Introduction to Engineering Systems, ESD.00

System Dynamics

Lecture 3

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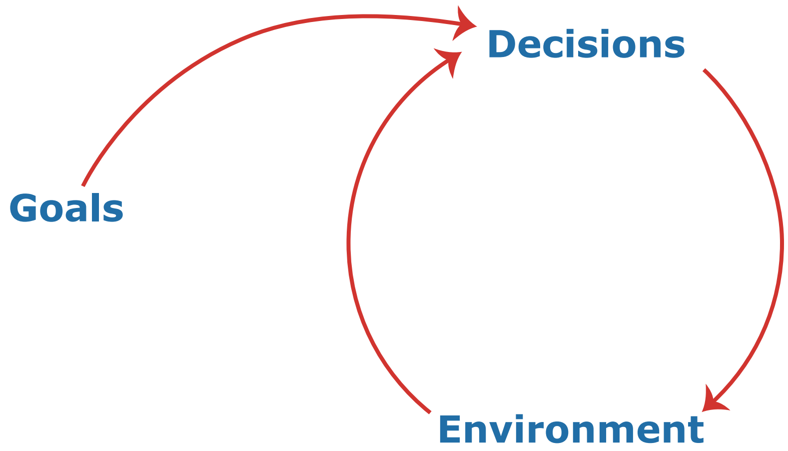
From Last Time: Systems Thinking

- "we can't do just one thing" – things are interconnected and our actions have numerous effects that we often do not anticipate or realize.

- Many times our policies and efforts aimed towards some objective fail to produce the desired outcomes, rather we often make matters worse

- *Systems Thinking* involves holistic consideration of our actions



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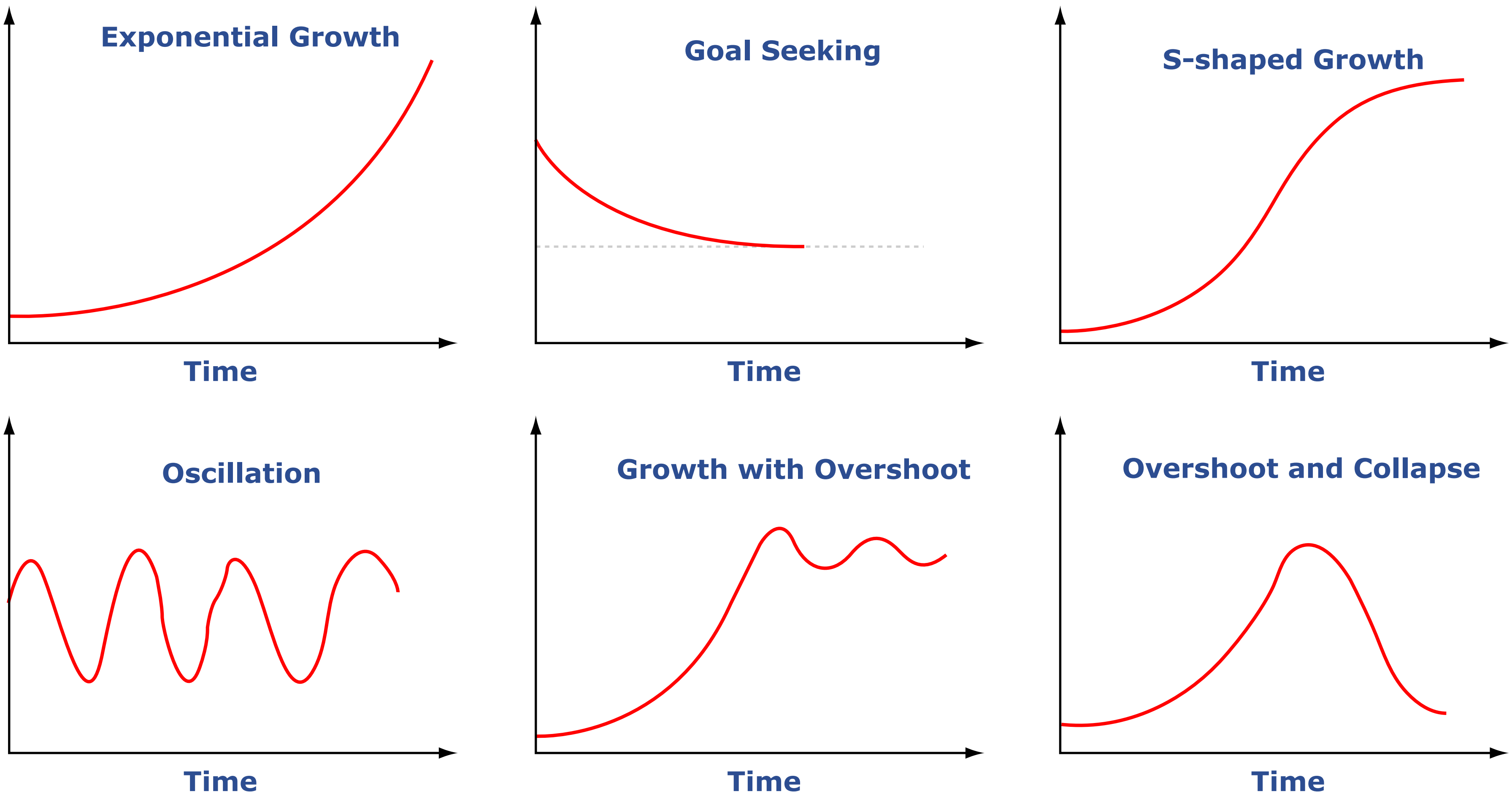
Ref: Figure 1-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Dynamic Complexity

- **Dynamic** (changing over time)
- **Governed by feedback** (actions feedback on themselves)
- **Nonlinear** (effect is rarely proportional to cause, and what happens locally often doesn't apply in distant regions)
- **History-dependent** (taking one road often precludes taking others and determines your destination, you can't unscramble an egg)
- **Adaptive** (the capabilities and decision rules of agents in complex systems change over time)
- **Counterintuitive** (cause and effect are distant in time and space)
- **Policy resistant** (many seemingly obvious solutions to problems fail or actually worsen the situation)
- **Characterized by trade-offs** (the long run is often different from the short-run response, due to time delays. High leverage policies often cause worse-before-better behavior while low leverage policies often generate transitory improvement before the problem grows worse.

Modes of Behavior



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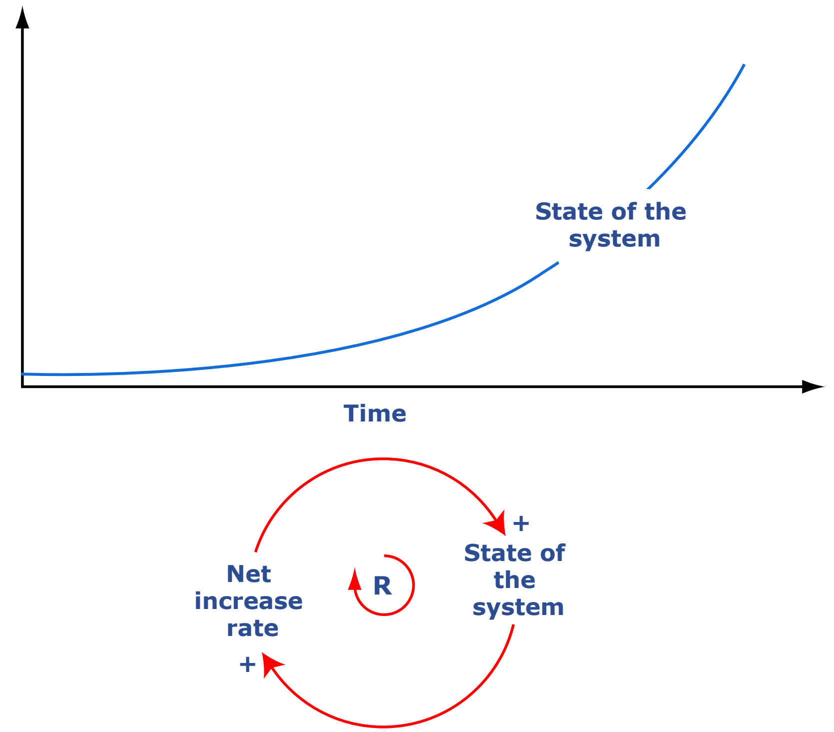
Ref: Figure 4-1, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Exponential Growth

- Arises from positive (self-reinforcing) feedback.
- In pure exponential growth the state of the system doubles in a fixed period of time.
  - Same amount of time to grow from 1 to 2, and from 1 billion to 2 billion!
- Self-reinforcing feedback can be a declining loop as well (e.g. stock prices)
- Common example: compound interest, population growth



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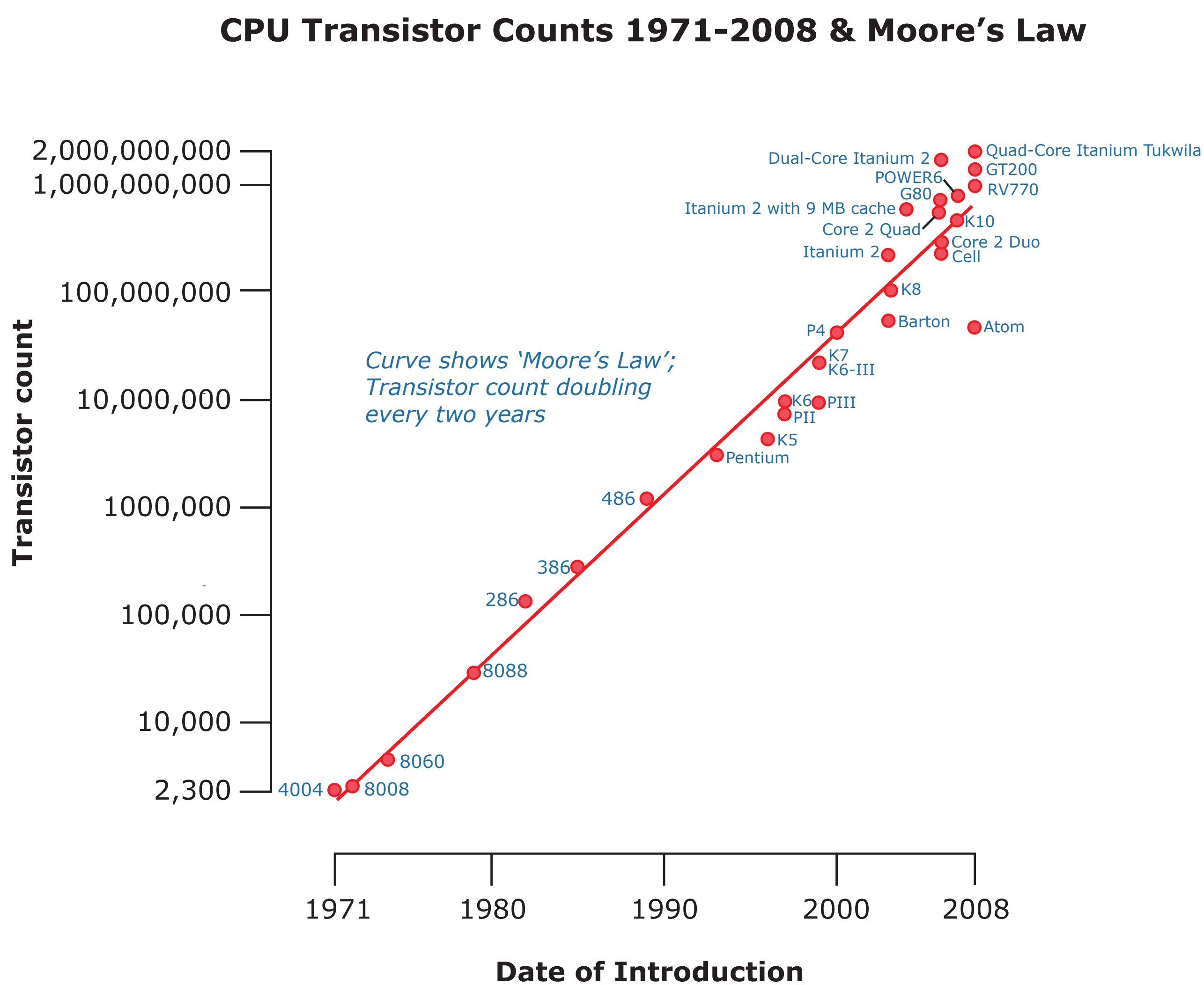
Ref: Figure 4-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Exponential Growth: Examples



Ref: wikipedia

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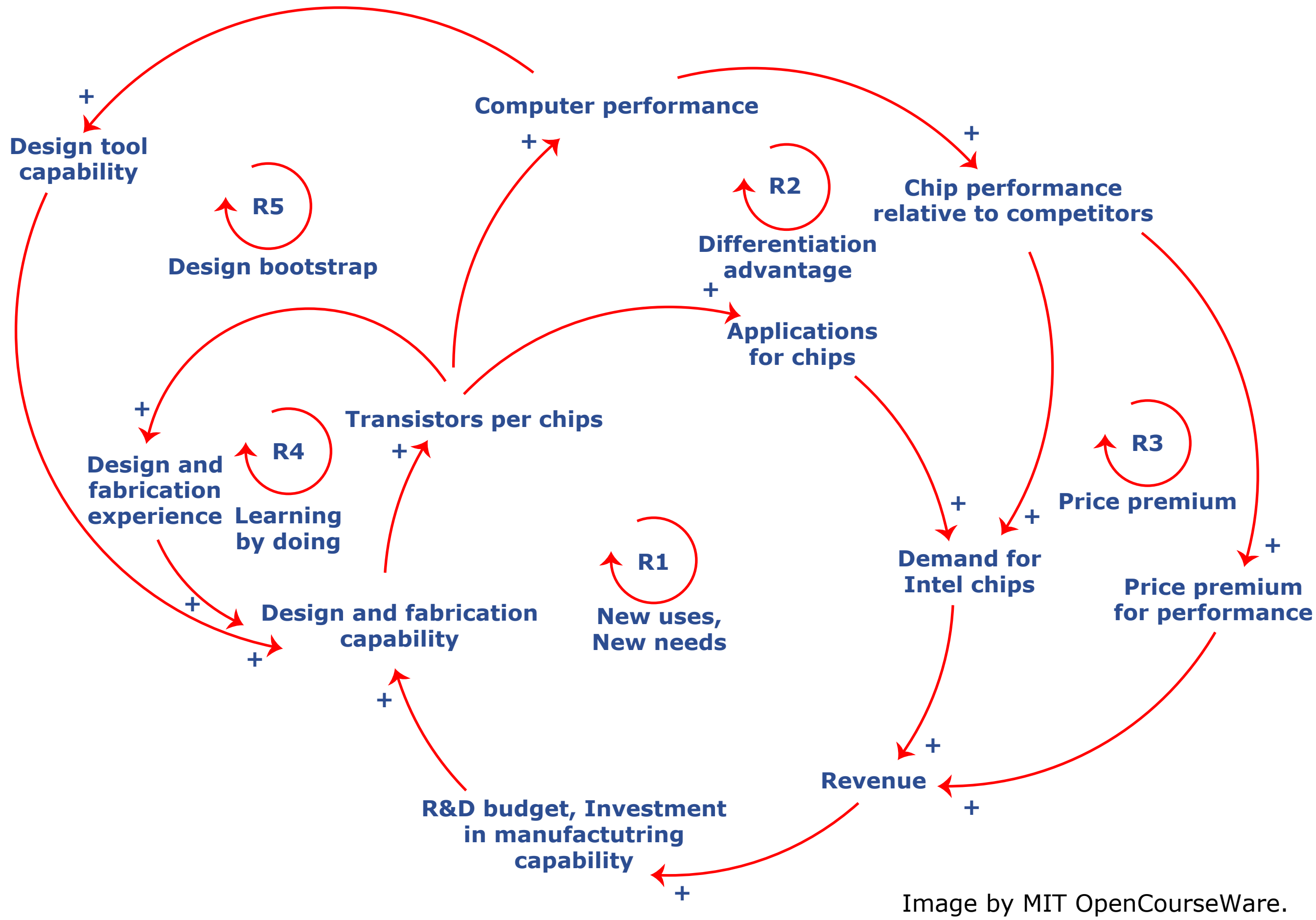
Some Positive Feedbacks underlying Moore's Law



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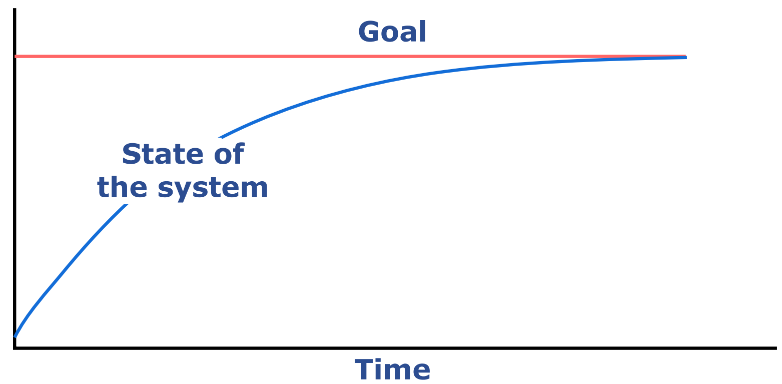
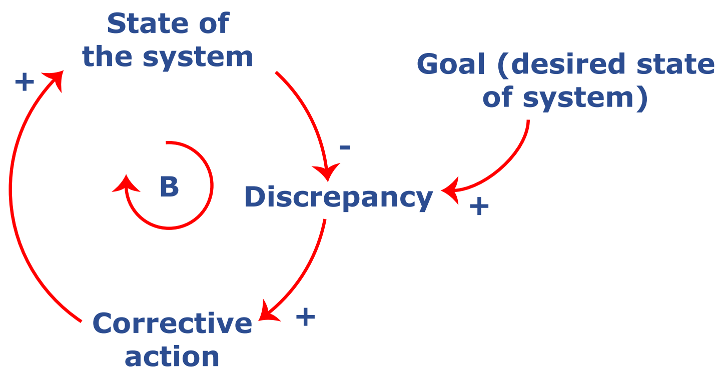
Ref: Exhibit 4-7, J. Sterman, Instructor's Manual, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Goal Seeking

- Negative loops seek balance, and equilibrium, and try to bring the system to a desired state (goal).
- Positive loops reinforce change, while negative loops counteract change or disturbances.
- Negative loops have a process to compare desired state to current state and take corrective action.
- Pure exponential decay is characterized by its half life – the time it takes for half the remaining gap to be eliminated.





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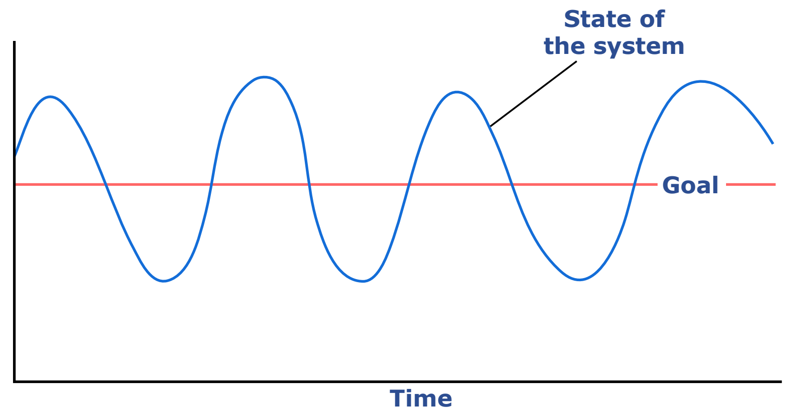
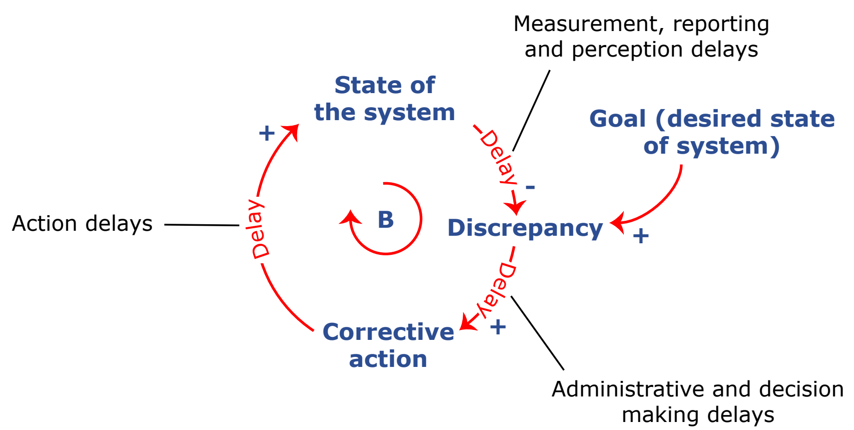
Ref: Figure 4-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Oscillation

- This is the third fundamental mode of behavior.
- It is caused by goal-seeking behavior, but results from constant 'over-shoots' and 'under-shoots'
- The over-shoots and under-shoots result due to time delays- the corrective action continues to execute even when system reaches desired state giving rise to the oscillations.





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Ref: Figure 4-6, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Interpreting Behavior

- Connection between structure and behavior helps in generating hypotheses
- If exponential growth is observed -> some reinforcing feedback loop is dominant over the time horizon of behavior
- If oscillations are observed, think of time delays and goal-seeking behavior.
- Past data shows historical behavior, the future maybe different. Dormant underlying structures may emerge in the future and change the 'mode'
- It is useful to think what future 'modes' can be, how to plan and manage them
- Exponential growth gets limited by negative loops kicking in/becoming dominant later on

Limits of Causal Loop Diagrams

- Causal loop diagrams (CLDs) help
  - in capturing mental models, and
  - showing interdependencies and
  - feedback processes.

- CLDs cannot
  - capture accumulations (stocks) and flows
  - help in determining detailed dynamics

Stocks, Flows and Feedback are central concepts in System Dynamics

Stocks

- Stocks are accumulations, aggregations, summations over time
- Stocks characterize/describe the state of the system
- Stocks change with inflows and outflows
- Stocks provide memory and give inertia by accumulating past inflows; they are the sources of delays.
- Stocks, by accumulating flows, decouple the inflows and outflows of a system and cause variations such as oscillations over time.

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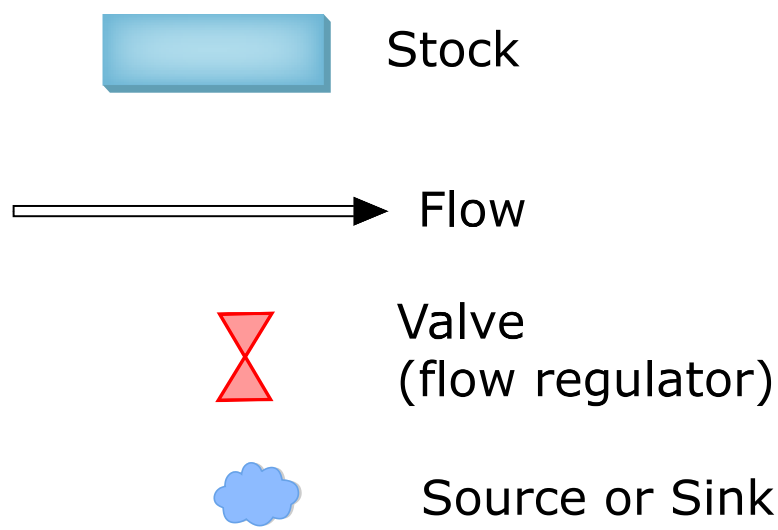

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Mathematics of Stocks

- Stock and flow diagramming were based on a hydraulic metaphor

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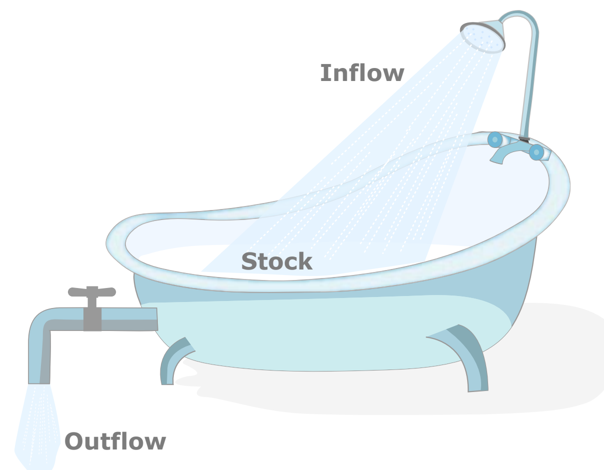
- Stocks integrate their flows:

$$\text{Stock}(t) = \int_{t_0}^{t} [\text{Inflow}(s) - \text{Outflow}(s)]ds + \text{Stock}(t_0)$$



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- The net flow is rate of change of stock:

$$d(\text{Stock})/dt = \text{Net Change in Stock} = \text{Inflow}(t) - \text{Outflow}(t)$$

Ref: Figure 6-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Stocks and Flows Examples

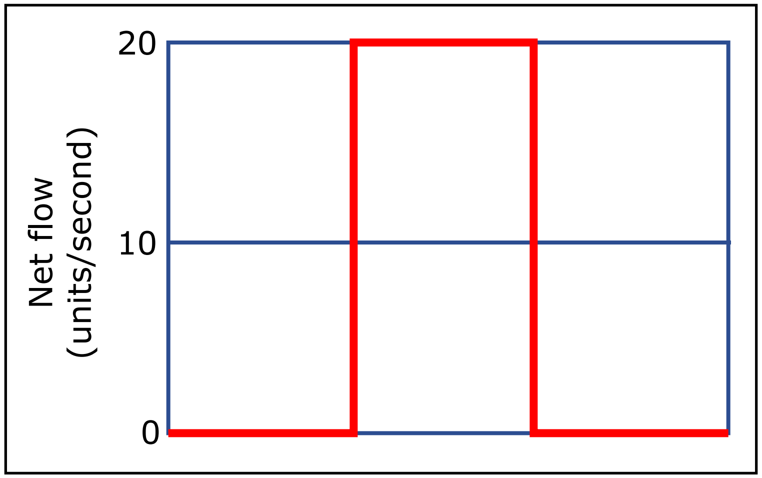
Field	Stocks	Flows
Mathematics, Physics and Engineering	Integrals, States, State variables, Stocks	Derivatives, Rates of change, Flows
Chemistry	Reactants and reaction products	Reaction Rates
Manufacturing	Buffers, Inventories	Throughput
Economics	Levels	Rates
Accounting	Stocks, Balance sheet items	Flows, Cash flow or Income statement items

Image by MIT OpenCourseWare.

Ref: Table 6-1, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Snapshot Test:
Freeze the system in time – things that are measurable in the snapshot are stocks.

Example



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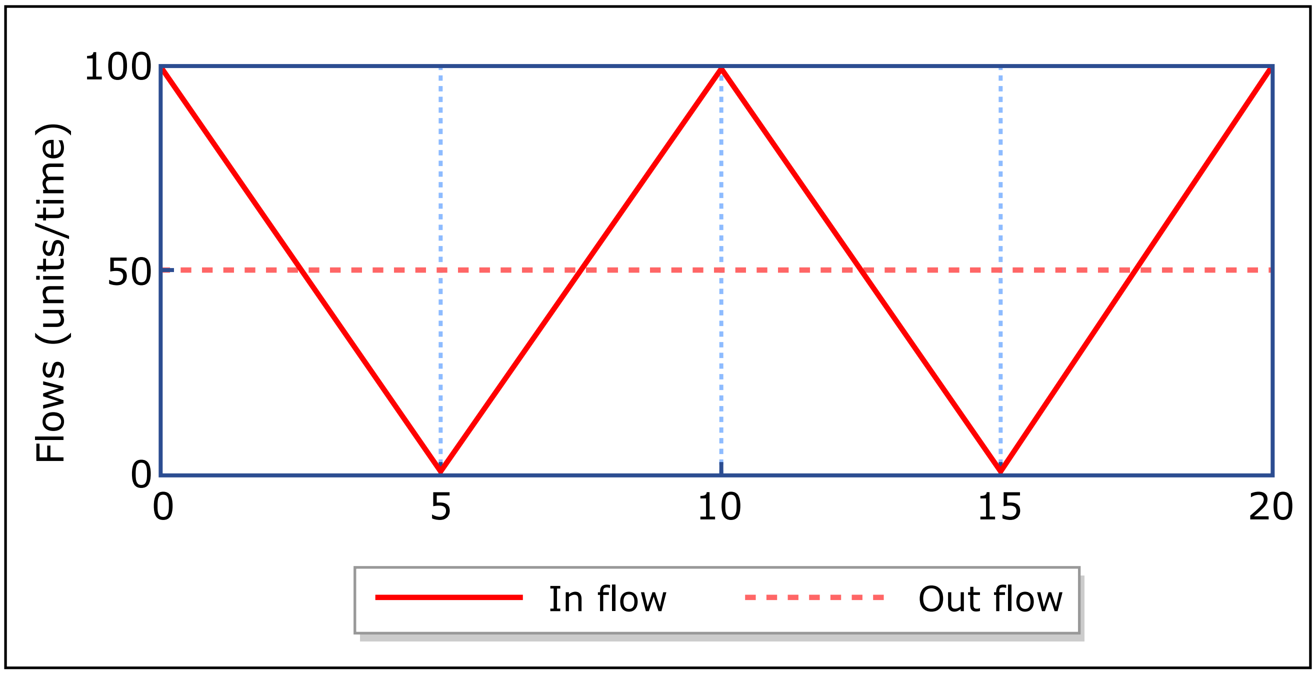
Ref: Figure 7-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Example



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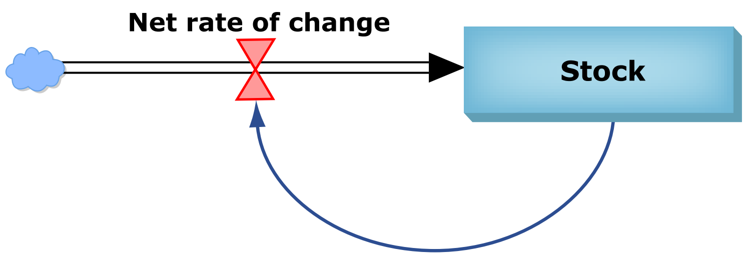
Ref: Figure 7-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Flow Rates

- Model systems as networks of stocks and flows linked by information feedbacks from the stocks to the rates.
- Rates can be influenced by stocks, other constants (variables that change very slowly) and exogenous variables (variables outside the scope of the model).
- Stocks only change via inflows and outflows.



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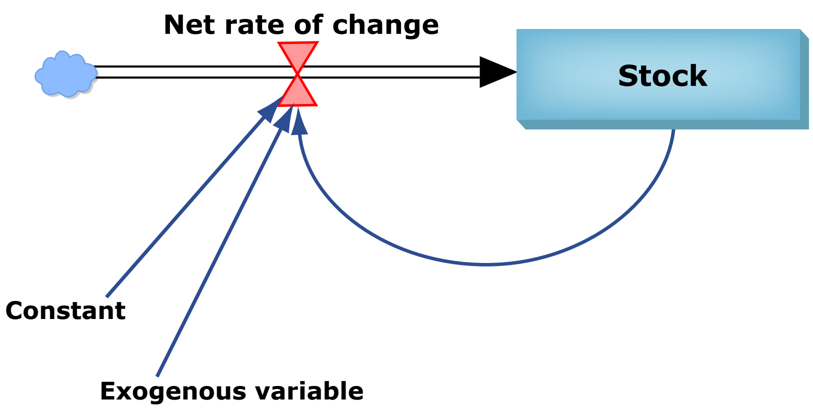

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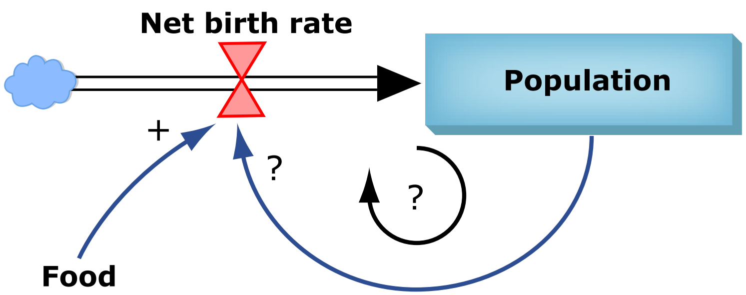
Auxiliary Variables



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Ref: Figure 8-?, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

- Auxiliary variables are neither stocks nor flows, but intermediate concepts for clarity
- Add enough structure to make polarities clear

Aggregation and Boundaries - I

- Identify main stocks in the system and then flows that alter them.
- Choose a level of aggregation and boundaries for the system
- Aggregation is number of internal stocks chosen
- Boundaries show how far upstream and downstream (of the flow) the system is modeled

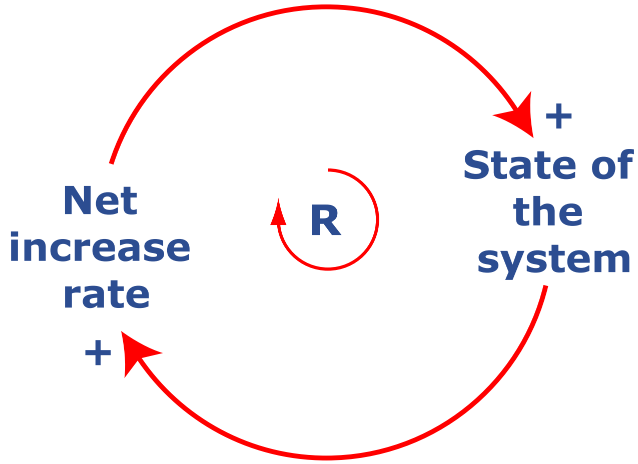
Aggregation and Boundaries - II

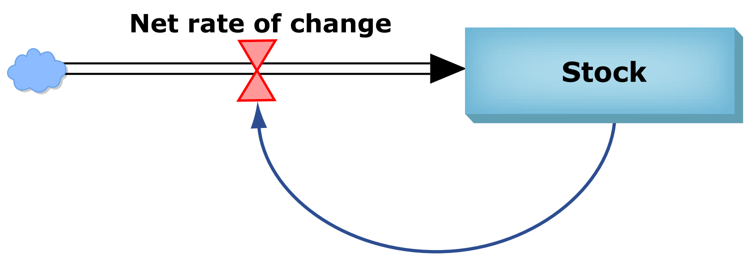
- One can 'challenge the clouds', i.e. make previous sources or sinks explicit.
- We can disaggregate our stocks further to capture additional dynamics.
- Stocks with short 'residence time' relative to the modeled time horizon can be lumped together
- Level of aggregation depends on purpose of model
- It is better to start simple and then add details.

From Structure to Behavior

- The underlying structure of the system defines the time-based behavior.

- Consider the simplest case: the state of the system is affected by its rate of change.





Ref: Figure 8-1, & 8-2 J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

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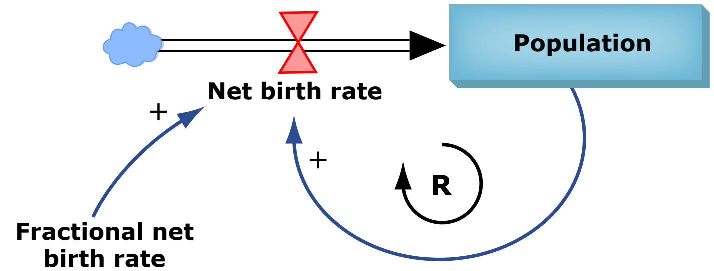
Population Growth

- Consider the population Model:



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- The mathematical representation of this structure is:

Net birth rate = fractional birth rate * population

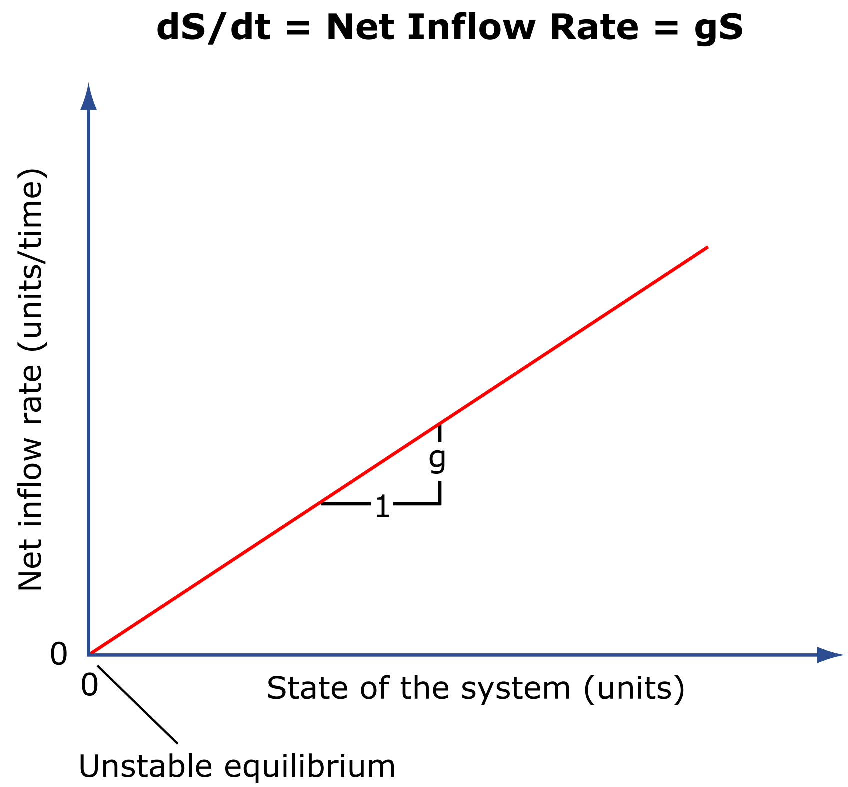
Ref: Figure 8-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Note: units of 'b', fractional growth rate are 1/time

Phase-Plots for Exponential Growth

- Phase plot is a graph of system state vs. rate of change of state
- Phase plot of a first-order, linear positive feedback system is a straight line
- If the state of the system is zero, the rate of change is also zero
- The origin however is an unstable equilibrium.

Ref: Figure 8-3, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000



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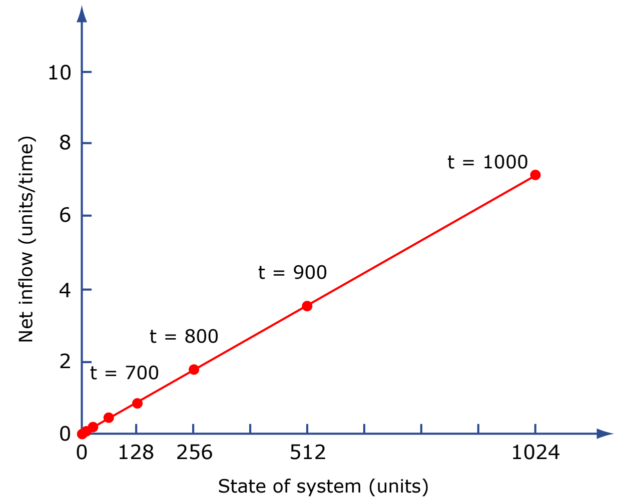
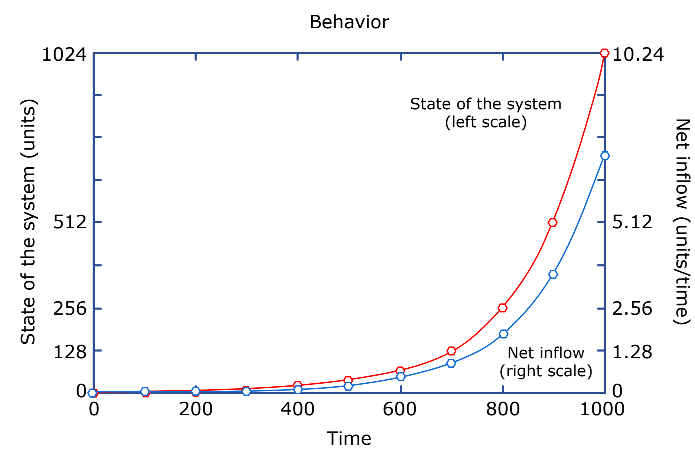
Time Plots

- Fractional growth rate g = 0.7%/time unit
- Initial state state = 1.
- State doubles every 100 time units
- Every time state of the system doubles, so too does the absolute rate of increase





Ref: Figure 8-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

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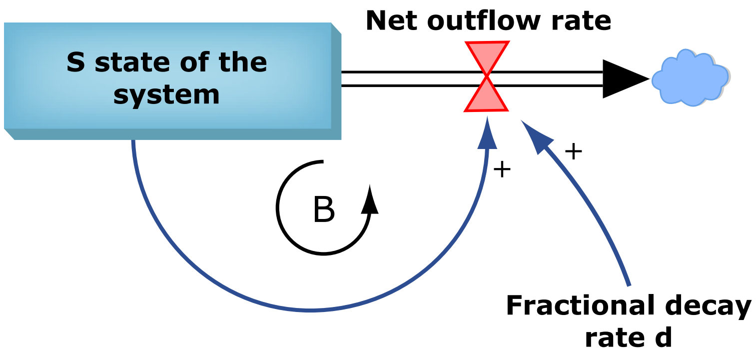
Rule of 70

- Exponential growth is one of the most powerful processes.
- The rate of increase grows as the state of the system grows.
- It has the remarkable property that the state of the system doubles in fixed period of time.
- If the doubling time is say 100 time units, it will take 100 units to go from 2 to 4, and another 100 units to go from 1000 to 2000 and so on.
- To find doubling time:

Negative Feedback and Exponential Decay

- First-order linear negative feedback systems generate exponential decay
- The net outflow is proportional to the size of the stock
- The solution is given by: $S(t) = S_o\, e^{-dt}$
- Examples:



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Net Inflow = -Net Outflow = -d*S

d: fractional decay rate [1/time]

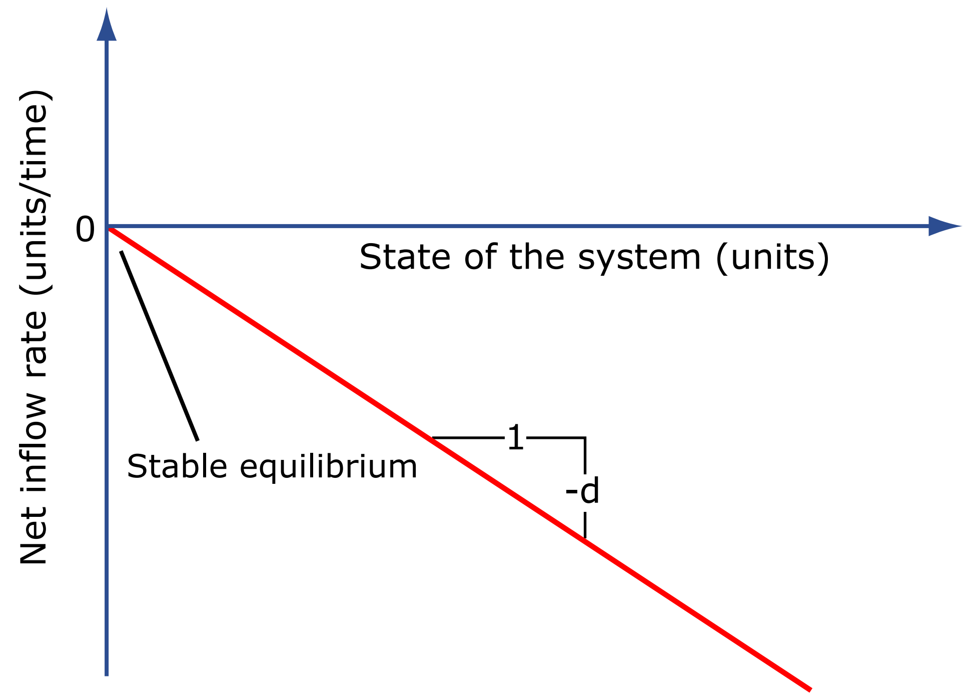
Reciprocal of d is average lifetime units in stock.

Ref: Figure 8-6, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Phase Plot for Exponential Decay

- In the phase-plot, the net rate of change is a straight line with negative slope
- The origin is a stable equilibrium, a minor perturbation in state S increases the decay rate to bring system back to zero – deviations from the equilibrium are self-correcting
- The goal in exponential decay is implicit and equal to zero

Net Inflow Rate = - Net Outflow Rate = -dS



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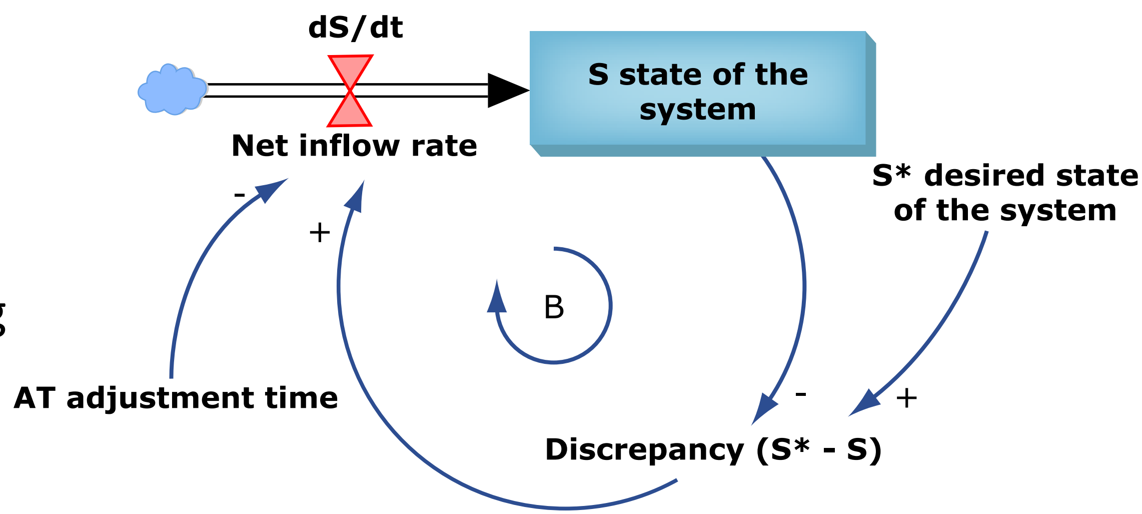
Ref: Figure 8-7, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Negative Feedback with Explicit Goals

- In general, negative loops have non-zero goals
- Examples:
- The corrective action determining net flow to the state of the system is : Net Inflow = *f* (S, S*)
- Simplest formulation is:
  Net Inflow = Discrepancy/adjustment time = (S*-S)/AT

General Structure



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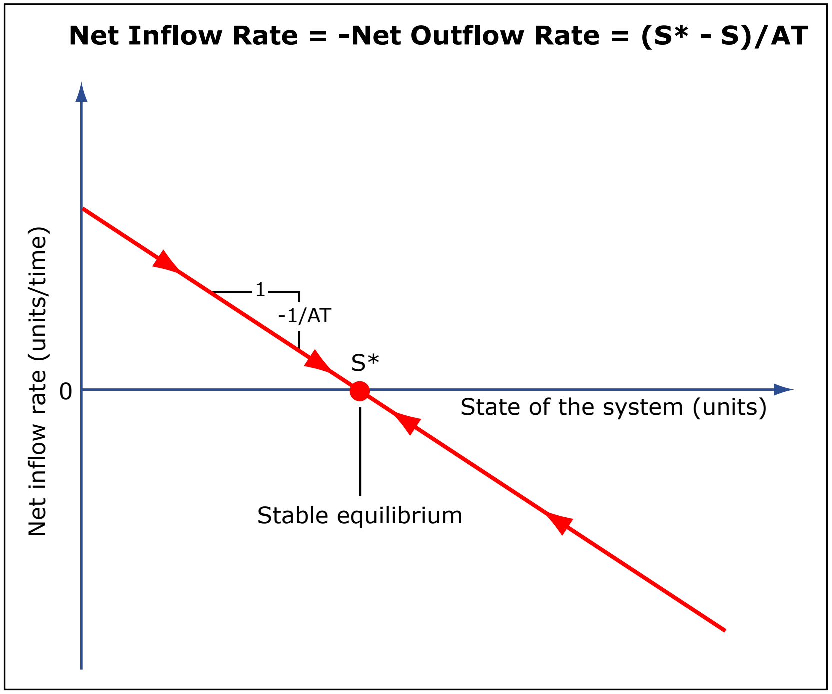
AT: adjustment time is also known as *time constant* for the loop

Ref: Figure 8-9, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Phase Plot for Negative Feedback with Non-Zero Goal

- In the phase-plot, the net rate of change is a straight line with slope -1/AT
- The behavior of the negative loop with an explicit goal is also exponential decay, in which the state reaches equilibrium when S=S*
- If the initial state is less than the desired state, the net inflow is positive and the state increases (at a diminishing rate) until S=S*. If the initial state is greater than S*, the net inflow is negative and the state falls until it reaches S*



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Ref: Figure 8-10, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Half-Lives

- Exponential decay cuts quantity remaining in half in fixed period of time
- The 'half-life' is calculated in similar way as doubling time.
- The system state as a function of time is given by:

$$\underbrace{S(t)}^{\text{State of system}} = \underbrace{S^*}_{\text{Desired State}} - \underbrace{\overbrace{\left(S^* - S_0\right)}^{\text{Initial Gap}} e^{-t/\tau}}_{\text{Gap Remaining}}$$

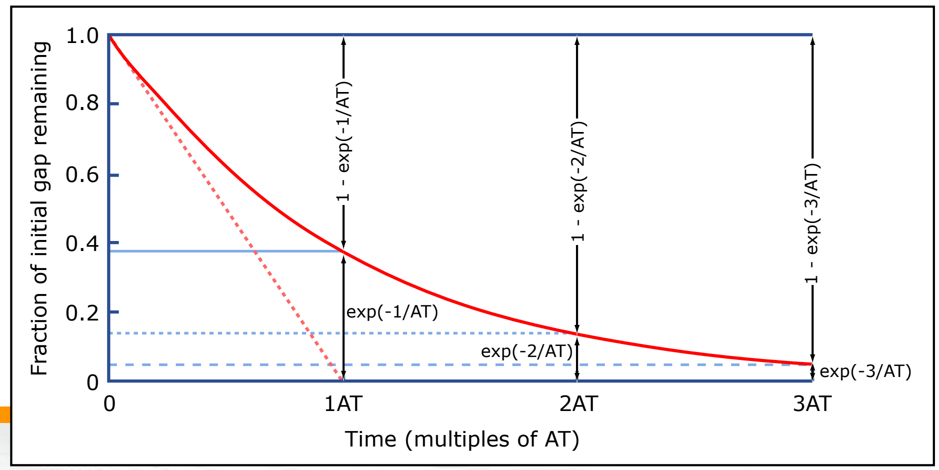
- The exponential term decays from 1 to zero as *t* tends to infinity.
- Half life is given by value of time t_h :

Time Constants and Settling Time

- For a first order, linear system with negative feedback, the system reaches 63% of its steady-state value in one time constant, and reaches 98% of its steady state value in 4 time constants.

- The steady state is not reached technically in finite time because the rate of adjustment keeps falling as the desired state is approached.

Time	Fraction of Initial Gap Remaining	Fraction of Initial Gap Corrected
0	$e^{-0} = 1$	$1-1 = 0$
τ	$e^{-1} = 0.37$	$1-e^{-1} = 0.63$
2τ	$e^{-2} = 0.14$	$1-e^{-2} = 0.87$
3τ	$e^{-3} = 0.05$	$1-e^{-3} = 0.95$
5τ	$e^{-5} = 0.007$	$1-e^{-5} = 0.993$



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Ref: Figure 8-12, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000