6.864: Lecture 21 (November 29th, 2005) Global Linear Models: Part II

Overview

- Log-linear models for parameter estimation
- Global and local features
 - The perceptron revisited
 - Log-linear models revisited

Three Components of Global Linear Models

- Φ is a function that maps a structure (x, y) to a **feature vector** $\Phi(x, y) \in \mathbb{R}^d$
- **GEN** is a function that maps an input *x* to a set of **candidates GEN**(*x*)
- W is a parameter vector (also a member of \mathbb{R}^d)
- $\bullet\,$ Training data is used to set the value of ${\bf W}$

Putting it all Together

- \mathcal{X} is set of sentences, \mathcal{Y} is set of possible outputs (e.g. trees)
- Need to learn a function $F : \mathcal{X} \to \mathcal{Y}$
- **GEN**, Φ , W define

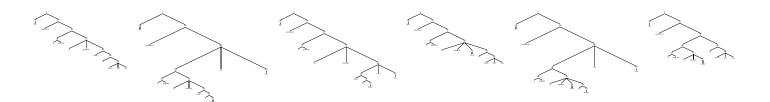
$$F(x) = \underset{y \in \mathbf{GEN}(x)}{\operatorname{arg max}} \Phi(x, y) \cdot \mathbf{W}$$

Choose the highest scoring candidate as the most plausible structure

• Given examples (x_i, y_i) , how to set W?

She announced a program to promote safety in trucks and vans

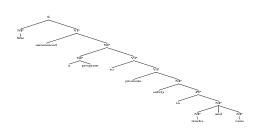
 \Downarrow **GEN**



 $\downarrow \Phi \qquad \downarrow \Phi$

 $\langle 1,1,3,5\rangle \qquad \langle 2,0,0,5\rangle \qquad \langle 1,0,1,5\rangle \qquad \langle 0,0,3,0\rangle \qquad \langle 0,1,0,5\rangle \qquad \langle 0,0,1,5\rangle$

 \Downarrow arg max



A Variant of the Perceptron Algorithm

Inputs:	Training set (x_i, y_i) for $i = 1 \dots n$
Initialization:	$\mathbf{W} = 0$
Define:	$F(x) = \operatorname{argmax}_{y \in \mathbf{GEN}(x)} \Phi(x, y) \cdot \mathbf{W}$
Algorithm:	For $t = 1 \dots T$, $i = 1 \dots n$ $z_i = F(x_i)$ If $(z_i \neq y_i)$ $\mathbf{W} = \mathbf{W} + \mathbf{\Phi}(x_i, y_i) - \mathbf{\Phi}(x_i, z_i)$
Output:	Parameters W

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Back to Maximum Likelihood Estimation [Johnson et. al 1999]

• We can use the parameters to define a probability for each parse:

$$P(y \mid x, \mathbf{W}) = \frac{e^{\mathbf{\Phi}(x, y) \cdot \mathbf{W}}}{\sum_{y' \in \mathbf{GEN}(x)} e^{\mathbf{\Phi}(x, y') \cdot \mathbf{W}}}$$

• Log-likelihood is then

$$L(\mathbf{W}) = \sum_{i} \log P(y_i \mid x_i, \mathbf{W})$$

• A first estimation method: take maximum likelihood estimates, i.e.,

$$\mathbf{W}_{ML} = \operatorname{argmax}_{\mathbf{W}} L(\mathbf{W})$$



- A first estimation method: take maximum likelihood estimates, i.e., $\mathbf{W}_{ML} = \operatorname{argmax}_{\mathbf{W}} L(\mathbf{W})$
- Unfortunately, very likely to "overfit": could use feature selection methods, as in boosting
- Another way of preventing overfitting: choose parameters as

$$\mathbf{W}_{MAP} = \operatorname{argmax}_{\mathbf{W}} \left(L(\mathbf{W}) - C \sum_{k} \mathbf{W}_{k}^{2} \right)$$

for some constant C

• Intuition: adds a penalty for large parameter values

The Bayesian Justification for Gaussian Priors

• In *Bayesian* methods, combine the log-likelihood $P(data \mid \mathbf{W})$ with a prior over parameters, $P(\mathbf{W})$

$$P(\mathbf{W} \mid data) = \frac{P(data \mid \mathbf{W})P(\mathbf{W})}{\int_{\mathbf{W}} P(data \mid \mathbf{W})P(\mathbf{W})d\mathbf{W}}$$

• The MAP (Maximum A-Posteriori) estimates are

$$\mathbf{W}_{MAP} = \operatorname{argmax}_{\mathbf{W}} P(\mathbf{W} \mid data)$$
$$= \operatorname{argmax}_{\mathbf{W}} \left(\underbrace{\log P(data \mid \mathbf{W})}_{\text{Log-Likelihood}} + \underbrace{\log P(\mathbf{W})}_{\text{Prior}} \right)$$

• Gaussian prior: $P(\mathbf{W}) \propto e^{-C \sum_{k} \mathbf{W}_{k}^{2}}$ $\Rightarrow \log P(\mathbf{W}) = -C \sum_{k} \mathbf{W}_{k}^{2} + C_{2}$

Summary

Choose parameters as:

$$\mathbf{W}_{MAP} = \operatorname{argmax}_{\mathbf{W}} \left(L(\mathbf{W}) - C \sum_{k} \mathbf{W}_{k}^{2} \right)$$

where

$$L(\mathbf{W}) = \sum_{i} \log P(y_i \mid x_i, \mathbf{W})$$
$$= \sum_{i} \log \frac{e^{\Phi(x_i, y_i) \cdot \mathbf{W}}}{\sum_{y' \in \mathbf{GEN}(x_i)} e^{\Phi(x_i, y') \cdot \mathbf{W}}}$$

Can use (conjugate) gradient ascent (see previous lectures on log-linear models)

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Global and Local Features

- So far: algorithms have depended on size of **GEN**
- Strategies for keeping the size of **GEN** manageable:
 - Reranking methods: use a baseline model to generate its top N analyses

Global and Local Features

- Global linear models are "global" in a couple of ways:
 - Feature vectors are defined over entire structures
 - Parameter estimation methods explicitly related to errors on entire structures
- Next topic: **global** training methods with **local features**
 - Our 'global' features will be defined through *local* features
 - Parameter estimates will be global
 - **GEN** will be large!
 - Dynamic programming used for search and parameter estimation: this is possible for some combinations of GEN and Φ

Tagging Problems

TAGGING: Strings to Tagged Sequences

a b e e a f h j \Rightarrow a/C b/D e/C e/C a/D f/C h/D j/C

Example 1: Part-of-speech tagging

Profi ts/N soared/V at/P Boeing/N Co./N ,/, easily/ADV topping/V forecasts/N on/P Wall/N Street/N ,/, as/P their/POSS CEO/N Alan/N Mulally/N announced/V fi rst/ADJ quarter/N results/N ./.

Example 2: Named Entity Recognition

Profits/NA soared/NA at/NA Boeing/SC Co./CC ,/NA easily/NA topping/NA forecasts/NA on/NA Wall/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP Mulally/CP announced/NA fi rst/NA quarter/NA results/NA ./NA

Tagging

Going back to tagging:

- Inputs x are sentences $w_{[1:n]} = \{w_1 \dots w_n\}$
- **GEN** $(w_{[1:n]}) = T^n$ i.e. all tag sequences of length n
- Note: **GEN** has an exponential number of members
- How do we define Φ ?

Representation: Histories

- A history is a 4-tuple $\langle t_{-1}, t_{-2}, w_{[1:n]}, i \rangle$
- t_{-1}, t_{-2} are the previous two tags.
- $w_{[1:n]}$ are the *n* words in the input sentence.
- *i* is the index of the word being tagged

Hispaniola/NNP quickly/RB became/VB an/DT important/JJ base/?? from which Spain expanded its empire into the rest of the Western Hemisphere .

- $t_{-1}, t_{-2} = DT, JJ$
- $w_{[1:n]} = \langle Hispaniola, quickly, became, \dots, Hemisphere, . \rangle$
- *i* = 6

Local Feature-Vector Representations

- Take a history/tag pair (h, t).
- φ_s(h, t) for s = 1...d are local features representing tagging decision t in context h.

Example: POS Tagging

- Word/tag features
- $\phi_{100}(h,t) = \begin{cases} 1 & \text{if current word } w_i \text{ is base and } t = \text{VB} \\ 0 & \text{otherwise} \end{cases}$ $\phi_{101}(h,t) = \begin{cases} 1 & \text{if current word } w_i \text{ ends in ing and } t = \text{VBG} \\ 0 & \text{otherwise} \end{cases}$
- Contextual Features

$$\phi_{103}(h,t) = \begin{cases} 1 & \text{if } \langle t_{-2}, t_{-1}, t \rangle = \langle \text{DT, JJ, VB} \rangle \\ 0 & \text{otherwise} \end{cases}$$

A tagged sentence with n words has n history/tag pairs

Hispaniola/NNP quickly/RB became/VB an/DT important/JJ base/NN

History		Tag		
t_{-2}	t_{-1}	$w_{[1:n]}$	i	t
*	*	$\langle Hispaniola, quickly, \dots, \rangle$	1	NNP
*	NNP	$\langle Hispaniola, quickly, \ldots, \rangle$	2	RB
NNP	RB	$\langle Hispaniola, quickly, \ldots, \rangle$	3	VB
RB	VB	$\langle Hispaniola, quickly, \ldots, \rangle$	4	DT
VP	DT	$\langle Hispaniola, quickly, \ldots, \rangle$	5	JJ
DT	JJ	$\langle Hispaniola, quickly, \dots, \rangle$	6	NN

A tagged sentence with n words has n history/tag pairs

Hispaniola/NNP quickly/RB became/VB an/DT important/JJ base/NN

History		Tag		
t_{-2}	t_{-1}	$w_{[1:n]}$	i	t
*	*	$\langle Hispaniola, quickly, \dots, \rangle$	1	NNP
*	NNP	$\langle Hispaniola, quickly, \ldots, \rangle$	2	RB
NNP	RB	$\langle Hispaniola, quickly, \ldots, \rangle$	3	VB
RB	VB	$\langle Hispaniola, quickly, \ldots, \rangle$	4	DT
VP	DT	$\langle Hispaniola, quickly, \ldots, \rangle$	5	JJ
DT	JJ	$\langle Hispaniola, quickly, \dots, \rangle$	6	NN

Define global features through local features:

$$\Phi(t_{[1:n]}, w_{[1:n]}) = \sum_{i=1}^{n} \phi(h_i, t_i)$$

where t_i is the *i*'th tag, h_i is the *i*'th history

Global and Local Features

• Typically, local features are indicator functions, e.g.,

$$\phi_{101}(h,t) = \begin{cases} 1 & \text{if current word } w_i \text{ ends in ing and } t = \text{VBG} \\ 0 & \text{otherwise} \end{cases}$$

• and global features are then counts,

 $\Phi_{101}(w_{[1:n]}, t_{[1:n]}) =$ Number of times a word ending in ing is tagged as VBG in $(w_{[1:n]}, t_{[1:n]})$

Putting it all Together

- **GEN** $(w_{[1:n]})$ is the set of all tagged sequences of length n
- **GEN**, Φ , W define

$$F(w_{[1:n]}) = \arg \max_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \mathbf{W} \cdot \mathbf{\Phi}(w_{[1:n]}, t_{[1:n]})$$
$$= \arg \max_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \mathbf{W} \cdot \sum_{i=1}^{n} \phi(h_i, t_i)$$
$$= \arg \max_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \sum_{i=1}^{n} \mathbf{W} \cdot \phi(h_i, t_i)$$

- Some notes:
 - Score for a tagged sequence is a sum of local scores
 - Dynamic programming can be used to find the argmax! (because history only considers the previous two tags)

A Variant of the Perceptron Algorithm

Inputs:	Training set (x_i, y_i) for $i = 1 \dots n$
Initialization:	$\mathbf{W} = 0$
Define:	$F(x) = \operatorname{argmax}_{y \in \mathbf{GEN}(x)} \Phi(x, y) \cdot \mathbf{W}$
Algorithm:	For $t = 1 \dots T$, $i = 1 \dots n$ $z_i = F(x_i)$ If $(z_i \neq y_i)$ $\mathbf{W} = \mathbf{W} + \mathbf{\Phi}(x_i, y_i) - \mathbf{\Phi}(x_i, z_i)$
Output:	Parameters W

Training a Tagger Using the Perceptron Algorithm

Inputs: Training set $(w_{[1:n_i]}^i, t_{[1:n_i]}^i)$ for $i = 1 \dots n$. **Initialization:** $\mathbf{W} = 0$

Algorithm: For $t = 1 \dots T, i = 1 \dots n$

$$z_{[1:n_i]} = \arg \max_{u_{[1:n_i]} \in \mathcal{T}^{n_i}} \mathbf{W} \cdot \mathbf{\Phi}(w_{[1:n_i]}^i, u_{[1:n_i]})$$

 $z_{[1:n_i]}$ can be computed with the dynamic programming (Viterbi) algorithm

If $z_{[1:n_i]} \neq t^i_{[1:n_i]}$ then $\mathbf{W} = \mathbf{W} + \Phi(w^i_{[1:n_i]}, t^i_{[1:n_i]}) - \Phi(w^i_{[1:n_i]}, z_{[1:n_i]})$ Output: Parameter vector \mathbf{W} .

An Example

Say the correct tags for i'th sentence are

the/DT man/NN bit/VBD the/DT dog/NN

Under current parameters, output is

the/DT man/NN bit/NN the/DT dog/NN

Assume also that features track: (1) all bigrams; (2) word/tag pairs Parameters incremented:

```
\langle NN, VBD \rangle, \langle VBD, DT \rangle, \langle VBD \rightarrow bit \rangle
```

Parameters decremented:

 $\langle NN, NN \rangle, \langle NN, DT \rangle, \langle NN \rightarrow bit \rangle$

Experiments

• Wall Street Journal part-of-speech tagging data

Perceptron = 2.89%, Max-ent = 3.28% (11.9% relative error reduction)

• [Ramshaw and Marcus, 1995] NP chunking data

Perceptron = 93.63%, Max-ent = 93.29% (5.1% relative error reduction)

How Does this Differ from Log-Linear Taggers?

- Log-linear taggers (in an earlier lecture) used very similar *local representations*
- How does the perceptron model differ?
- Why might these differences be important?

Log-Linear Tagging Models

- Take a history/tag pair (h, t).
- $\phi_s(h,t)$ for $s = 1 \dots d$ are features \mathbf{W}_s for $s = 1 \dots d$ are parameters
- Conditional distribution:

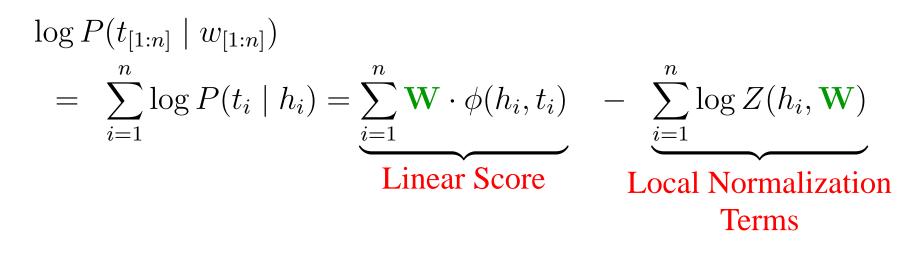
$$P(t|h) = \frac{e^{\mathbf{W} \cdot \phi(h,t)}}{Z(h,\mathbf{W})}$$

where $Z(h, \mathbf{W}) = \sum_{t' \in \mathcal{T}} e^{\mathbf{W} \cdot \phi(h, t')}$

• Parameters estimated using maximum-likelihood e.g., iterative scaling, gradient descent

Log-Linear Tagging Models

- Word sequence $w_{[1:n]} = [w_1, w_2 \dots w_n]$
- Tag sequence $t_{[1:n]}$ $= [t_1, t_2 \dots t_n]$ Histories h_i $= \langle t_{i-1}, t_{i-2}, w_{[1:n]}, i \rangle$



• Compare this to the perceptron, where GEN, Φ , W define

$$F(w_{[1:n]}) = \arg_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \underbrace{\sum_{i=1}^{n} \mathbf{W} \cdot \phi(h_i, t_i)}_{\text{Linear score}}$$

Problems with Locally Normalized models

- "Label bias" problem [Lafferty, McCallum and Pereira 2001] See also [Klein and Manning 2002]
- Example of a conditional distribution that locally normalized models can't capture (under bigram tag representation):

$$a b c \Rightarrow \begin{vmatrix} A & - B & - C \\ A & b & c \end{vmatrix} \text{ with } P(A B C \mid a b c) = 1$$
$$a b e \Rightarrow \begin{vmatrix} A & - D & - E \\ A & b & e \end{vmatrix} \text{ with } P(A D E \mid a b e) = 1$$

• Impossible to find parameters that satisfy

$$P(A \mid a) \times P(B \mid b, A) \times P(C \mid c, B) = 1$$
$$P(A \mid a) \times P(D \mid b, A) \times P(E \mid e, D) = 1$$

Overview

- Recap: global linear models, and boosting
- Log-linear models for parameter estimation
- An application: LFG parsing
- Global and local features
 - The perceptron revisited
 - Log-linear models revisited

Global Log-Linear Models

• We can use the parameters to define a probability for each tagged sequence:

$$P(t_{[1:n]} \mid w_{[1:n]}, \mathbf{W}) = \frac{e^{\sum_{i} \mathbf{W} \cdot \phi(h_{i}, t_{i})}}{Z(w_{[1:n]}, \mathbf{W})}$$

where

$$Z(w_{[1:n]}, \mathbf{W}) = \sum_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} e^{\sum_{i} \mathbf{W} \cdot \phi(h_i, t_i)}$$

is a **global** normalization term

• This is a global log-linear model with

$$\Phi(w_{[1:n]}, t_{[1:n]}) = \sum_{i} \phi(h_i, t_i)$$

Now we have:

$$\log P(t_{[1:n]} \mid w_{[1:n]})$$

$$= \underbrace{\sum_{i=1}^{n} \mathbf{W} \cdot \phi(h_i, t_i)}_{\text{Linear Score}} - \underbrace{\log Z(w_{[1:n]}, \mathbf{W})}_{\text{Global Normalization}}$$

When finding highest probability tag sequence, the global term is irrelevant:

$$\operatorname{argmax}_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \sum_{i=1}^{n} \left(\mathbf{W} \cdot \phi(h_i, t_i) - \log Z(w_{[1:n]}, \mathbf{W}) \right)$$
$$= \operatorname{argmax}_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \sum_{i=1}^{n} \mathbf{W} \cdot \phi(h_i, t_i)$$

Parameter Estimation

• For parameter estimation, we must calculate the gradient of

$$\log P(t_{[1:n]} \mid w_{[1:n]}) = \sum_{i=1}^{n} \mathbf{W} \cdot \phi(h_i, t_i) - \log \sum_{\substack{t'_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})}} e^{\sum_i \mathbf{W} \cdot \phi(h'_i, t'_i)}$$

with respect to \mathbf{W}

• Taking derivatives gives

$$\frac{dL}{d\mathbf{W}} = \sum_{i=1}^{n} \phi(h_i, t_i) - \sum_{\substack{t'_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})}} P(t'_{[1:n]} \mid w_{[1:n]}, \mathbf{W}) \sum_{i=1}^{n} \phi(h'_i, t'_i)$$

• Can be calculated using dynamic programming! (very similar to forward-backward algorithm for EM training)

Summary of Perceptron vs. Global Log-Linear Model

• Both are global linear models, where

 $GEN(w_{[1:n]}) = \text{the set of all possible tag sequences for } w_{[1:n]}$ $\Phi(w_{[1:n]}, t_{[1:n]}) = \sum_{i} \phi(h_i, t_i)$

• In both cases,

$$\begin{aligned} \boldsymbol{F}(w_{[1:n]}) &= \operatorname{argmax}_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \mathbf{W} \cdot \boldsymbol{\Phi}(w_{[1:n]}, t_{[1:n]}) \\ &= \operatorname{argmax}_{t_{[1:n]} \in \mathbf{GEN}(w_{[1:n]})} \sum_{i} \mathbf{W} \cdot \phi(h_{i}, t_{i}) \end{aligned}$$

can be computed using dynamic programming

- Dynamic programming is also used in training:
 - Perceptron requires highest-scoring tag sequence for each training example
 - Global log-linear model requires gradient, and therefore "expected counts"

Results

From [Sha and Pereira, 2003]

• Task = shallow parsing (base noun-phrase recognition)

Model	Accuracy
SVM combination	94.39%
Conditional random field	94.38%
(global log-linear model)	
Generalized winnow	93.89%
Perceptron	94.09%
Local log-linear model	93.70%

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