# Massachusetts Institute of Technology Department of Electrical Engineering and Computer Science 

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High Speed Communication Circuits and Systems Spring 2003

## Homework \#4: Narrowband LNA's and Mixers

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Reading: Chapters 11 and 12 of Thomas H. Lee's book. Chapter 6 and pages 138-146 of Behzad Razavi's book.

1. This problem will focus on design of the LNA shown in Figure 1. In the figure, $L_{\text {deg }}$ $\left(L_{s}\right)$ is assumed to have infinite $Q$, but inductors $L_{g}$ and $L_{d}$ have associated resistances due to their finite $Q$ (i.e., the values of $R_{p g}$ and $R_{p d}$ are set by the $Q$ and inductance value of $L_{g}$ and $L_{d}$, respectively). Assume the following

- Design of the LNA should be focused on its performance at frequency $w_{o}=$ $(2 \pi) 2.5 \mathrm{GHz}$
- If an external inductor is used for $L_{g}$, its Q is 40 . For integrated inductors, assume the Q to be 5 (other than for $L_{\text {deg }}$ ).
- The LNA input impedance, $Z_{i n}$, must be designed to match that of the source resistance of 50 Ohms at frequency $w_{o}$ (Note: assume $C_{\text {big }}$ is a short at frequency $w_{o}$, and ignore the influence of the 5 kOhm bias resistor)
- The power dissipation of the core amplifier must be no greater than 10 mW . (Ignore the bias circuitry here)
- All transistor devices are the same size and have length $\mathrm{L}=0.18 \mu \mathrm{~m}$
- The influence of the cascode device $M_{2}$ can be ignored for noise figure and gain calculations.
- Ignore $C_{g d}, C_{d b}, r_{o}$, and backgate effects for all transistors.
- Ignore the noise contribution of transistors $M_{2}$ and $M_{3}$ and the $5 \mathrm{k} \Omega$ bias resistor.
- Assume the following process parameters
$-c=\mathrm{j} 0.55$
$-\gamma=3$
$-\delta=6$
$-g_{m} / g_{d o}=0.68$
$-w_{t}=(2 \pi) 42.8 \mathrm{GHz}$

$$
\begin{aligned}
& -I_{d e n}=100 \mu \mathrm{~A} / \mu \mathrm{m} \\
& -g_{m}=\frac{W}{1.8} 0.78 \mathrm{mS}, \text { where } W \text { is transistor width in microns } \\
& -V_{d d}=1.8 \mathrm{~V}
\end{aligned}
$$



Figure 1: Narrowband LNA.
(a) Design the amplifier to achieve the minimum noise figure given the conditions stated above. State the resulting noise figure (at $w_{o}$ ), $I_{\text {bias }}$, transistor widths, component values, and gain (at $w_{o}$ ) for the amplifier. For intermediate calculations, determine $\eta, \chi_{d}$, and $Z_{g s w}$.
(b) How does the noise figure change for part (a) if $\mathrm{c}=0$ ? (a numerical answer is required here)
(c) How does the noise figure change for part (a) if $\mathrm{c}=\mathrm{j} 1$ ? (a numerical answer is required here)
2. The following is based on Problem 4 in Chapter 12 of Thomas Lee's book. Here we consider the ideal multiplying mixer shown in Figure 2, whose input RF signal is $A \sin \left(w_{r f} t\right)$ and whose LO signal is either a square-wave or sinusoidal waveform.


Figure 2: Ideal multiplying mixer.
(a) Derive the conversion gain of the mixer under the following conditions:
i. A square-wave LO waveform with peak-to-peak amplitude of $2 B$ whose frequency is chosen such that the mixer output occurs at a nonzero IF frequency.
ii. A sine-wave LO waveform with peak-to-peak amplitude of $2 B$ whose frequency is chosen such that the mixer output occurs at a nonzero IF frequency.
iii. A square-wave LO waveform with peak-to-peak amplitude of $2 B$ whose frequency is chosen such that the mixer output occurs at zero IF frequency.
iv. A sine-wave LO waveform with peak-to-peak amplitude of $2 B$ whose frequency is chosen such that the mixer output occurs at zero IF frequency.
(b) Comparing the above cases, is there a difference in conversion gain if the IF output occurs at zero or nonzero frequency?
(c) Which type of LO signal achieves a higher conversion gain (assuming the same peak-to-peak amplitude)?
(d) Given a non-zero IF frequency, compare the relative requirements on a post-mixer filter to reject undesired frequency components for square-wave and sine-wave LO signals. Present your arguments using pictures of the appropriate Fourier transforms of the relevant signals.
3. This problem is based on Problems 5 and 6 in Chapter 12 of Thomas Lee's book. Here we focus on the double-balanced mixer shown in Figure 3. It will be assumed that a nonzero IF output frequency is sought in all cases.


Figure 3: A double-balanced passive CMOS mixer.
(a) Given that $R_{L}$ is set equal to $R_{S}$, what is the conversion gain of the mixer? Assume infinitely fast switching, neglect switch resistance, and assume that the post-mixer RC filter formed by $R_{L}$ and $C_{L}$ has higher bandwidth than the desired IF output.
(b) Repeat part (a) given non-zero switch resistance (denoted as $R_{s w}$ ). What is the maximum acceptable switch resistance if the degradation in conversion gain (relative to part (a)) is not to exceed 1 dB ?
(c) How does the filter requirement change if the LO drive does not possess a perfect $50 \%$ duty cycle? Express your answer using pictures of the appropriate Fourier transforms where you have considered the impact of the non- $50 \%$ duty cycle square wave. Based on this exercise, how important is the achievement of symmetry in device switching for the mixer?
4. This problem focuses on the analysis and simulation of the Weaver image rejection mixer architecture shown in Figure 4.


Figure 4: The Weaver image rejection mixer architecture.
(a) Given the RF signal Fourier transform shown in Figure 4, what condition is required for the signal centered at $f_{i}$ to be the image of the desired signal centered at $f_{d}$ (i.e., what is the relationship between $f_{i}, f_{d}$, and $f_{1}$ ?).
(b) Given the relationship in part (a), what value should be chosen for $f_{2}$ so that the overall mixer output is at baseband frequencies (i.e., has an IF frequency of zero)?
(c) Given the conditions in part (a) and (b), sketch the Fourier transforms of the signals at nodes A, B, C, D, and the overall output of the mixer.
(d) Verify operation of the mixer by simulating the system in Figure 4 using CppSim given $f_{1}=1 \mathrm{GHz}, f_{d}=900 \mathrm{MHz}$, the lowpass cutoff frequencies are at 150 MHz ,
and $f_{2}$ is appropriately chosen for baseband conversion. Rather than inputing an RF signal as shown in the figure (which would be rather challenging), simply input two sinusoidal signals. One of the signals should be placed at the center of the desired frequency band, and the other should be placed just slightly offset from its respective image frequency location (offset so that you can view it distinctly from the desired location if it were to not be completely rejected in the final output). Note that you'll need to create new CppSim module blocks for the mixers and lowpass filter.
Turn in the module code and plots at A, C, and the Output. Hint: Make sure that your input is real-valued.
(e) Now modify your CppSim simulation to observe the impact of phase mismatch between the cosine and sine waves at frequency $f_{1}$. Specifically, phase shift the sine wave by 1 degree and then measure the corresponding image rejection ratio at DC based on the simulation results.

