a random variable & converges to 0 only in probabilistic sense (Valiant – PAC learning)

ε–learnability & "locking sequence/data set"



Ball of radius ε

Locking sequence:
If (finite) sequence l_ε gets within ε of target & then it <u>stays</u> there

Locking sequence theorem

- Thm 1 (Blum & Blum, 1975, ε version)
  If A identifies a target grammar g in the limit, then, for every $\varepsilon>0$, $\exists$ a locking sequence $l_e \in D$ s.t.

  (i) $l_e \subseteq L_g$ (ii) $d(A(l_e),g) < \varepsilon$ &

  (iii) $d(A(l_e.\sigma),g) < \varepsilon$, $\forall \sigma \in D$, $\sigma \subseteq L_g$

- Proof by contradiction. Suppose no such l_e

Proof…

- If no such l_e, then ∃ some σ_l s.t.
  $$d(A(l \bullet \sigma_l, g) \geq \varepsilon$$
- Use this to construct a text q on which A will not identify the target L_g
- Evil daddy: every time guesses get ε close to the target, we'll tack on a piece of σ_l that pushes it outside that ε–ball – so, conjectures on q greater than ε infinitely often

The adversarial parent…

- Remember: $d(A\ (l \bullet \sigma_l, g) \geq \varepsilon$
- Easy to be evil: construct $r = s_1, s_2, \ldots, s_n \ldots$ for L_g
- Let $q_1 = s_1$. If $d(A\ (q_i, g) < \varepsilon$, then pick a σ_{qi} and tack it onto the text sequence,

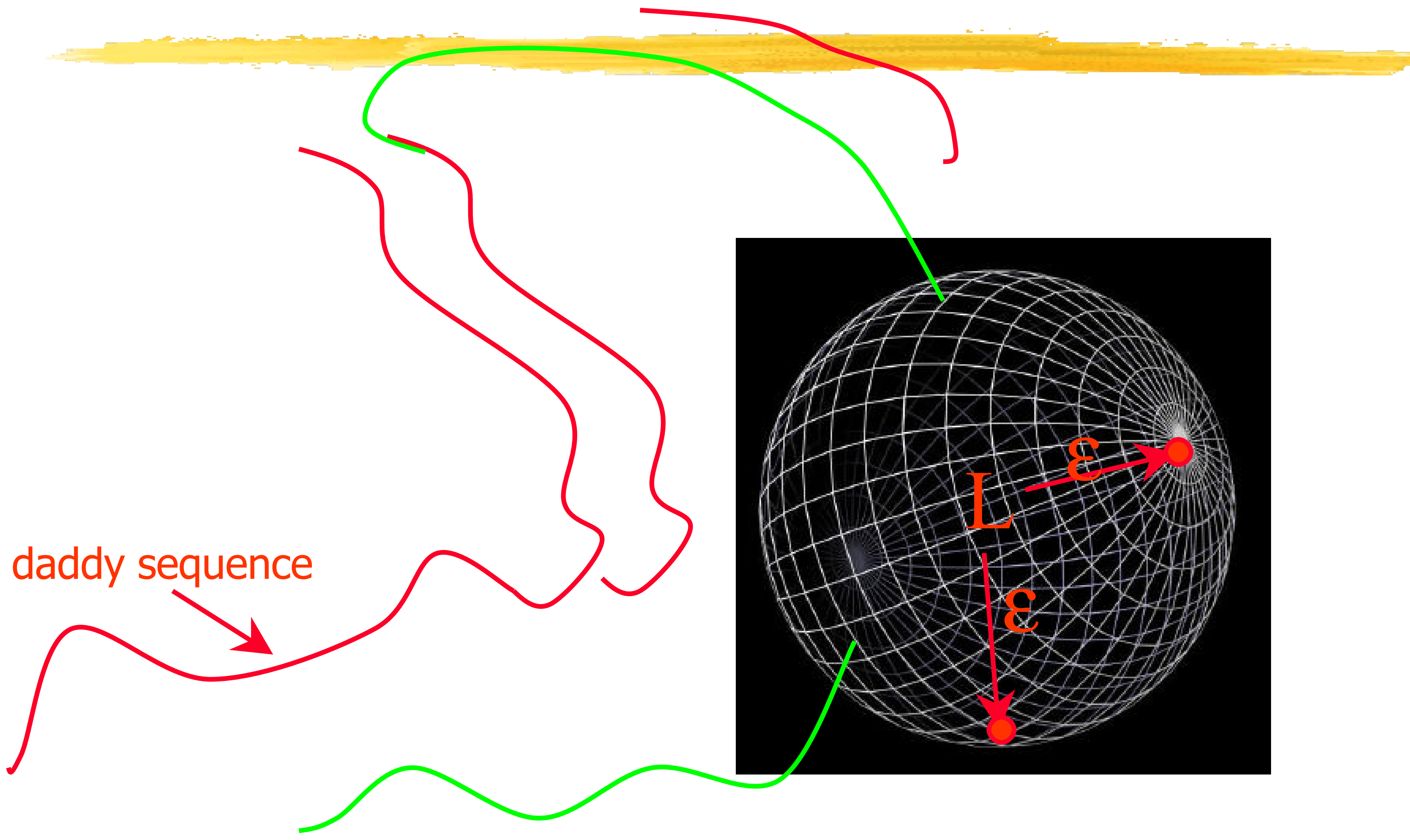
$$q_{i+1} = q_i\, \sigma_{qi}\, s_{i+1}$$

o.w. , d is already too large ($>\varepsilon$) , so can leave q_{i+1} sequence as q_i followed by s_{i+1}

$$q_{i+1} = q_i\, s_{i+1}$$

Pinocchio sequence…

Evil daddy sequence



Gold's theorem

- Suppose *A* is able to identify the family *L*. Then it must identify the infinite language, L_{inf} .
- By Thm, a locking sequence exists, σ_{inf}
- Construct a finite language $L_{\sigma_{inf}}$ from this locking sequence to get locking sequence for $L_{\sigma_{inf}}$ - a different language from L_{inf}
- *A* can't identify $L_{\sigma_{inf}}$, a contradiction

Example of identification (learning) in the limit – whether TM halts or not

<u>Dfn of learns</u>: ∃ some point m after which (i) algorithm *A* outputs correct answer; and (ii) no longer changes its answer.

The following *A* will work:

Given any Turing Machine M_j , at each time i , run the machine for i steps.

If after i steps, if M has not halted, output 0 (i.e., "NO"), o.w., output 1 (i.e, "Yes")

Suppose TM halts:

1 2 3 4 5 … m m+1 …

NO NO NO NO NO … NO YES YES YES …

Suppose TM does not halt:

1 2 3 4 5 …

NO NO NO NO NO … NO NO NO NO …

Exact learning seems too stringent

- Why should we have to speak perfect French forever?
- Can't we say "MacDonald's" once in a while?
- Or what about this:
- You say potato; I say pohtahto; You say potato; I say pohtahto;…

Summary of learnability given Gold

- With positive-only evidence, no interesting families of languages are learnable
- Even if given (sentence, meaning)
- Even if a stochastic grammar (mommy is talking via some distribution μ)
  - BUT if learner knew what the distribution was, they could learn in this case – however, this is almost like knowing the language anyway

If a parent were to provide true negative evidence of the type specified by Gold, interactions would look like the Osbournes:

Child:	me want more.
Father:	ungrammatical.
Child:	want more milk.
Father:	ungrammatical.
Child:	more milk !
Father:	ungrammatical.
Child:	cries
Father:	ungrammatical

When is learnability possible?

- Strong constraints on distribution
- Finite number of languages/grammars
- Both positive and (lots of) negative evidence
  - the negative evidence must also be 'fair' – in the sense of covering the distribution of possibilities (not just a few pinpricks here and there…)

Positive results from Gold

- Active learning: suppose learner can query membership of arbitrary elts of Σ^*
- Then DFAs learnably in poly time, <u>but</u> CFGs still unlearnable
- So, does enlarge learnability possibilities – but arbitrary query power seems questionable

Relaxing the Gold framework constraints: toward the statistical framework

- Exact identification $\rightarrow$ ε–identification
- Identification on all texts $\rightarrow$ identification only on > 1-δ (so lose, say, 1% of the time)
  - This is called a (ε, δ) framework

Statistical learning theory approach

- Removes most of the assumptions of the Gold framework –
- It does not ask for convergence to exactly the right language
- The learner receives positive and negative examples
- The learning process has to end after a certain number of examples
- Get bounds on the # of examples sentences needed to converge with high probability
- Can also remove assumption of arbitrary resources: efficient (poly time) [Valiant/PAC]

Modern statistical learning: VC dimension & Vapnik-Chervonenkis theorem (1971,1991)

- Distribution-free (no assumptions on the source distribution)
- No assumption about learning algorithm
- TWO key results:

1. Necessary & sufficient conditions for learning to be possible at all ("capacity" of learning machinery)
2. Upper & lower bounds on # of examples needed

Statistical learning theory goes further – but same results

- Languages defined as before:

  $1_L(s)=1$ if $s \in L$, 0 o.w. (an 'indicator function')

- Examples provided by some distribution P on set of all sentences
- Distances between languages defined as well by the probability measure P

  $d(L_1 - L_2) = \Sigma_S \mid 1_{L1}(s) - 1_{L2}(s) \mid P(s)$

This is a 'graded distance' - $L_1(P)$ topology

Learnability in statistical framework

Model:

- Examples drawn randomly, depending on P
- After l data pts, learner conjectures hypothesis h_l - note, this is now a random variable, because it depends on the randomly generated data
- Dfn: Learner's hypothesis h_l converges to the target (1_L) with probability 1, iff for every $\varepsilon > 0$

$$\text{Prob}[d(h_l\,,\,1_L) > \varepsilon\,] \rightarrow_{l \rightarrow \infty} 0$$

P is not known to the learner except through the draws

(What about how h is chosen? We might want to minimize error from target…)

Standard P(robably) A(approximately) C(orrect) formulation (PAC learning)

- If h_l converges to the target 1_L in a weak sense, then for every $\varepsilon>0$ there exists an $m(\varepsilon,\delta)$ s.t. for all $l > m(\varepsilon,\delta)$

$$Prob[d(h_l , 1_L) > \varepsilon] < \delta$$

With high probability (> $1-\delta$) the learner's hypothesis is approximately close (within ε in this norm) to the target language

m is the # of samples the learner must draw

$m(\varepsilon,\delta)$ is the sample complexity of learning

Vapnik- Chervonenkis result

- Gets lower & upper bounds on $m(\varepsilon,\delta)$
- Bounds depend on ε, δ and a measure of the "capacity" of the hypothesis space *H* called VC-dimension, d

$$m(\varepsilon,\delta) > f(\varepsilon,\delta, d)$$

- What's this d ??
- Note: distribution free!

VC dimension,"d"

- Measures how much info we can pack into a set of hypotheses, in terms of its discriminability – its learning capacity or flexibility
- Combinatorial complexity
- Defined as the largest d s.t. there exists a set of d points that H can shatter, and ∞ otherwise
- Key result: L is learnable iff it has finite VC dimension (d finite)
- Also gives lower bound on # of examples needed
- Defined in terms of "shattering"

Shattering

- Suppose we have a set of points $x_1, x_2, \ldots, x_n$
- If for every <u>different</u> way of partitioning the set of n points into two classes (labeled 0 & 1), a function in *H* is able to <u>implement</u> the partition (the function will be different for every different partition) we say that the set of points <u>is shattered</u> by *H*
- This says "how rich" or "how powerful" *H* is – its representational or informational capacity for learning

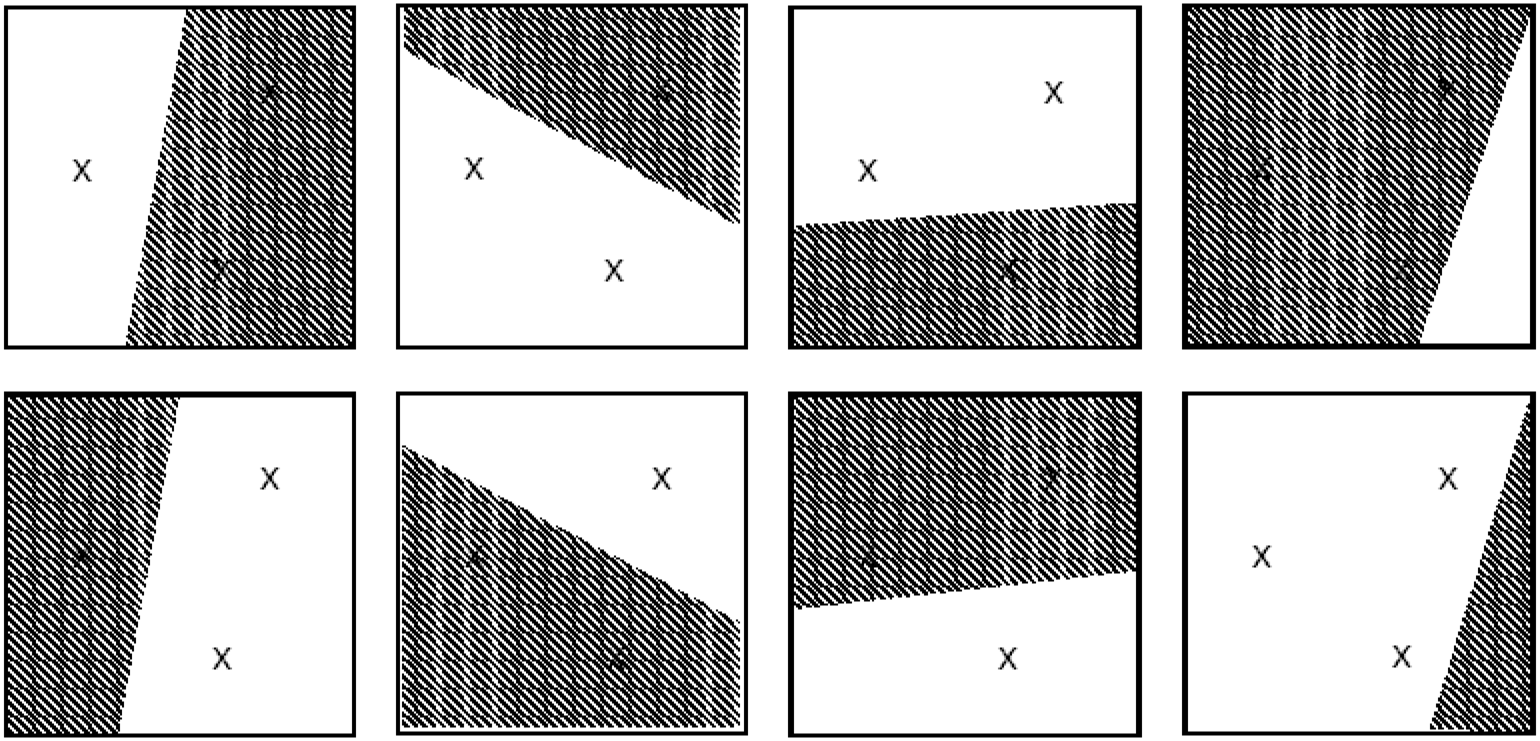
Shattering – alternative 'view'

- *H* can shatter a set of points iff for every possible training set, there are some way to twiddle the *h*'s such that the training error is 0

Example 1

- Suppose *H* is the class of linear separators in 2-D (half-plane slices)
- We have 3 points. With +/- (or 0/1) labels, there are 8 partitions (in general: with m pts, 2^m partitions)
- Then any partition of 3 points in a plane can be separated by a half-plane:

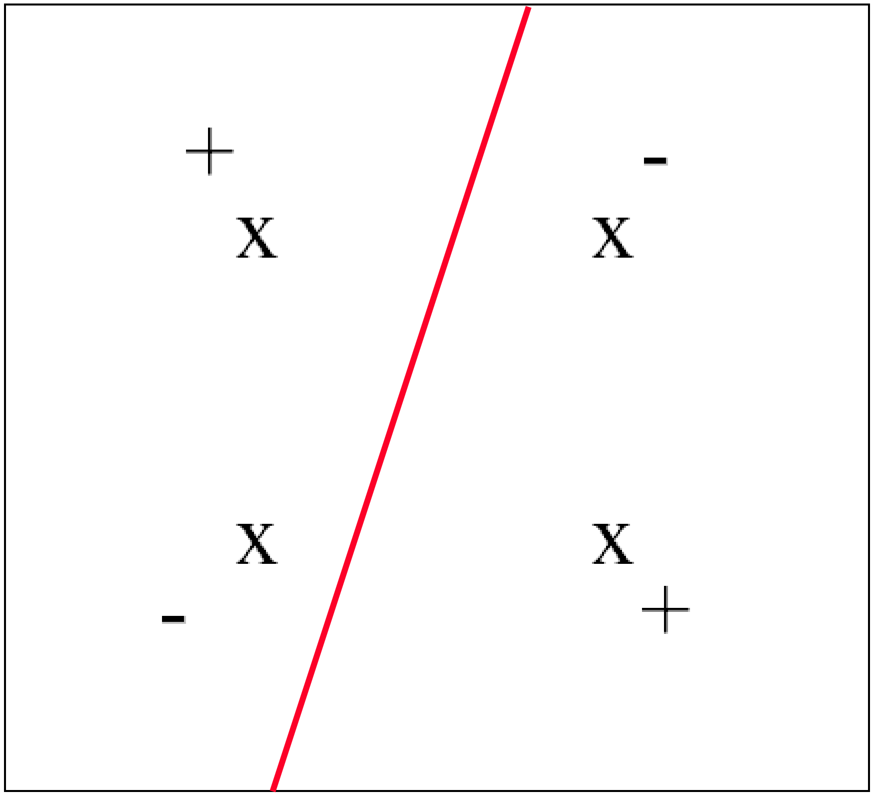
Half-planes can shatter <u>any</u> 3 point partition in 2-D: white=0; shaded =1 (there are 8 labelings)



BUT NOT…

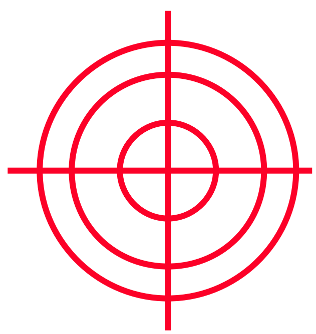
But not 4 points – this labeling can't be done by a half-plane:

…so, VC dimension for *H* = half-planes is 3

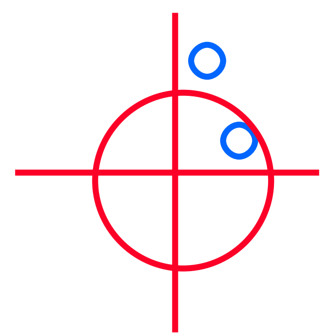


Another case: class *H* is circles (of a restricted sort)

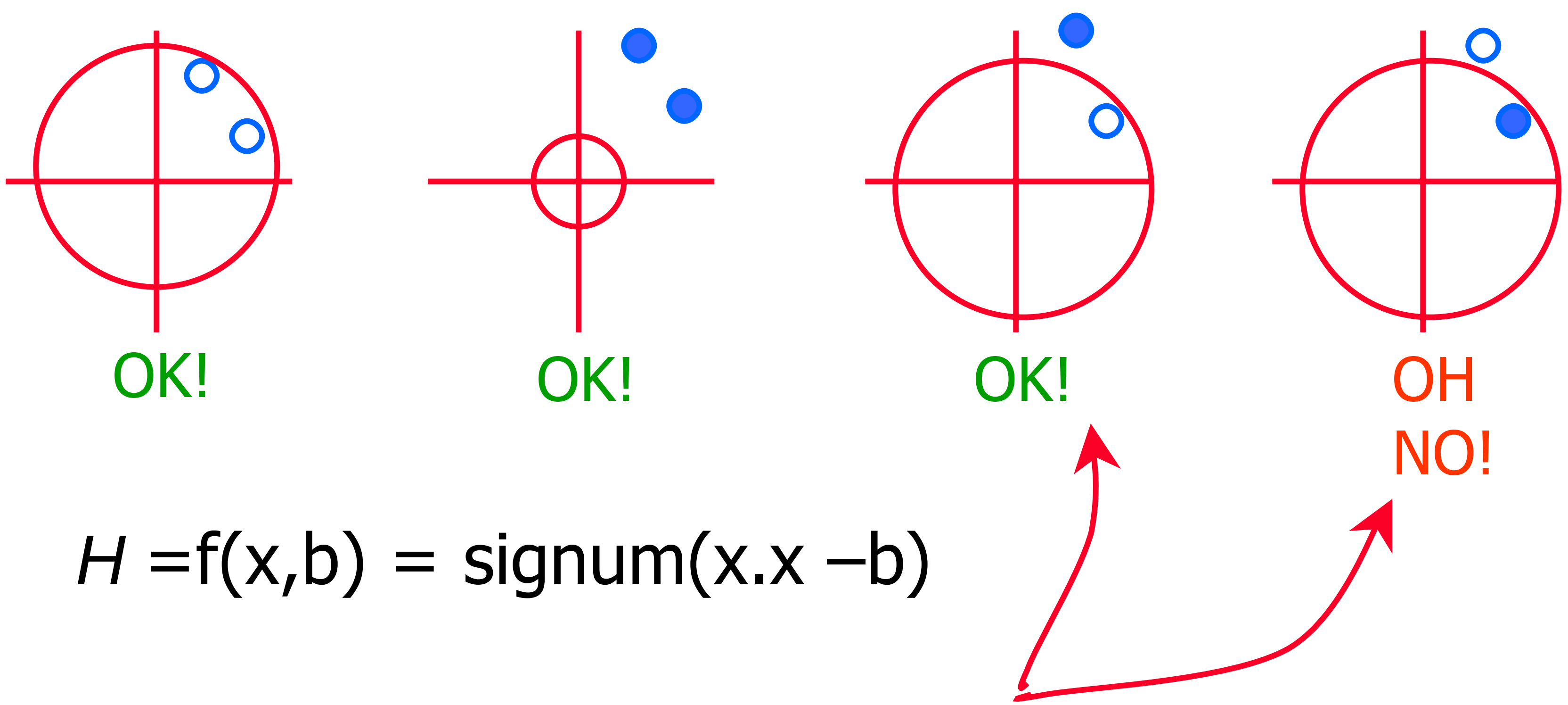
H =f(x,b) = sign(x.x −b)



Can this f shatter the following points?



Is this *H* powerful enough to separate 2 points?



H =f(x,b) = signum(x.x −b)

Same circle can't yield <u>both</u> + and -

This *H* can separate one point…

VC dimension intuitions

- How many distinctions hypothesis can exhibit
- # of <u>effective</u> degrees of freedom
- Maximum # of points for which *H* is unbiased

Main VC result & learning

- If H has VC-dimension d, then $m(\varepsilon, \delta)$, the # of samples required to guarantee learning within ε of the target language, $1-\delta$ of the time, is greater than:

$$\log(2)\left(\frac{d}{4}\log(\frac{3}{2})+\log(\frac{1}{8\delta})\right)$$

This implies

- Finite VC dimension of *H* is necessary for (potential) learnability!
- This is true no matter what the distribution is
- This is true no matter what the learning algorithm is
- This is true even for positive and negative examples

Applying VC dimension to language learning

- For *H* (or *L*) to be learnable, it must have <u>finite</u> VC dimension

- So what about some familiar classes?

- Let's start with the class of all <u>finite</u> languages (each L generates only sentences less than a certain length)

VC dimension of finite languages

- is infinite! So the family of finite languages is not learnable (in (ε,δ) or PAC learning terms)!
- Why? the set of finite languages is infinite - the # of states can grow larger and larger as we grow the fsa's for them
- It is the # of states that distinguish between different equivalence classes of symbols
- This ability to partition can grow without bound – so, for every set of d points one can partition – shatter – there's another of size $d+1$ one can also shatter – just add one more state

Gulp!

- If class of all finite languages is not PAC learnable, then neither are:
- fsa’s, cfg’s,…- pick your favorite general set of languages
- What’s a mother to do?

- Well: posit *a priori* restrictions – or make the class *H* finite in some way

FSAs with *n* states

- DO have finite VC dimension…
- So, as before, they are learnable
- More precisely: their VC dimension is
  $O(n \log n)$, n = # states

Lower bound for learning

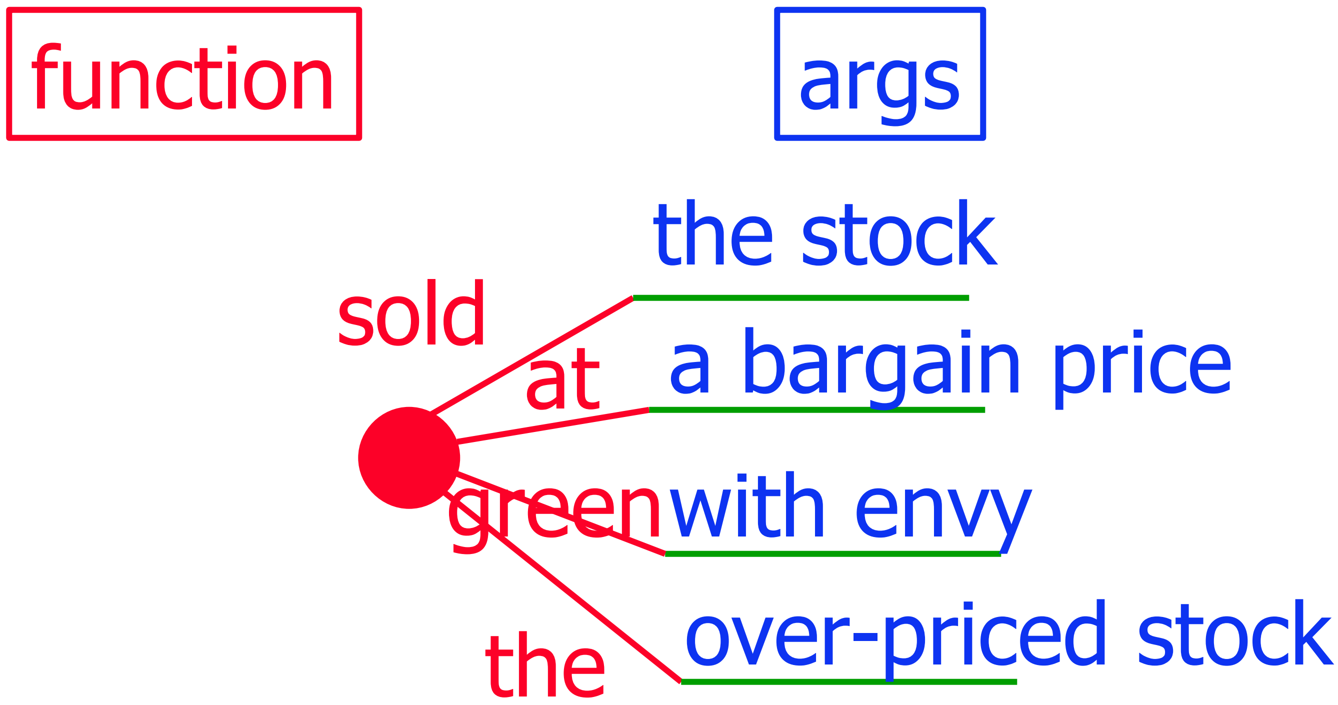
- If H has VC-dimension d then $m(\varepsilon, \delta)$, the # of samples required to guarantee learning within ε of the target language, 1-δ of the time, is at least:

$$m(\mathrm{e},\mathrm{d}) > \log(2)\left(\frac{d}{4}\log(\frac{3}{2}) + \log(\frac{1}{8\delta})\right)$$

OK, smarty: what can we do?

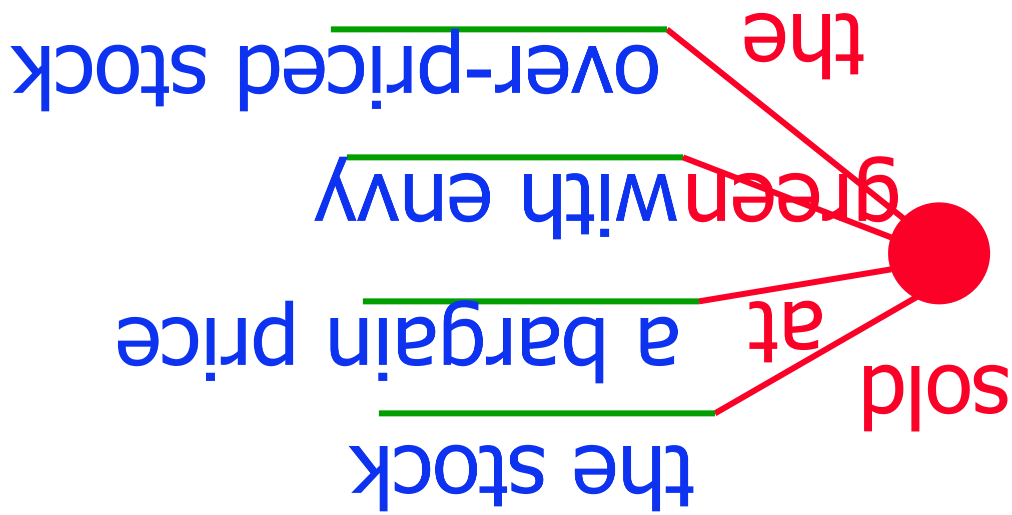
- Make the hypothesis space finite, small, and ‘easily separable’
- One solution: parameterize set of possible grammars (languages) according to a small set of parameters
- We’ve seen the head-first/final parameter

English is function-argument form

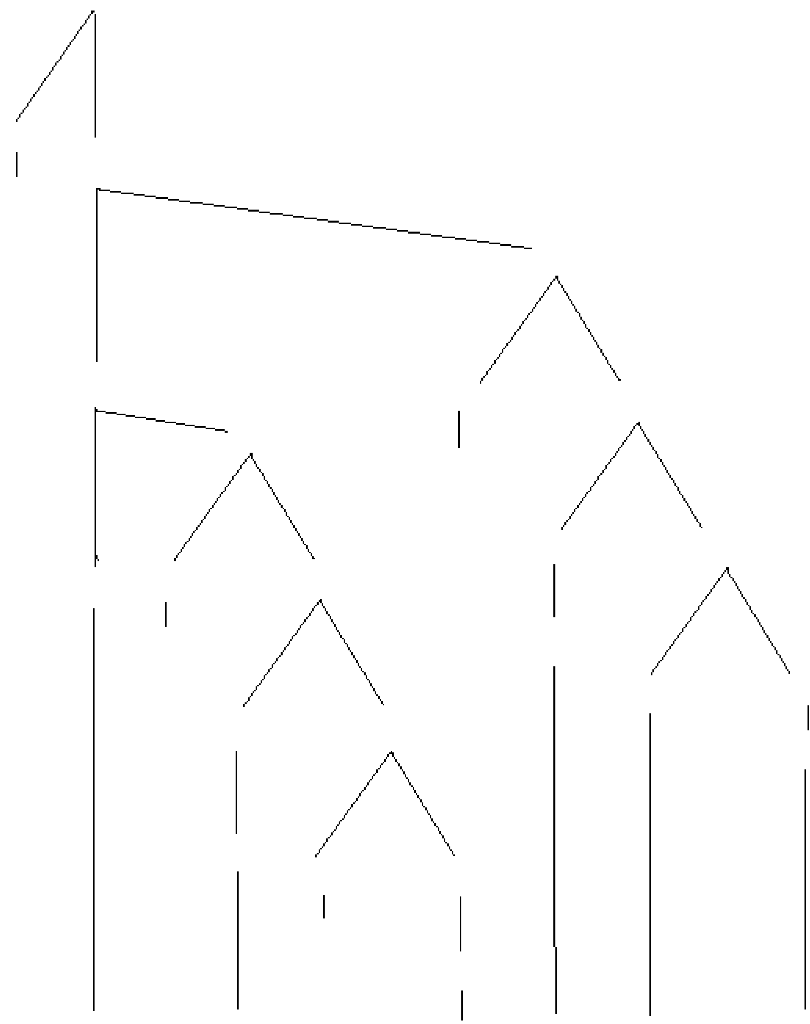


Other languages are the mirror-inverse: arg-function

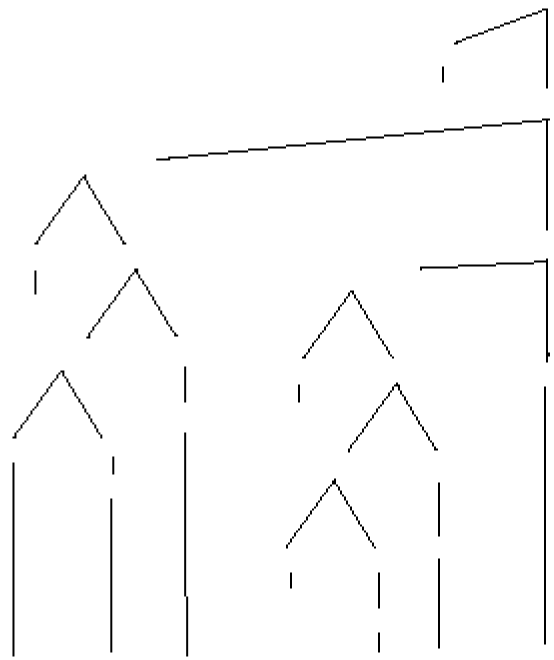
This is like Japanese



English form

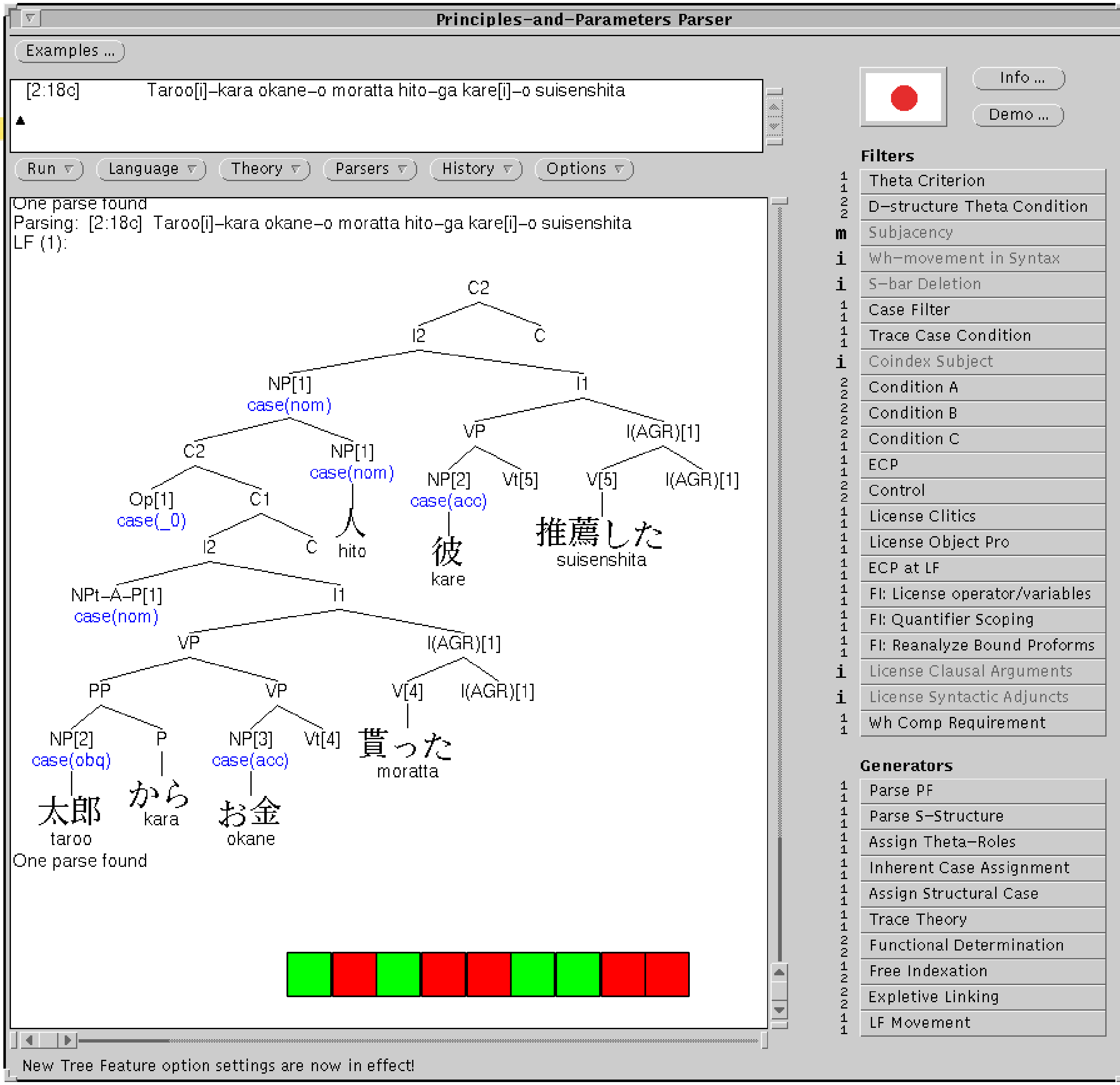


Bengali, German, Japanese form



Variational space of languages

Language	S-H	C-H	C	Ng	Va	Vt	SR	Scr	NP Scr	Op Scr	LD Sc	V2	Wh	Pro
Arabic	green	green	green	red	green	green	red	green	green	red	red	red	green	green
Dutch	green	red	green	red	green	green	green	green	green	green	red	green	green	red
English	green	green	green	green	red	red	green	red	red	red	red	red	green	red
French	green	green	green	green	green	green	green	red	red	red	red	red	green	red
German	green	red	green	red	green	green	green	green	green	green	green	green	green	red
Hindi	green	red	green	green	green	green	green	green	green	green	green	red	green	red
Icelandic	green	green	green	green	green	green	green	green	green	green	green	green	green	red
Irish	green	green	green	green	green	green	red	red	red	red	red	red	green	red
Italian	green	green	green	green	green	green	green	red	red	red	red	red	green	green
Japanese	green	red	red	green	green	green	green	green	green	red	green	red	red	green
Malay	green	green	green	green	green	green	green	red	red	red	red	red	green	green
Mandarin	green	green	red	green	green	green	green	red	red	red	red	red	red	green
Swedish	green	green	green	green	green	green	green	green	red	green	red	green	green	red
Tamil	green	red	green	green	green	green	green	green	green	red	green	red	green	green


Principles–and–Parameters Parser
Examples ...
[2:18c] Taroo[i]–kara okane–o moratta hito–ga kare[i]–o suisenshita
Info ...
Demo ...
Run
Language
Theory
Parsers
History
Options
One parse found
Parsing: [2:18c] Taroo[i]–kara okane–o moratta hito–ga kare[i]–o suisenshita
LF (1):
C2
I2
C
NP[1]
case(nom)
I1
C2
NP[1]
case(nom)
VP
I(AGR)[1]
Op[1]
case(_0)
C1
人
hito
NP[2]
case(acc)
Vt[5]
V[5]
I(AGR)[1]
I2
C
彼
kare
推薦した
suisenshita
NPt–A–P[1]
case(nom)
I1
VP
I(AGR)[1]
PP
VP
V[4]
I(AGR)[1]
NP[2]
case(obq)
P
NP[3]
case(acc)
Vt[4]
貰った
moratta
太郎
taroo
から
kara
お金
okane
One parse found
Filters
Theta Criterion
D–structure Theta Condition
Subjacency
Wh–movement in Syntax
S–bar Deletion
Case Filter
Trace Case Condition
Coindex Subject
Condition A
Condition B
Condition C
ECP
Control
License Clitics
License Object Pro
ECP at LF
FI: License operator/variables
FI: Quantifier Scoping
FI: Reanalyze Bound Proforms
License Clausal Arguments
License Syntactic Adjuncts
Wh Comp Requirement
Generators
Parse PF
Parse S–Structure
Assign Theta–Roles
Inherent Case Assignment
Assign Structural Case
Trace Theory
Functional Determination
Free Indexation
Expletive Linking
LF Movement
New Tree Feature option settings are now in effect!

Actual (prolog) code for this diff

```
% parametersEng.pl
%% X-Bar Parameters
specInitial.
specFinal :- \+ specInitial.

headInitial(_).
headFinal(X) :- \+ headInitial(X).

agr(weak).

%% V2 Parameters
% Q is available as adjunction site
boundingNode(i2).
boundingNode(np).

%% Case Adjacency Parameter
CaseAdjacency. % holds

%% Wh In Syntax Parameter
whInSyntax.

%% Pro-Drop Parameter
no proDrop.
```

```
%% X-Bar Parameters
specInitial.
specFinal :- \+ specInitial.

headFinal.
headInitial :- \+ headFinal.
headInitial(X) :- \+ headFinal(X).
headFinal(_) :- headFinal.

agr(strong).
%% V2 Parameters
%% Subjacency Bounding Nodes
boundingNode(i2).
boundingNode(np).

%% Case Adjacency Parameter
no caseAdjacency.

%% Wh In Syntax Parameter
no whInSyntax.

%% Pro-Drop
proDrop.
```

Learning in parameter space

- Greedy algorithm: start with some randomized parameter settings

1. Get example sentence, s
2. If s is parsable (analyzable) by current parameter settings, keep current settings; o.w.,
3. Randomly flip a parameter setting & go to Step 1.

More details

- 1-bit different example that moves us from one setting to the next is called a <u>trigger</u>
- Let's do a simple model – 3 parameters only, so 8 possible languages

Tis a gift to be simple…

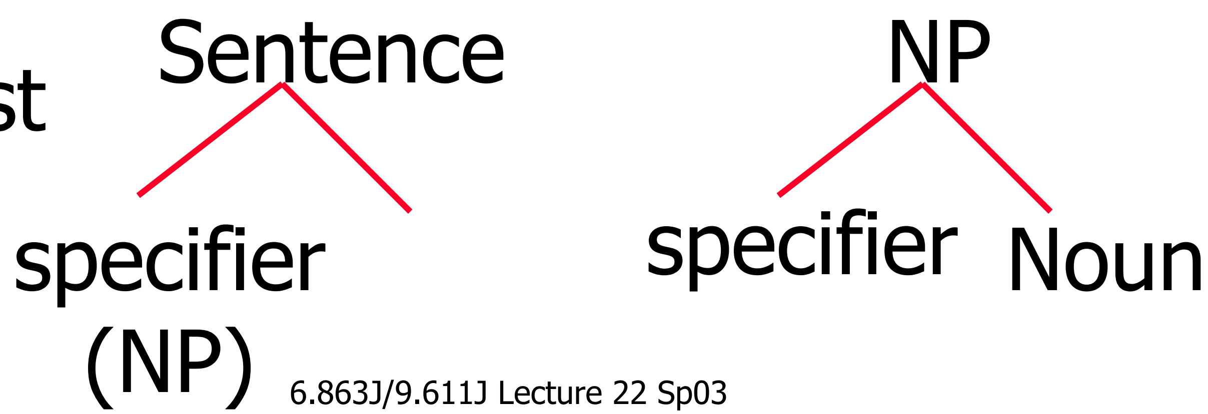
- Just 3 parameters, so 8 possible languages (grammars) – set 0 or 1

- Complement first/final (dual of Head 1^{st})
  - English: Complement final (value = 1)
- Specifier first/final (determiner on right or left, Subject on right or left)
- Verb second or not (German/not German)

3-parameter case

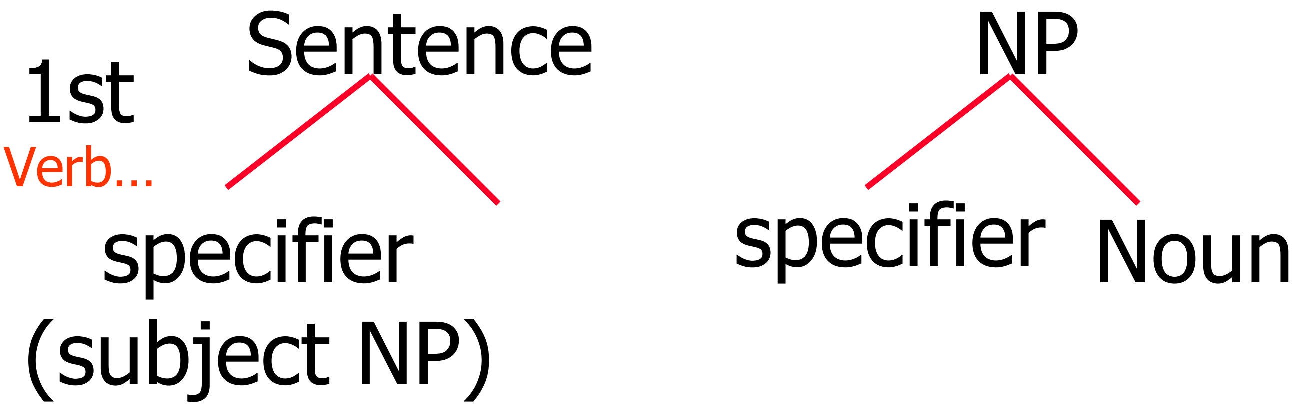
1. Specifier first or final
2. Complement (Arguments) first/final
3. Verb 2nd or not

Spec 1st



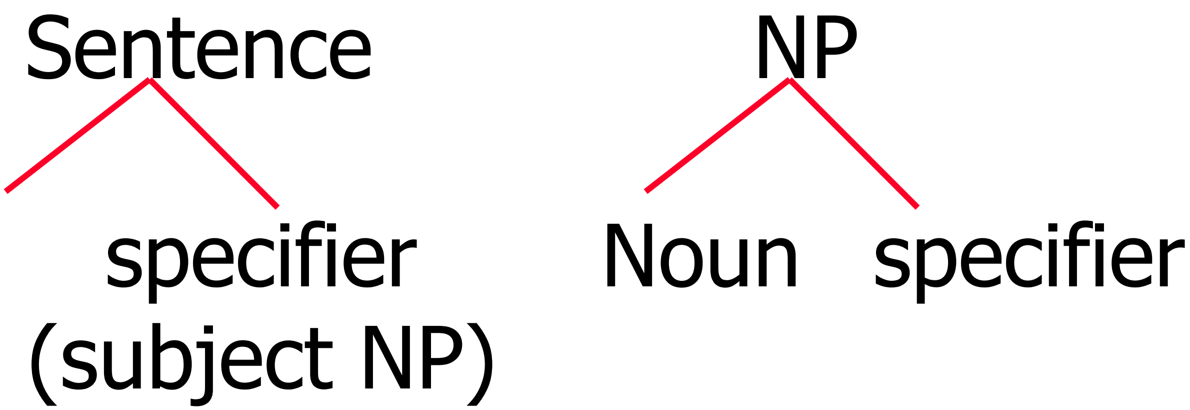
Parameters

Spec 1st
Subject Verb…



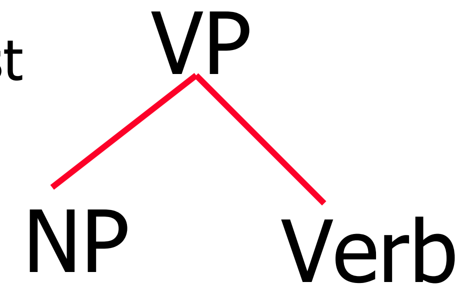
Spec final

Verb Subject…



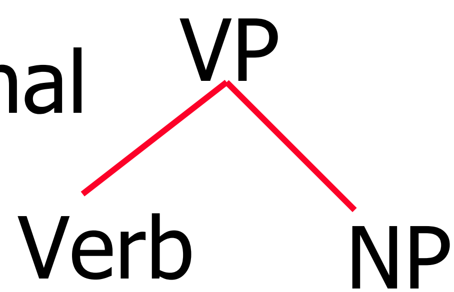
Comp(lement) Parameter

Comp 1st



…Object Verb

Comp final

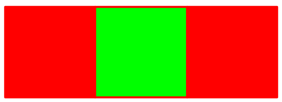


Verb Object…

Verb second (V2)

- Finite (tensed) verb must appear in exactly 2nd position in main sentence

English / German



[0 1 0] = 'English'

spec 1st/final comp 1st/final –V2/+V2

[0 0 1] = 'German'

Even this case can be hard…

- German: dass Karl da Buch kauft

  (that Karl the book buys)

  Karl kauft das Buch
- OK, what are the parameter settings?
- Is German comp-1st ? (as the first example suggests) or comp-last?
- Ans: V2 parameter – in main sentence, this moves verb kauft to 2nd position

Input data – 3 parameter case

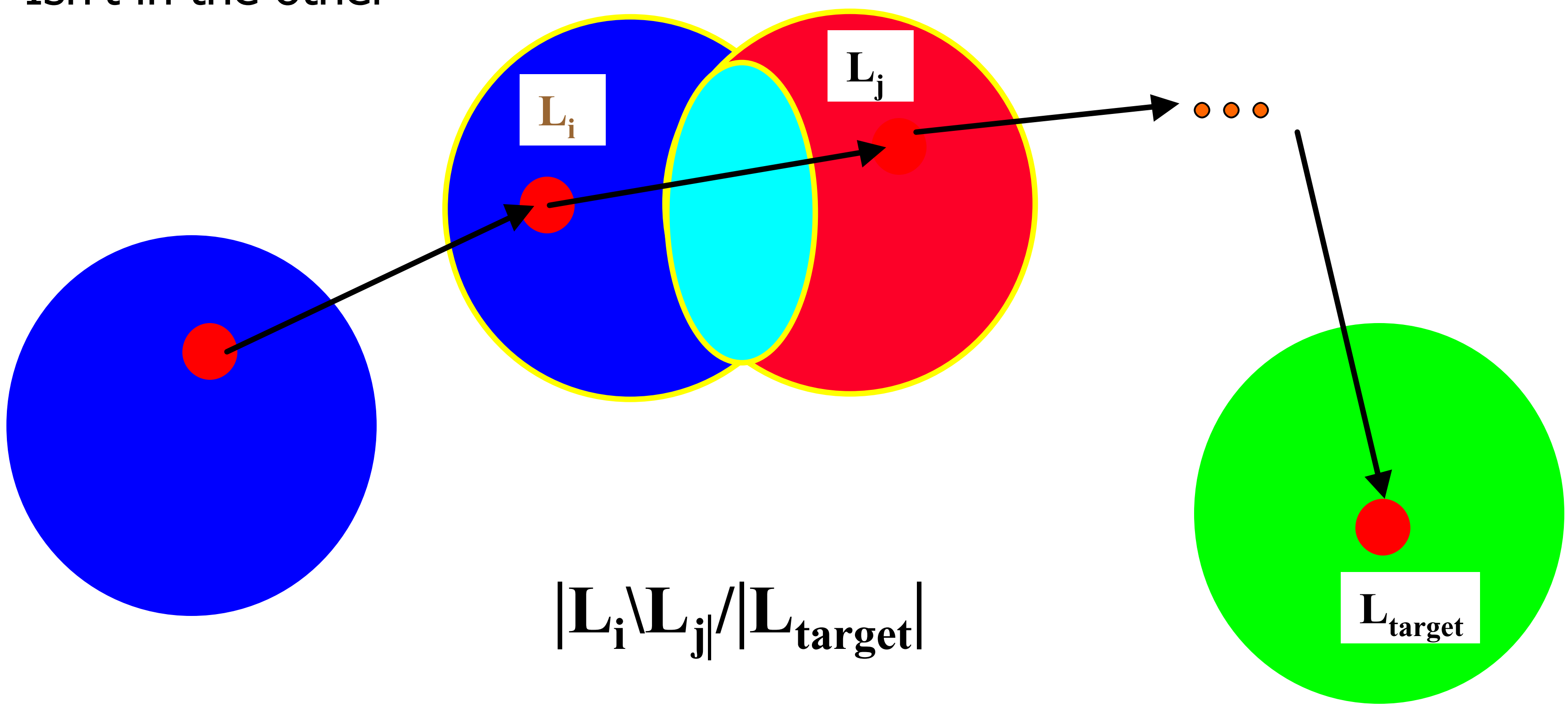
- Labels: S, V, Aux, O, O1, O2
- All unembedded sentences (psychological fidelity)
- Possible English sentences:
  - S V, S V O1 O2; S Aux V O; S Aux V O1 O2; Adv S V; Adv S V O; Adv S V O1 O2; Adv S Aux V; Adv S Aux V O; Adv S Aux V O1 O2
- Too simple, of course: collapses many languages together…
- Like English and French…oops!

Sentences drawn from target

- Uniformly
- From possible target patterns
- Learner starts in random initial state, 1,…8
- What drives learner?
- Errors

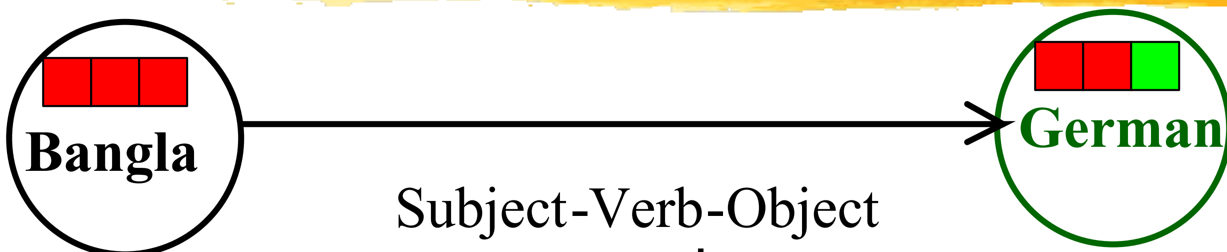
Learning driven by language triggering set differences

A trigger is a sentence in one language that
Isn't in the other



$$|L_i \backslash L_{j|}/|L_{target}|$$

How to get there from here



Spec-first,
Comp first,
–Verb 2nd
[0 0 0]

Hans boi-ta kene
Hans the book buys

Subject-Verb-Object
example
sentence *w*

Hans kauft das Buch

Spec-first,
Comp-first
+Verb 2nd
[0 0 1]

Hans kauft das Buch
Hans buys the book

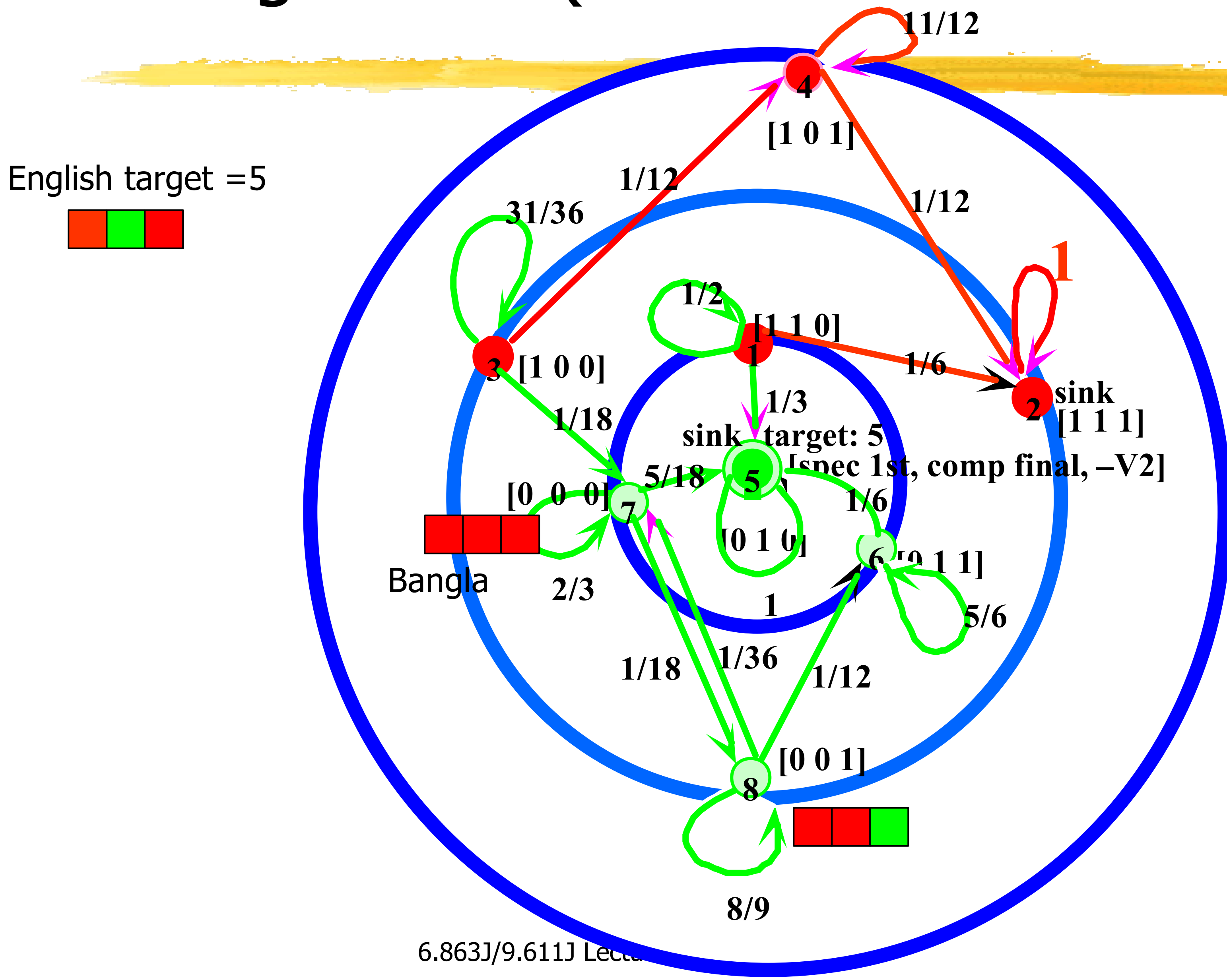
- transitions based on example sentence

  Prob(transition) based on set differences between languages, normalized by target language $|L_{target}|$ examples (in our case, if t=English,36 of them)

Formalize this as…

- A Markov chain relative to a target language, as matrix M, where M(i,j) gives the transition pr of moving from state i to state j (given target language strings)
- Transition pr's based on cardinality of the set differences
- M x M = pr's after 1 example step; in the limit, we find M^{∞}
- Here is M when target is L_5 = 'English'

The Ringstrasse (Pax Americana version)



Markov matrix, target = 5 (English)

$$
\textbf{From}\quad
\begin{array}{c}
\\ L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8
\end{array}
\begin{array}{c}
\textbf{To} \\
\begin{array}{cccccccc}
L_1 & L_2 & L_3 & L_4 & L_5 & L_6 & L_7 & L_8
\end{array} \\
\left[\begin{array}{cccccccc}
\frac{1}{2} & \frac{1}{6} & & & \frac{1}{3} & & & \\
 & 1 & & & & & & \\
 & & \frac{3}{4} & \frac{1}{12} & & & \frac{1}{6} & \\
 & \frac{1}{12} & & \frac{11}{12} & & & & \\
 & & & & 1 & & & \\
 & & & & \frac{1}{6} & \frac{5}{6} & & \\
 & & & & \frac{5}{18} & & \frac{2}{3} & \frac{1}{18} \\
 & & & & & \frac{1}{12} & \frac{1}{36} & \frac{8}{9}
\end{array}\right]
\end{array}
$$