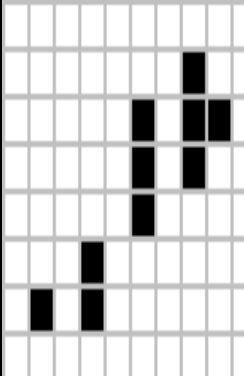






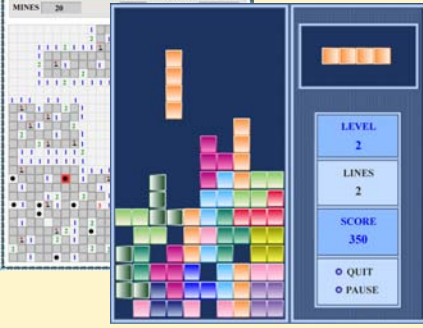

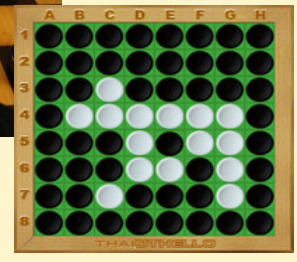



# Complexity of Games & Puzzles

[Demaine, Hearn & many others]

unbounded

bounded

	 <p>Courtesy of <a href="#">BigJ Smack</a></p>  <p>Courtesy of <a href="#">Sam Cancilla</a>. Used with permission.</p>	 <p>Image courtesy of <a href="#">Herman Hiddema</a></p>  <p>Image courtesy of <a href="#">Nguyen Dai</a>.</p>	<p>Rengo Kriegspiel?</p>  <p>Courtesy of <a href="#">Glenn Peters</a>. Used with permission.</p>
<p>PSPACE</p>	<p>PSPACE</p>	<p>EXPTIME</p>	<p>Undecidable</p>
 <p>Image courtesy of <a href="#">Jason Whittaker</a>.</p>	 <p>Images by MIT OpenCourseWare.</p>	 <p>Image courtesy of <a href="#">PartsnPieces</a>.</p>  <p>Image is in the public domain.</p>	<p>bridge?</p>  <p>Image courtesy of <a href="#">Marie-Lan Nguyen</a>.</p>
<p>P</p>	<p>NP</p>	<p>PSPACE</p>	<p>NEXPTIME</p>

0 players  
(simulation)

1 player  
(puzzle)

2 players  
(game)

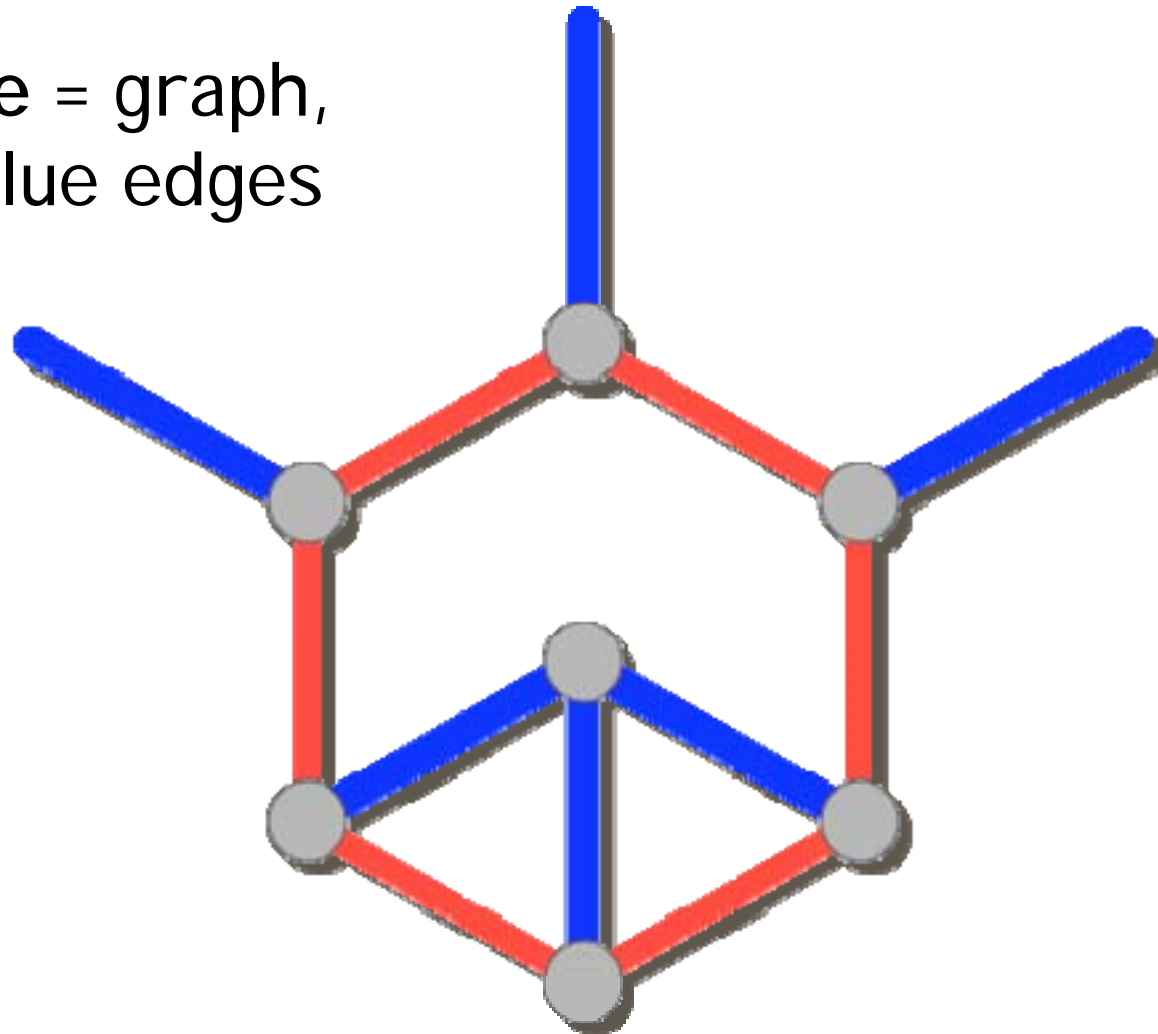
team,  
imperfect info



# Constraint Graphs

---

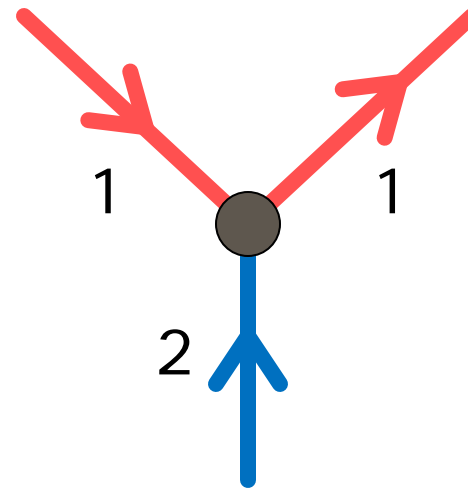
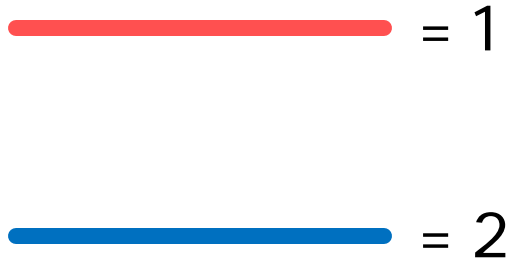
Machine = graph,  
red & blue edges





# Constraint Logic

---

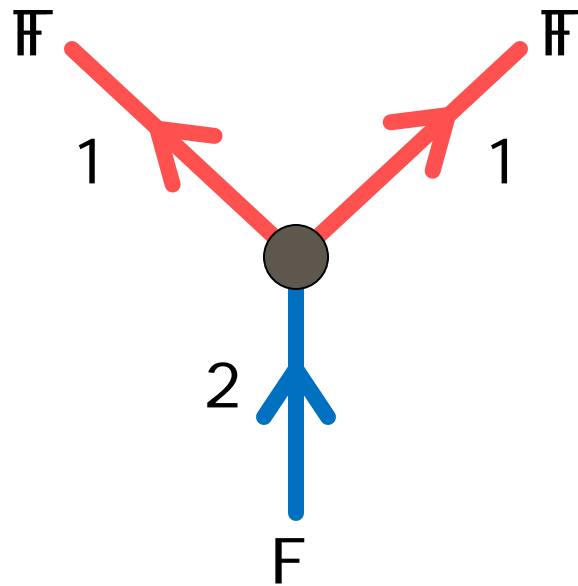


**Rule:** at least 2 units incoming at a vertex

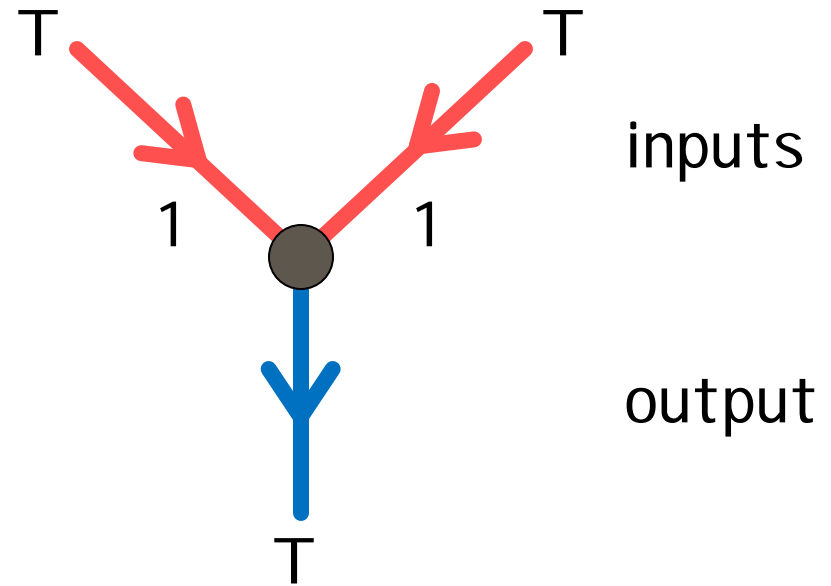
**Move:** reverse an edge, preserving Rule

# AND vertex

---



not your usual  
AND gate!



inputs

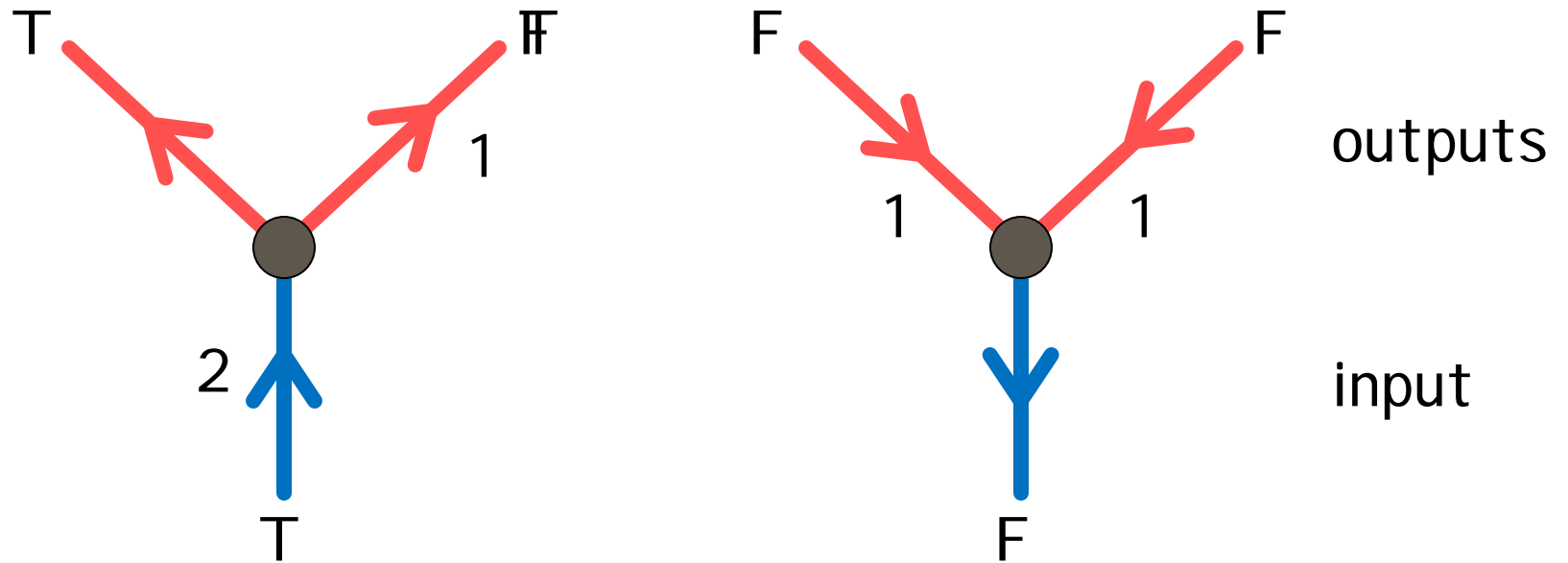
output

**Rule:** at least 2 units  
incoming at a vertex

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# SPLIT vertex

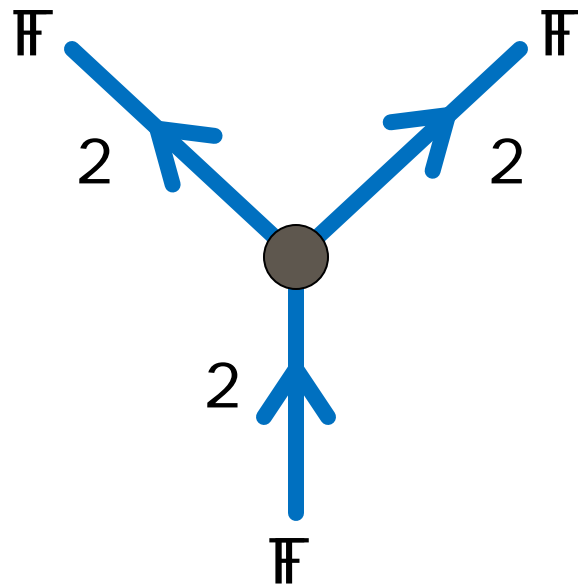
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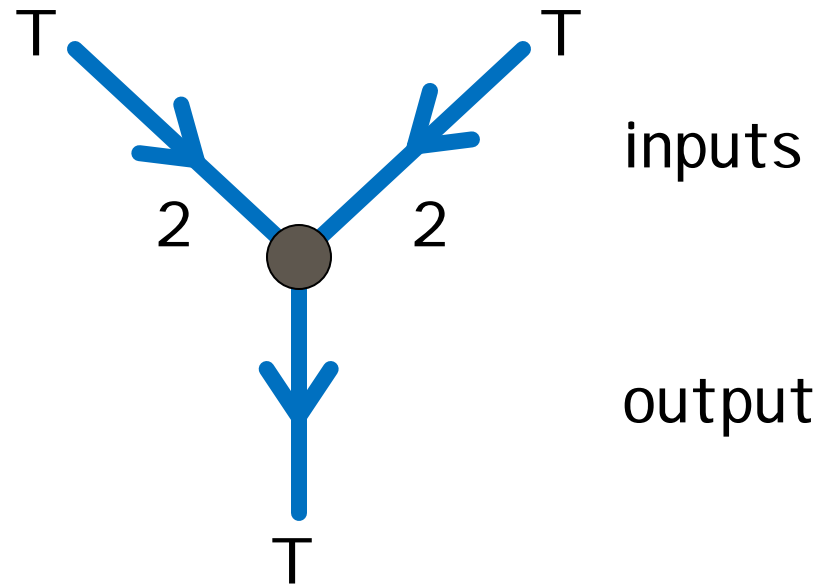
**Rule:** at least 2 units incoming at a vertex

# OR vertex

---



not your usual  
OR gate!



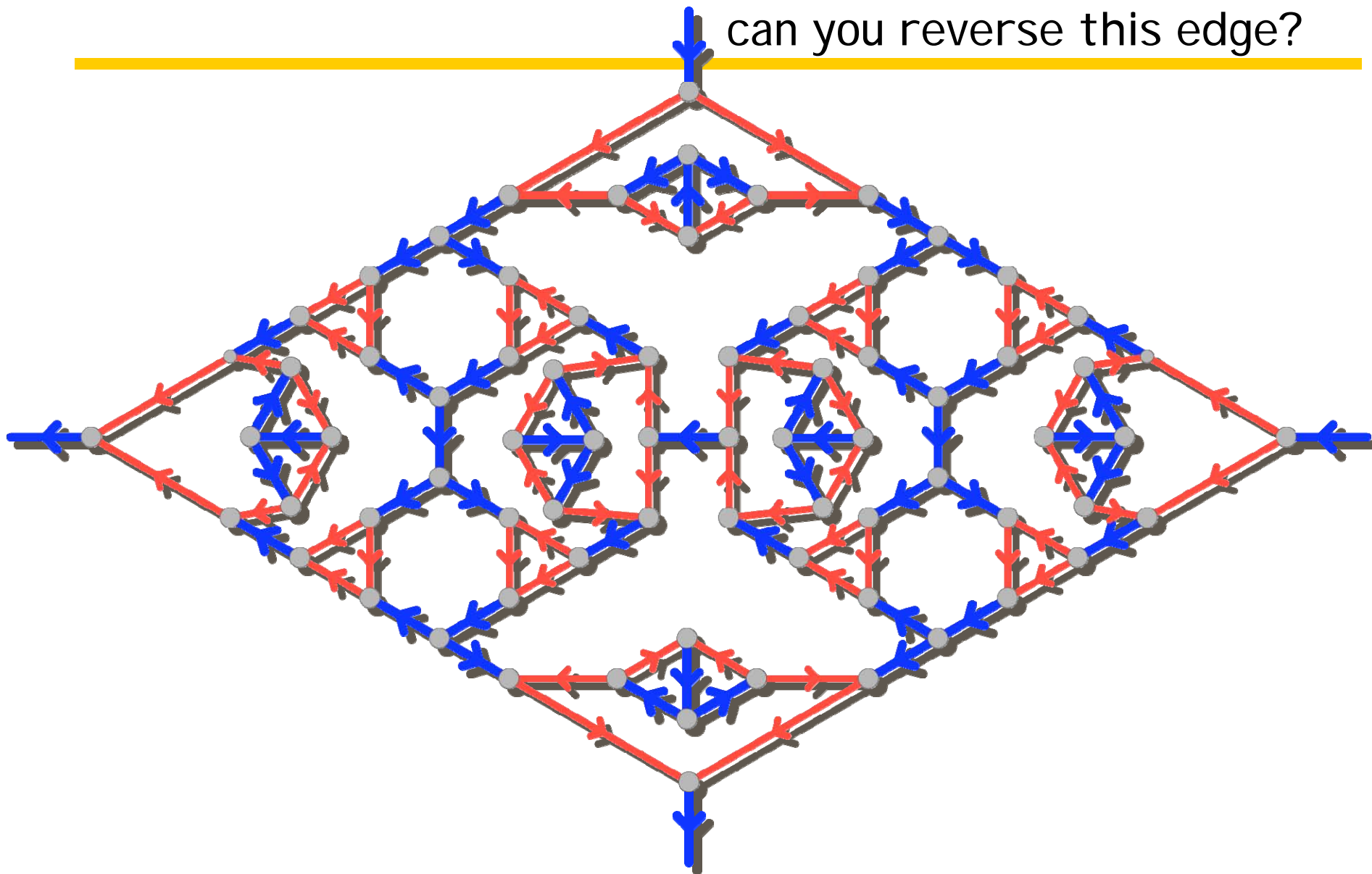
**Rule:** at least 2 units  
incoming at a vertex

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# Decision Problem

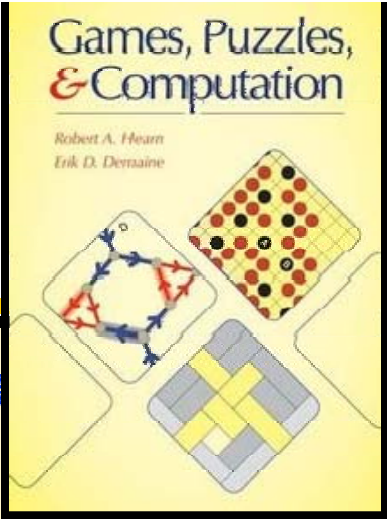
can you reverse this edge?



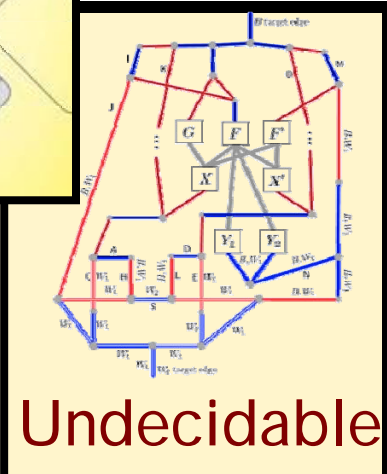
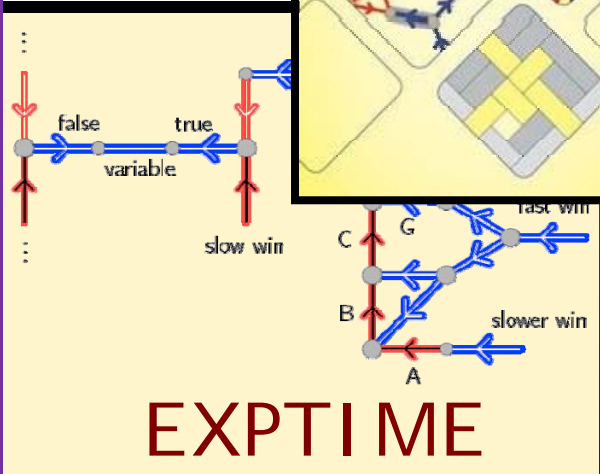
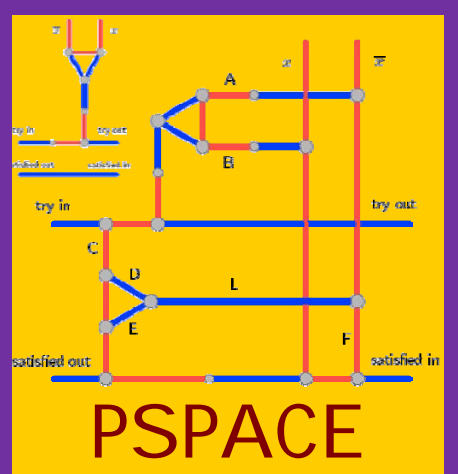
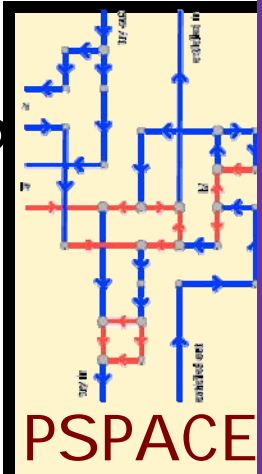
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# Constraint Logic

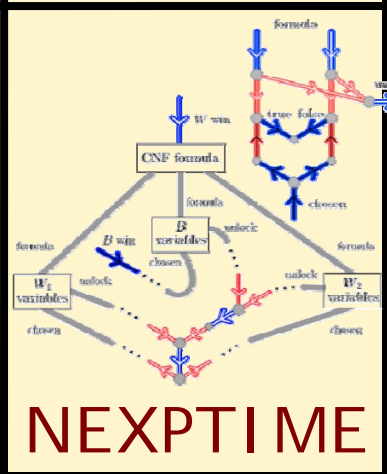
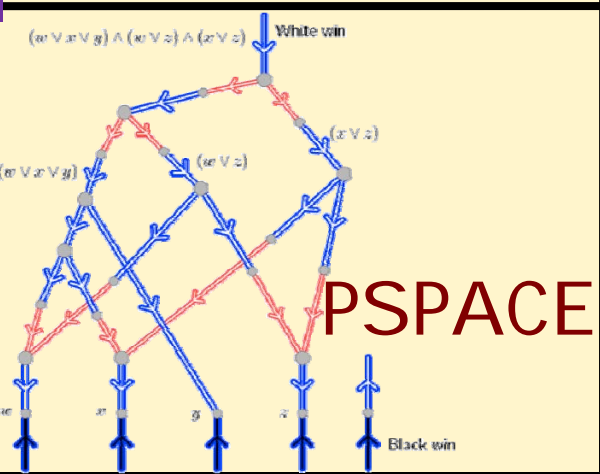
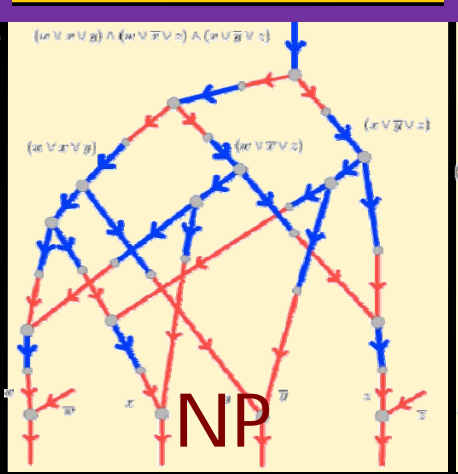
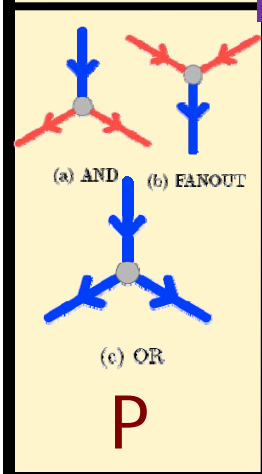
[Hearn & Demaine 2009]



unbounded



bounded



0 players  
(simulation)

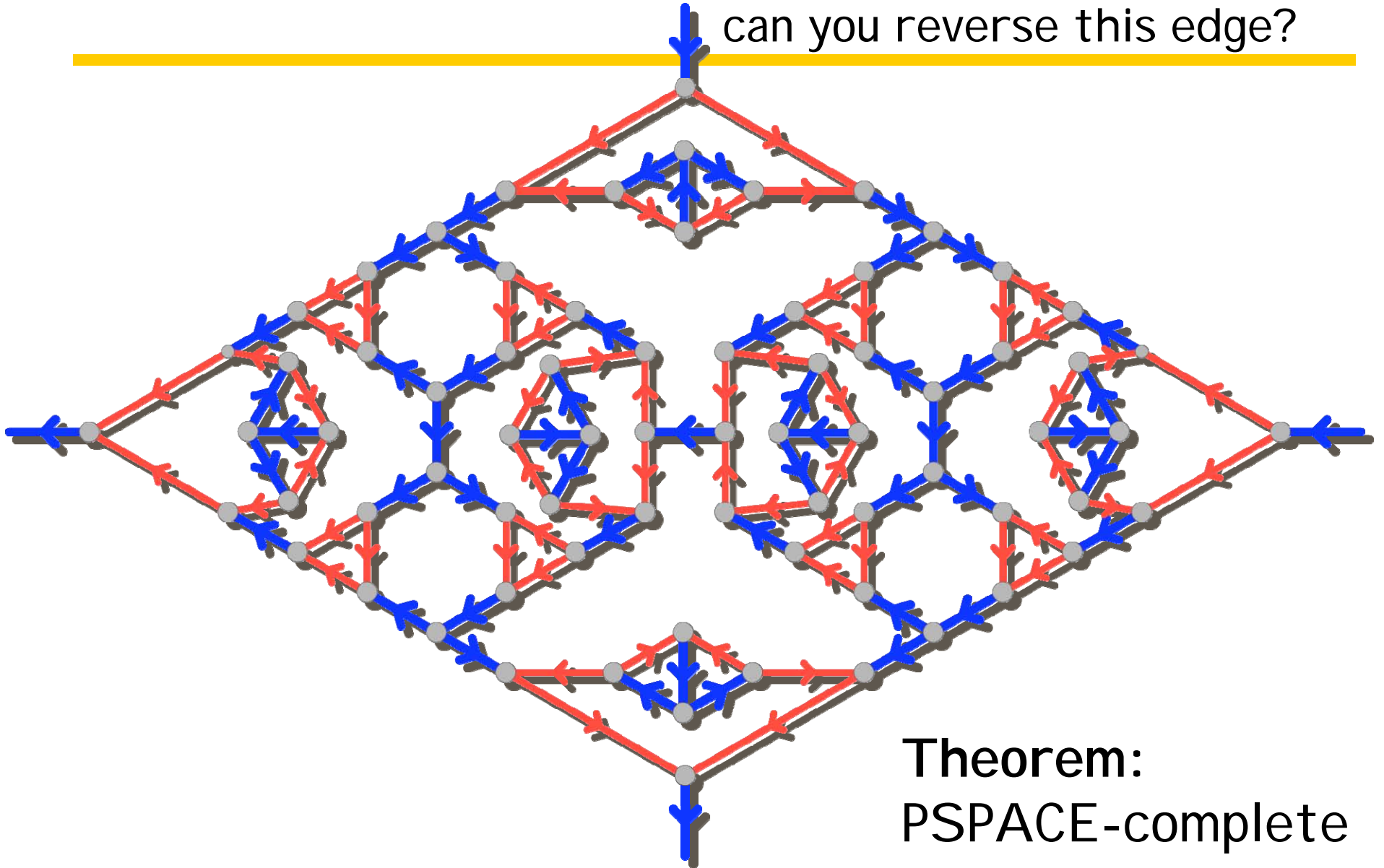
1 player  
(puzzle)

2 players  
(game)

team,  
imperfect info

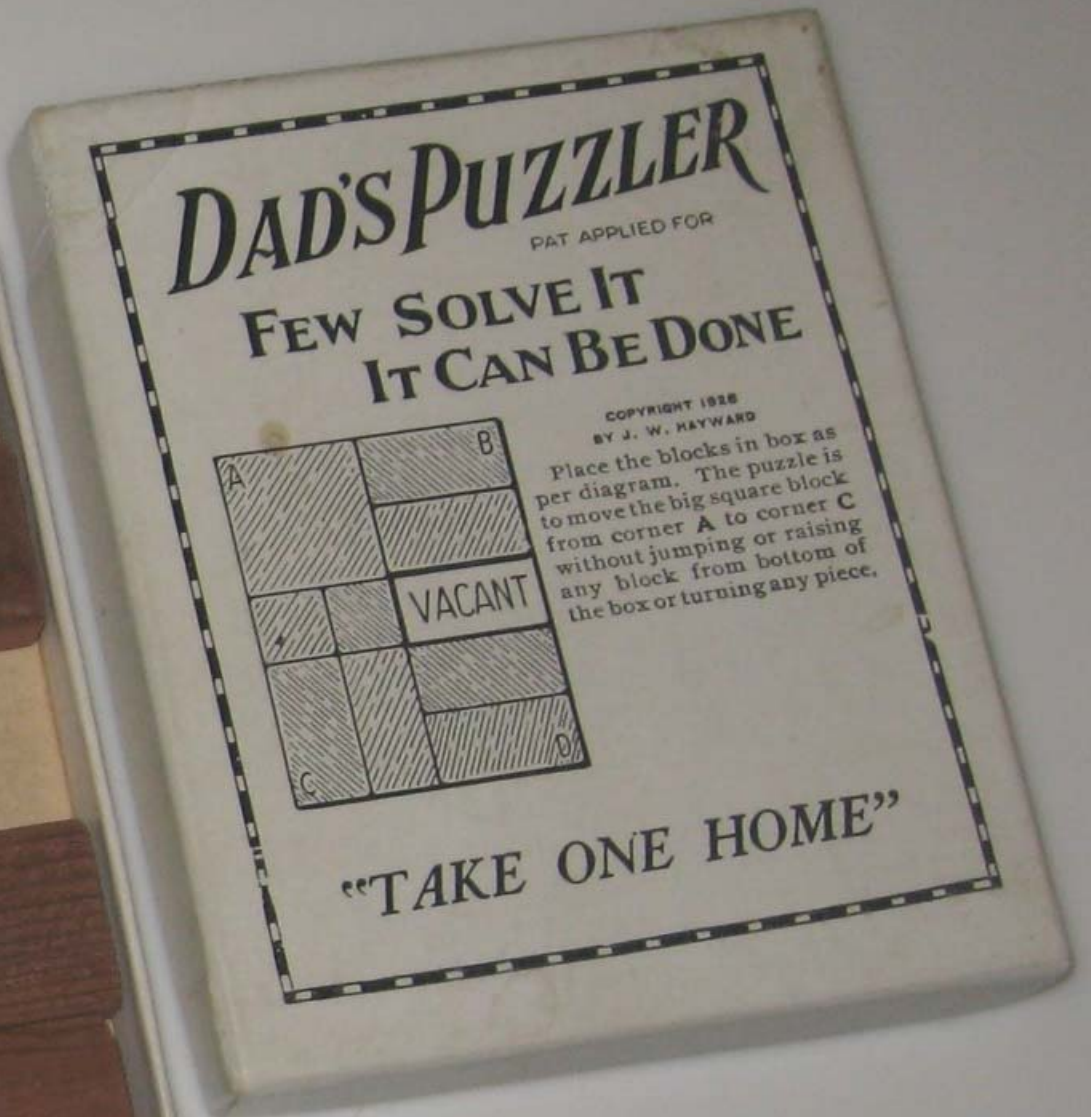
# Decision Problem

can you reverse this edge?



Theorem:  
PSPACE-complete

# Sliding-Block Puzzles



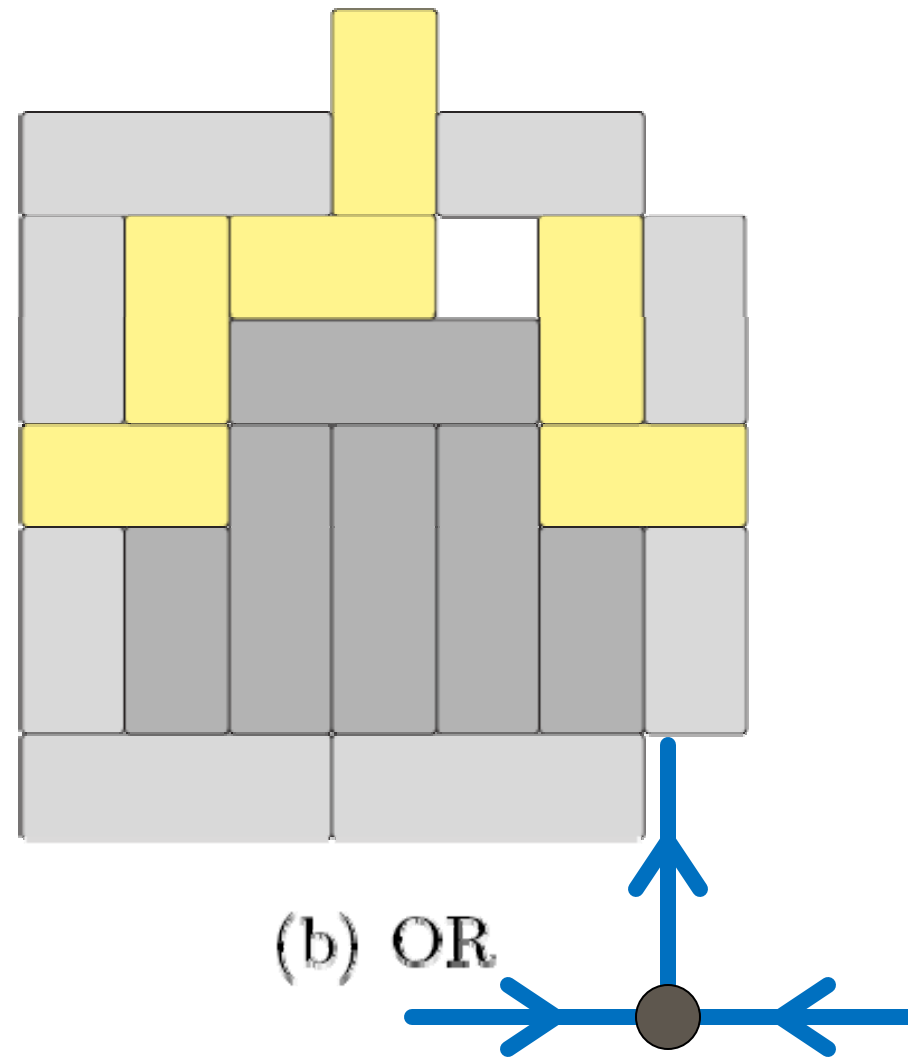
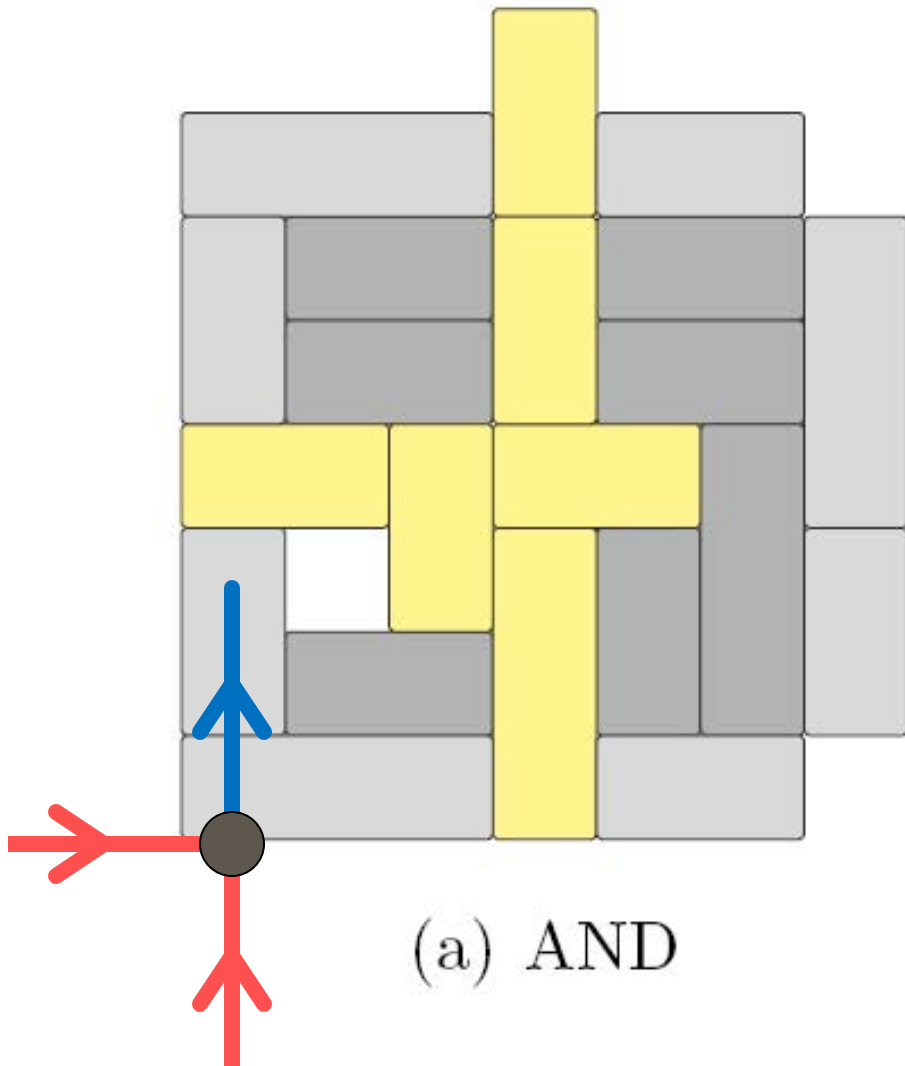
Courtesy of [Dr. Jim Storer](#). Used with permission.



# Sliding-Block Puzzles

[Hearn & Demaine 2002]

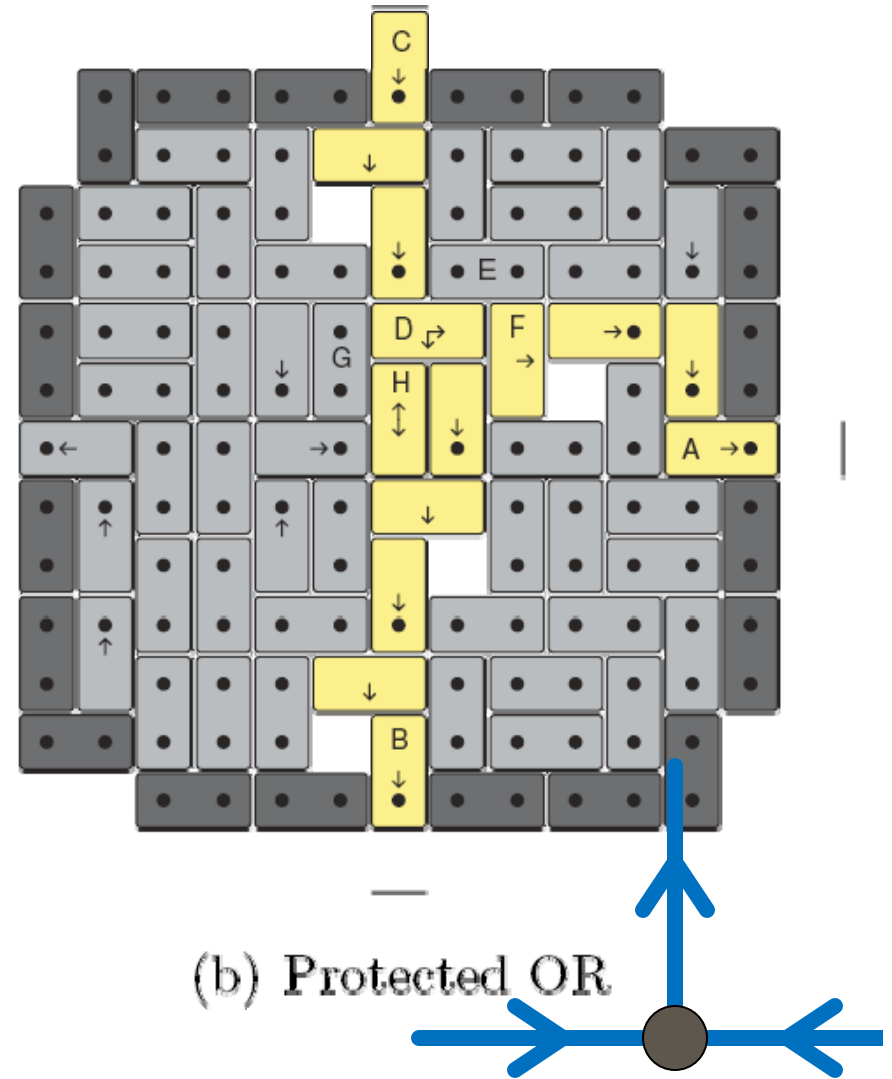
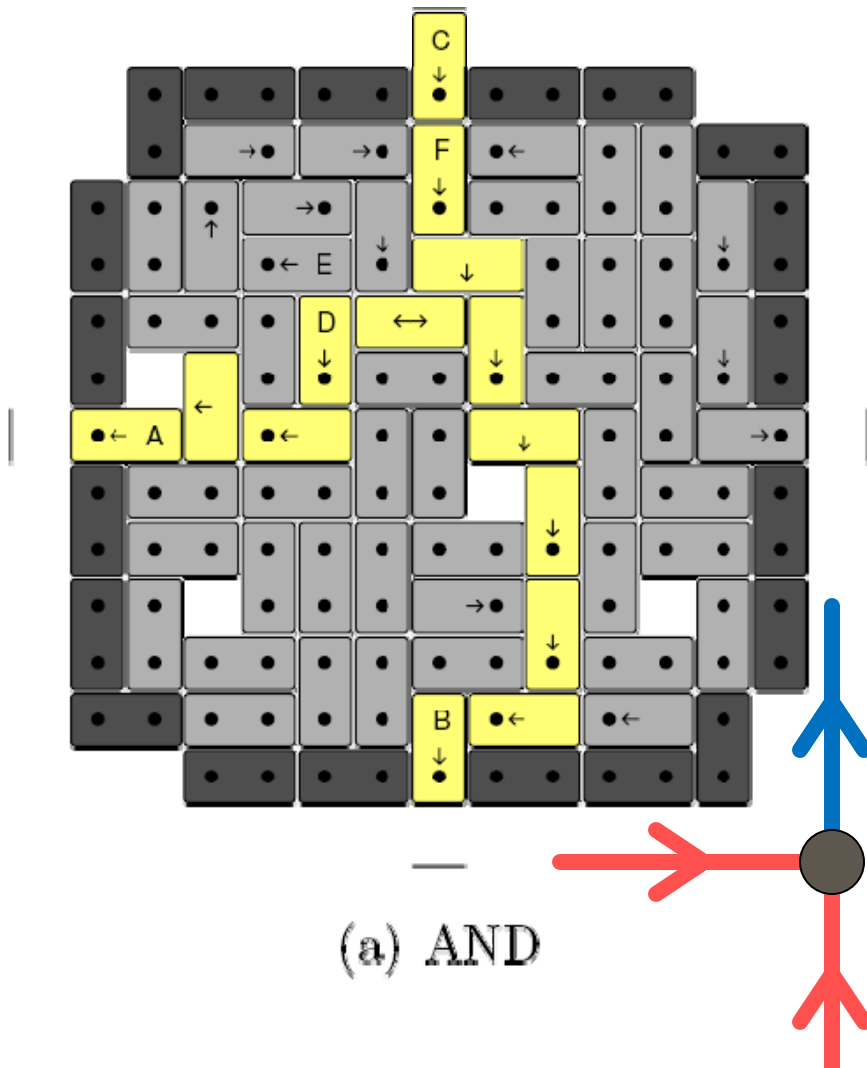
Corollary:  
PSPACE-complete



# Sliding-Block Puzzles

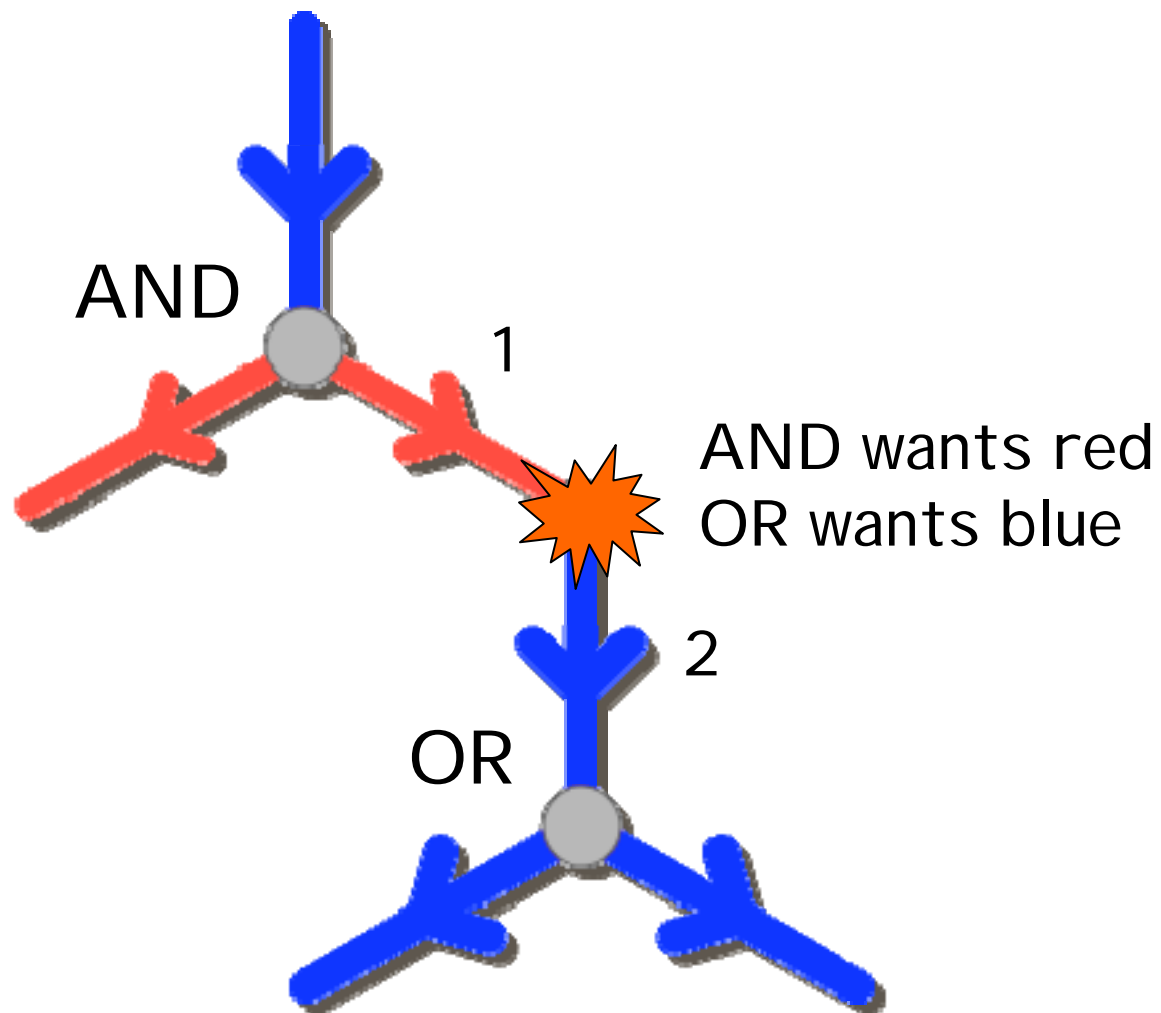
[Hearn & Demaine 2002]

Corollary:  
PSPACE-complete



# Wiring Vertices Together

---



# Red-Blue Conversion

---

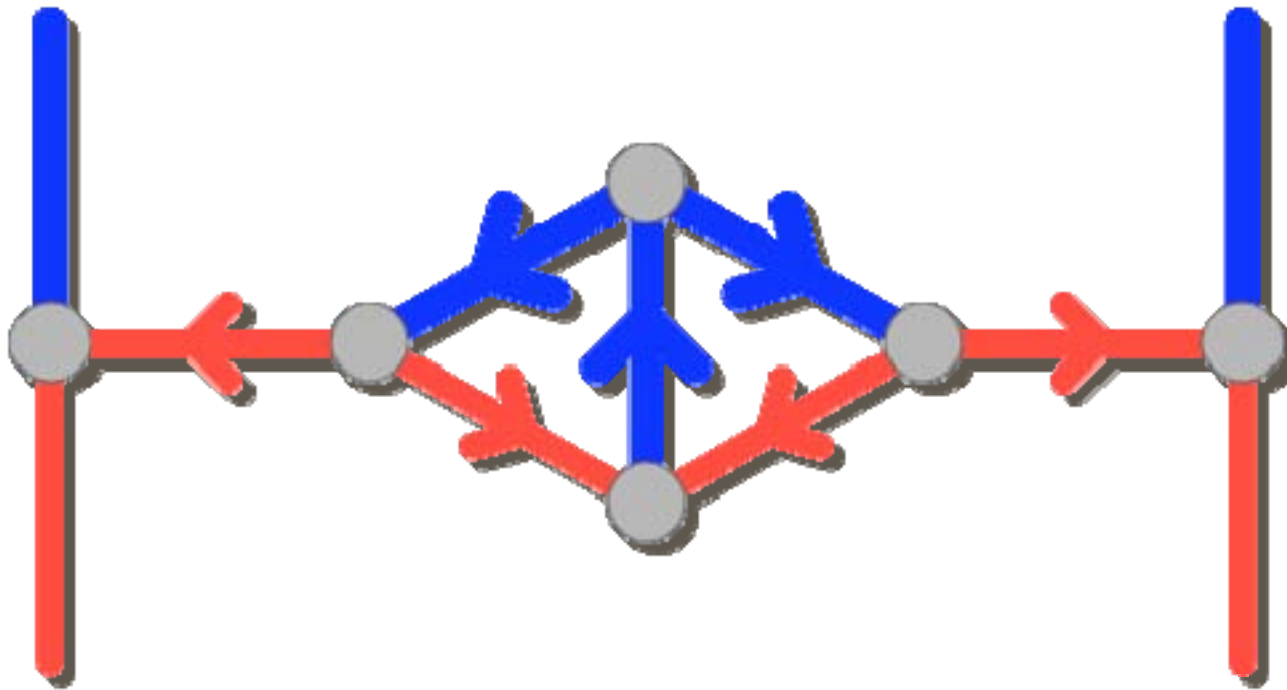


assume an even number of conversions



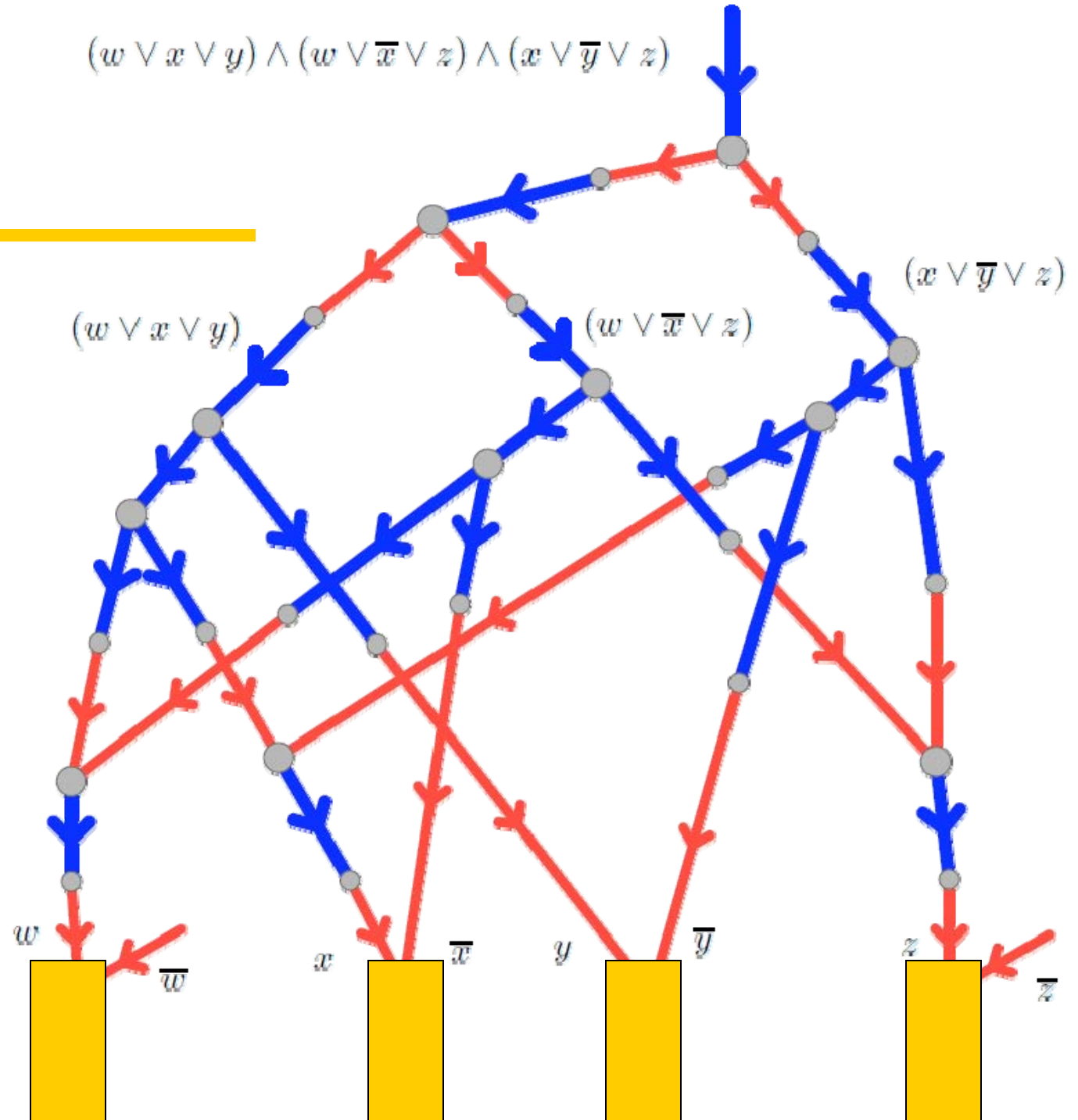
# Red-Blue Conversion

---



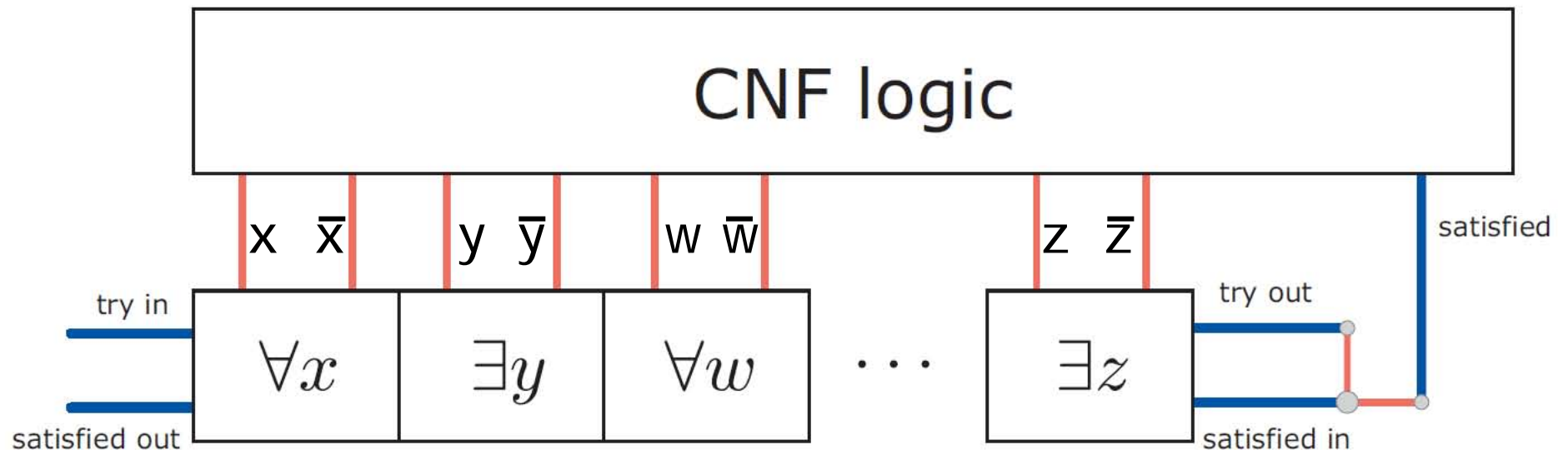
assume an even number of conversions

# Boolean Formulas



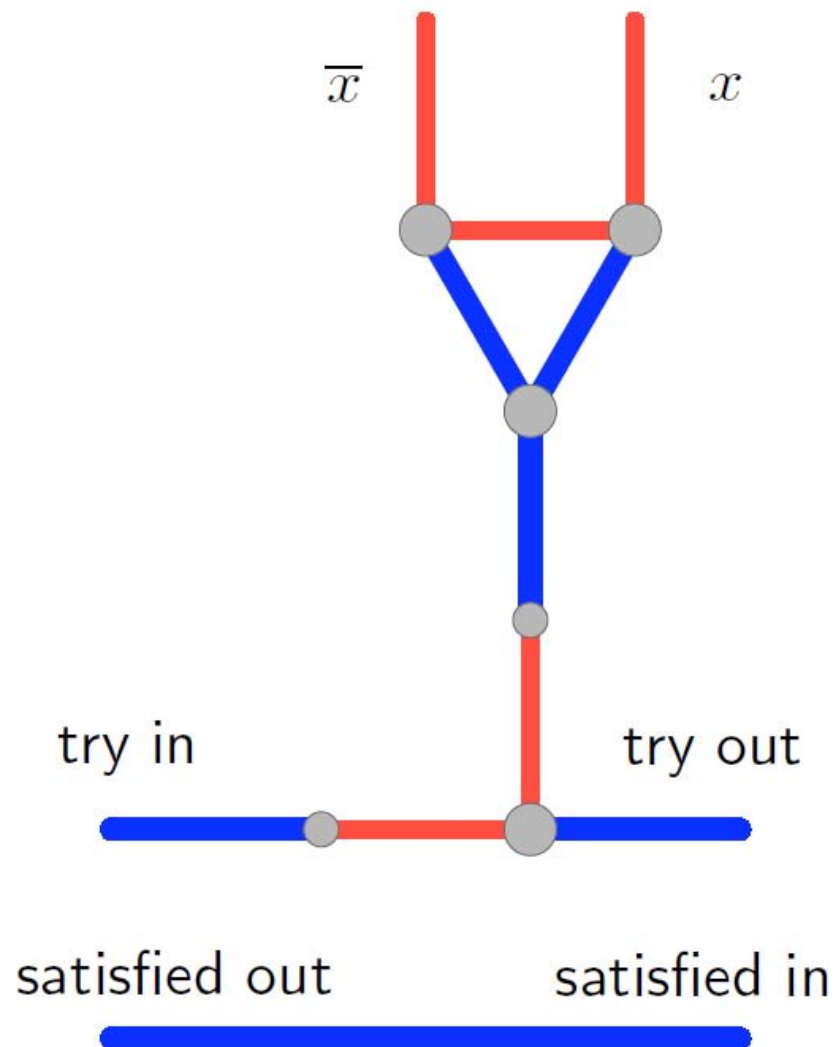
# Quantified Boolean Formulas (QBF)

$$\forall x \exists y \forall w \dots \exists z [(x \vee y) \wedge \dots \wedge (\bar{z} \vee x \vee \bar{w})]$$



# Existential Quantifier

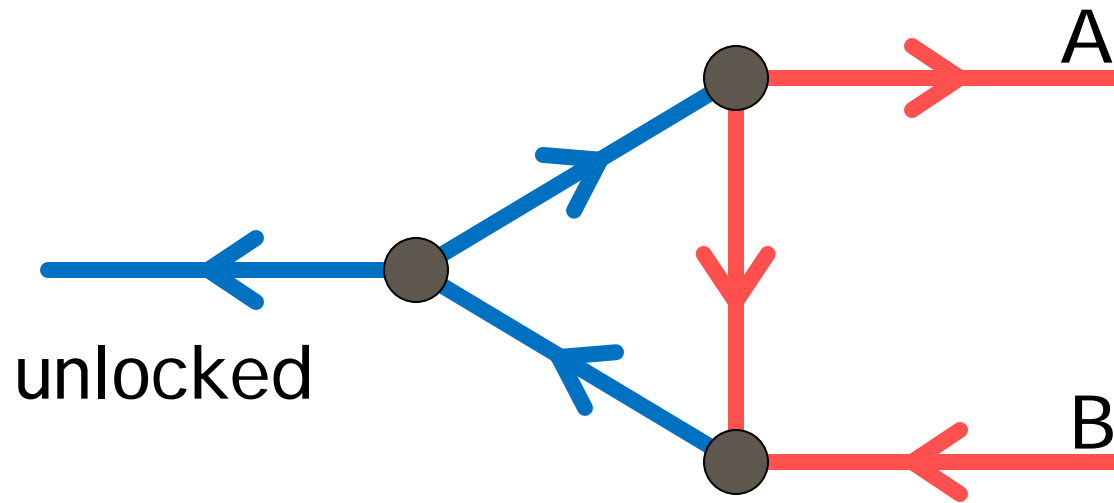
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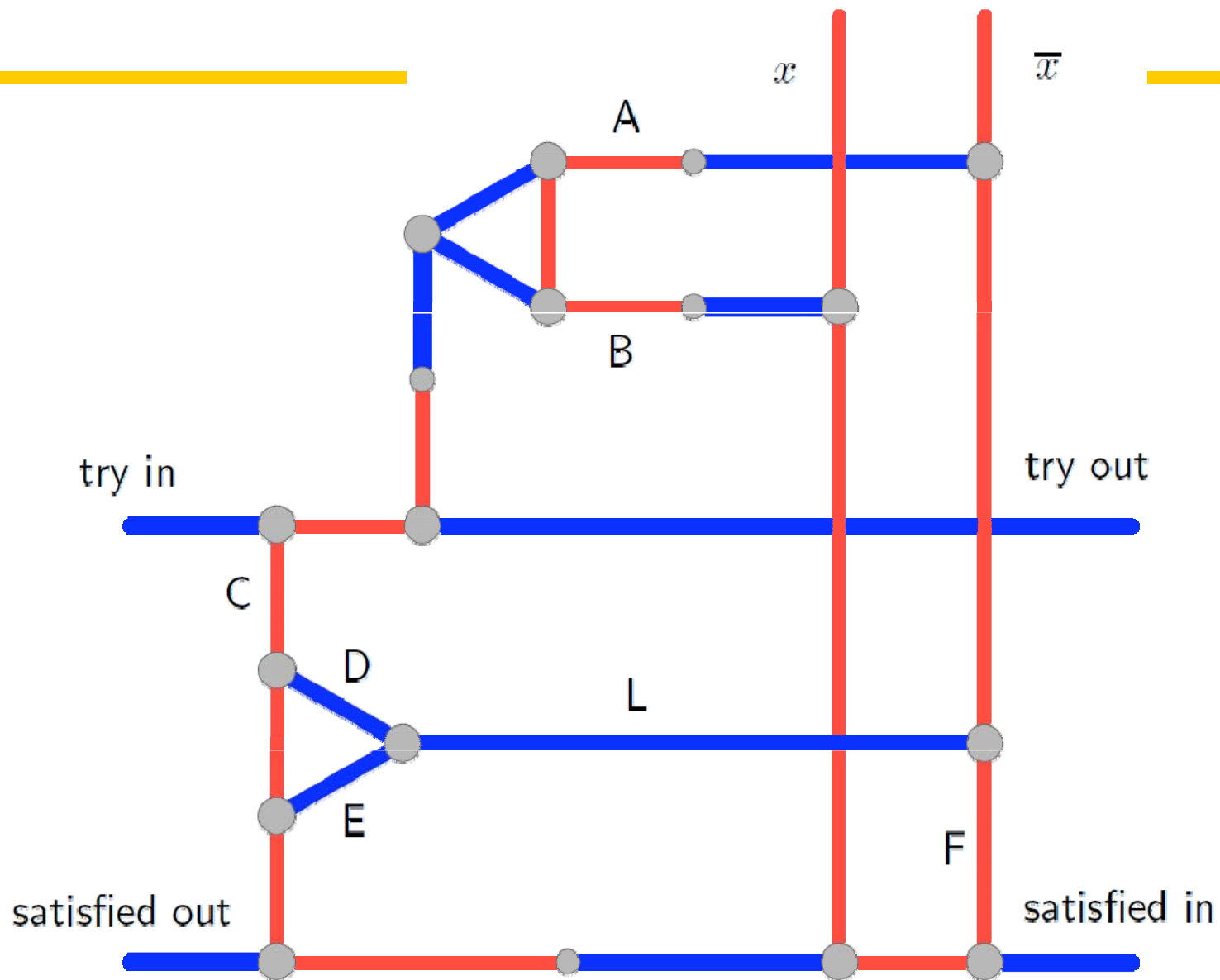
# Latch

---



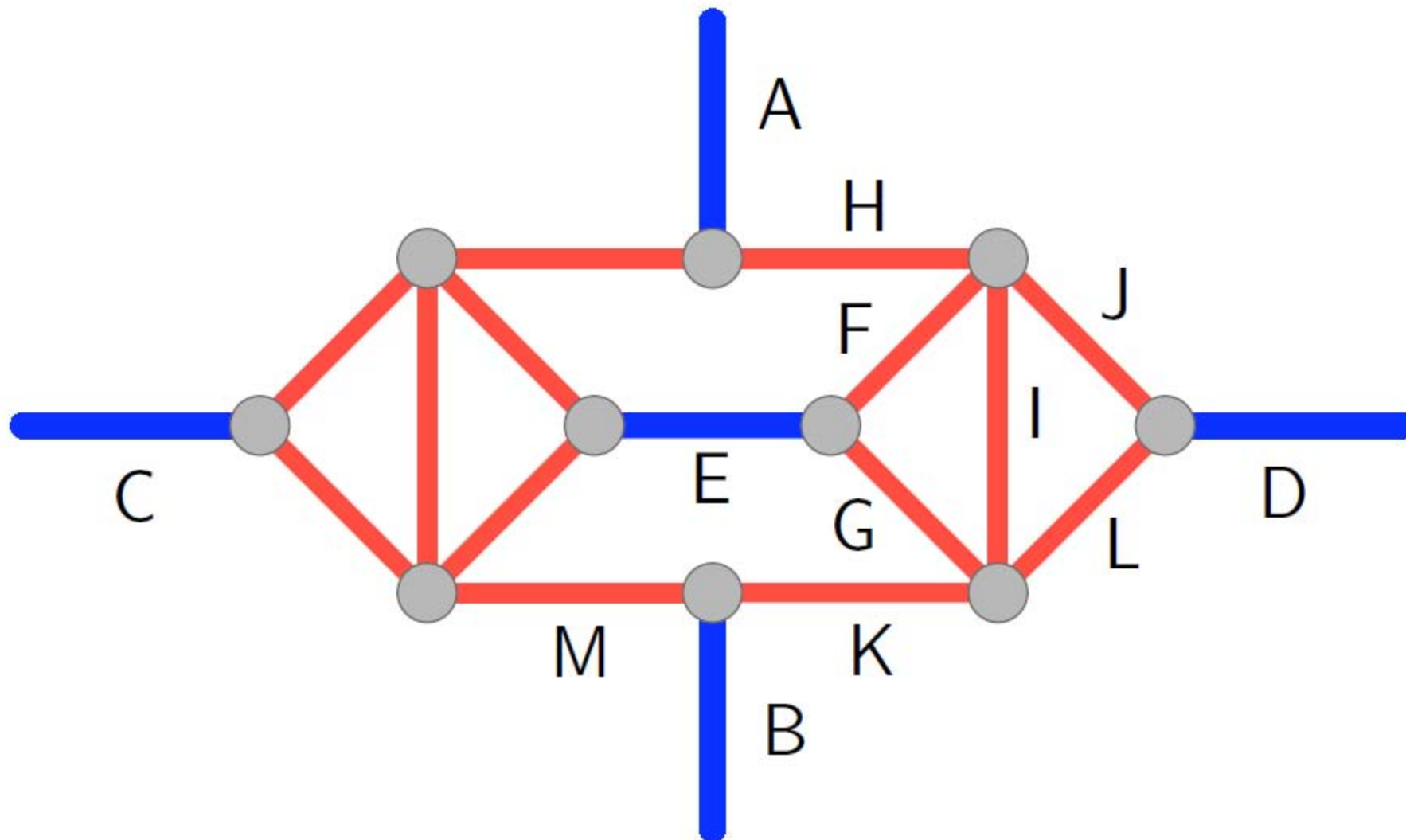
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# Universal Quantifier



# Crossover Gadget

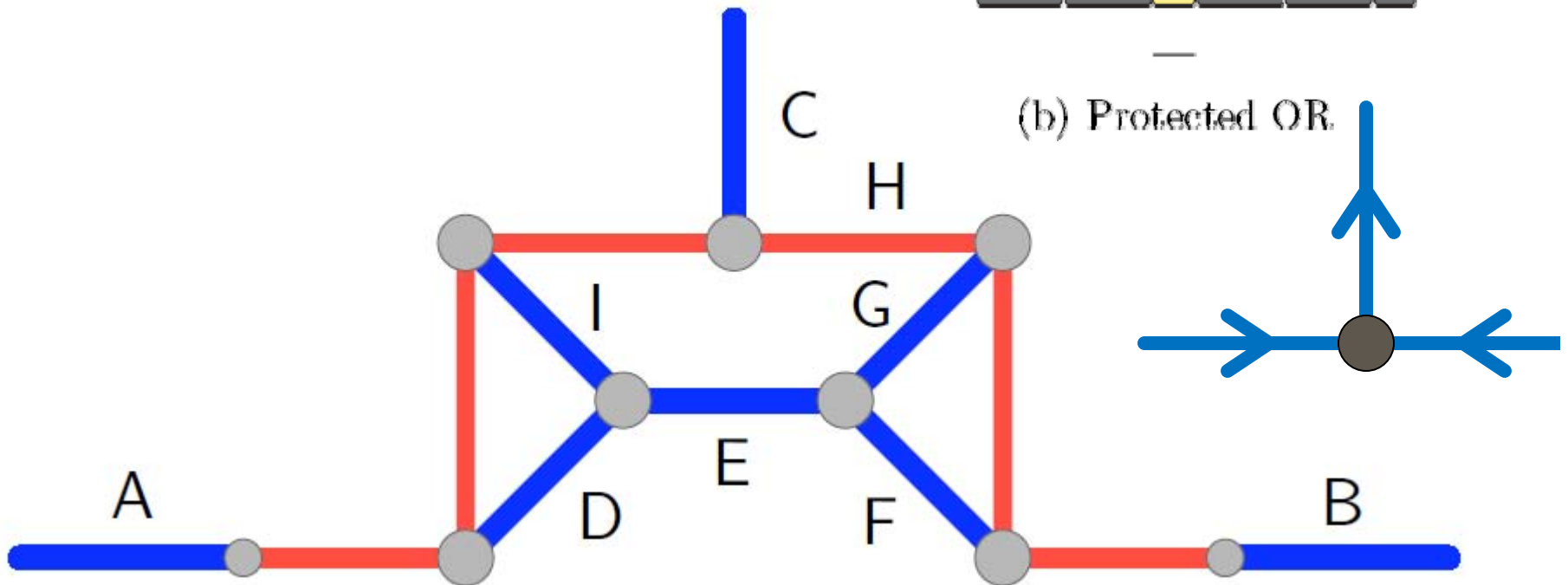
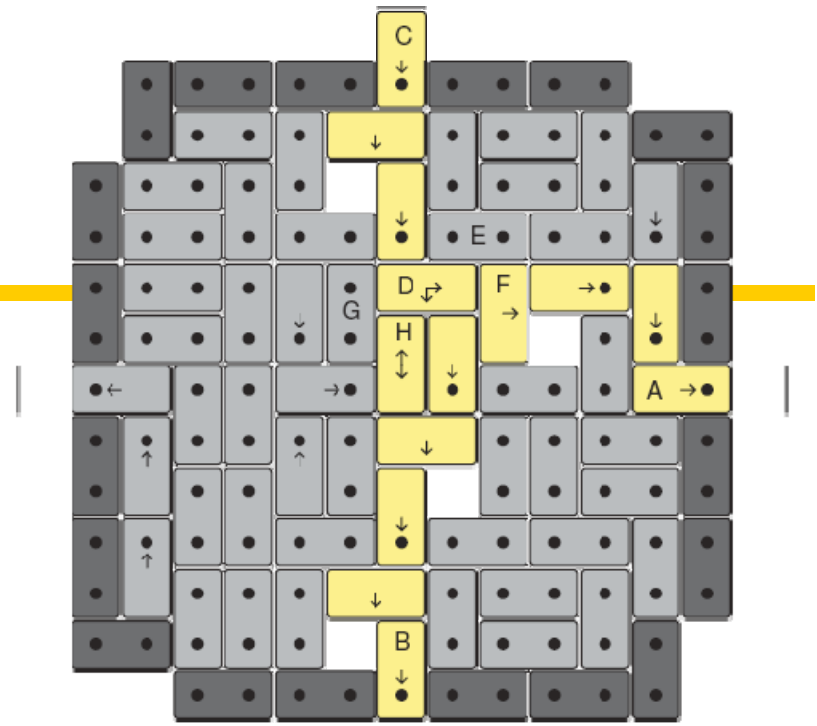
---



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# OR from Protector OR

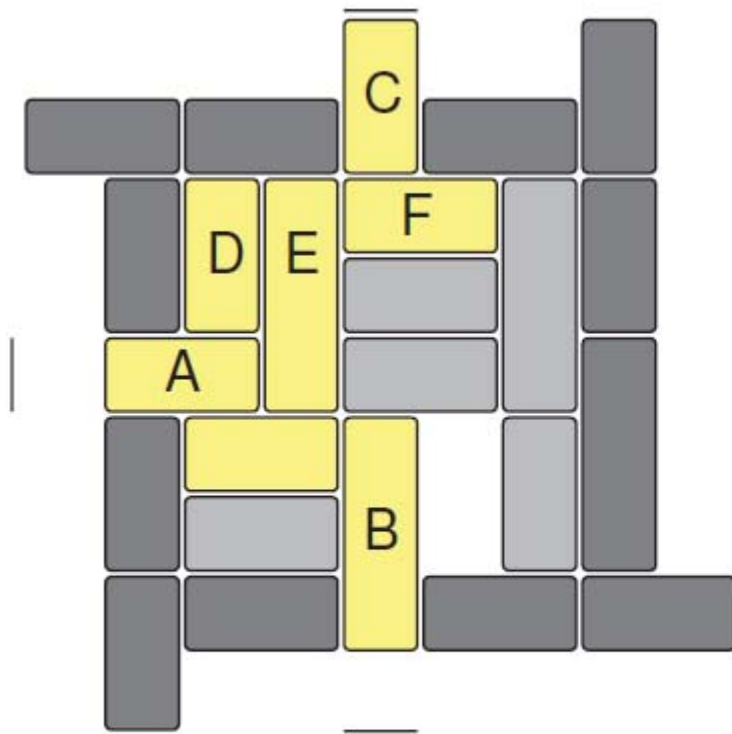


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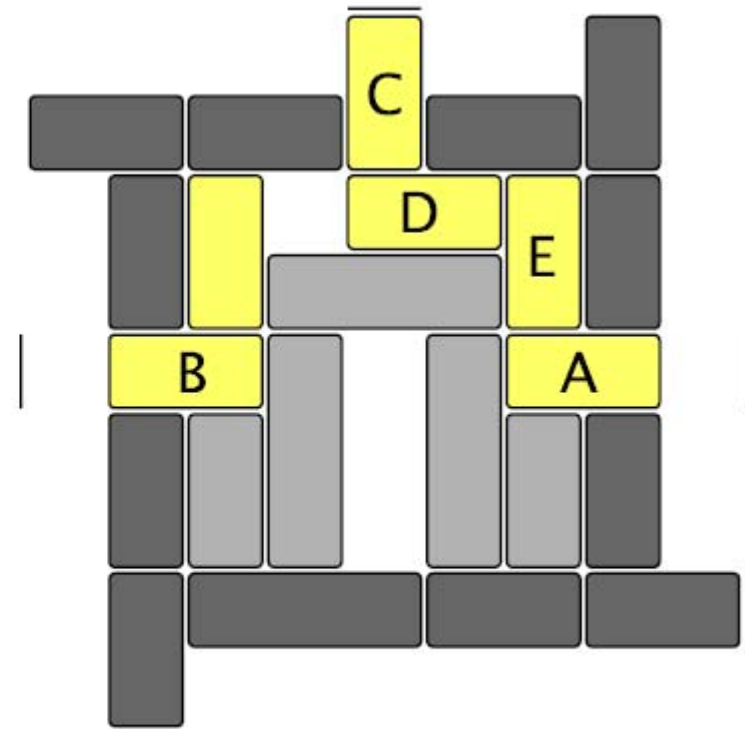
# Rush Hour

[Hearn & Demaine 2002]

---



(b) AND



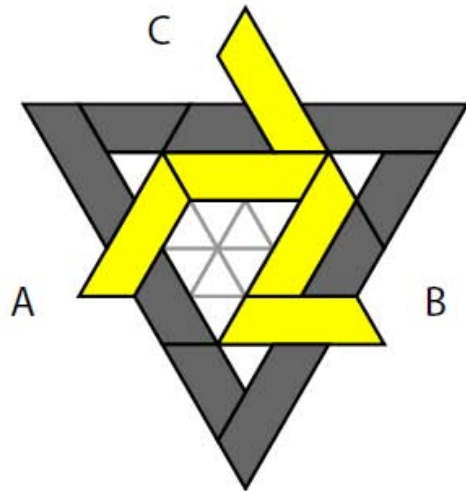
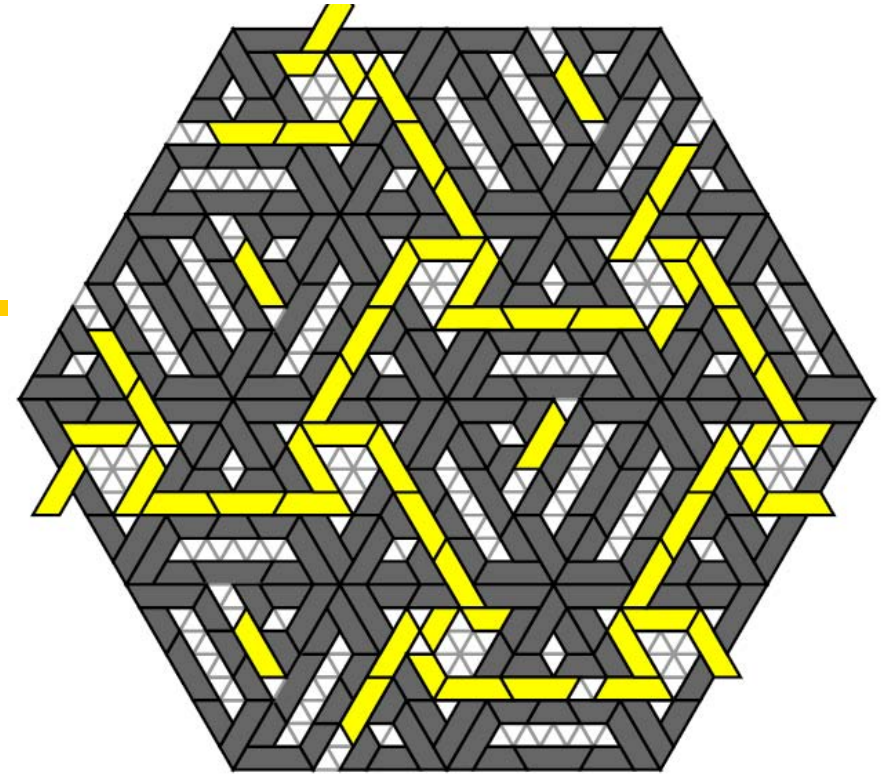
(c) Protected OR

PSPACE-completeness known [Flake & Baum 2002]

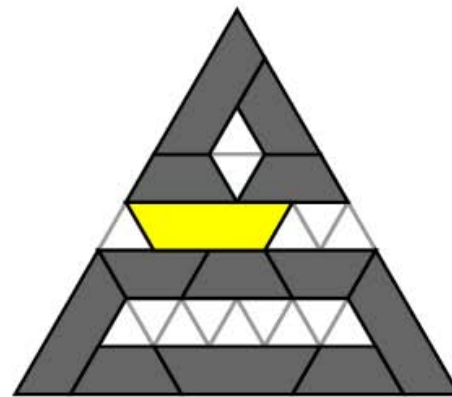
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# Triangular Rush Hour

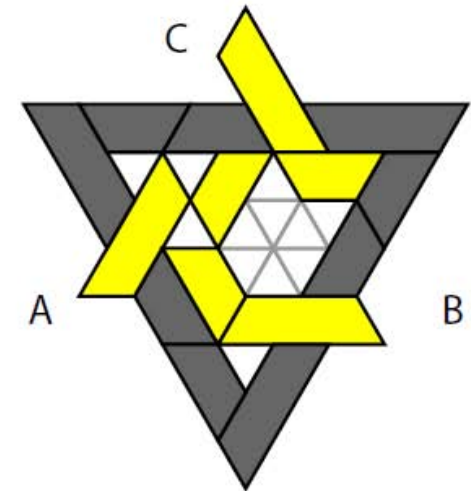
[Hearn & Demaine 2009]



(a) AND vertex



(b) Connector

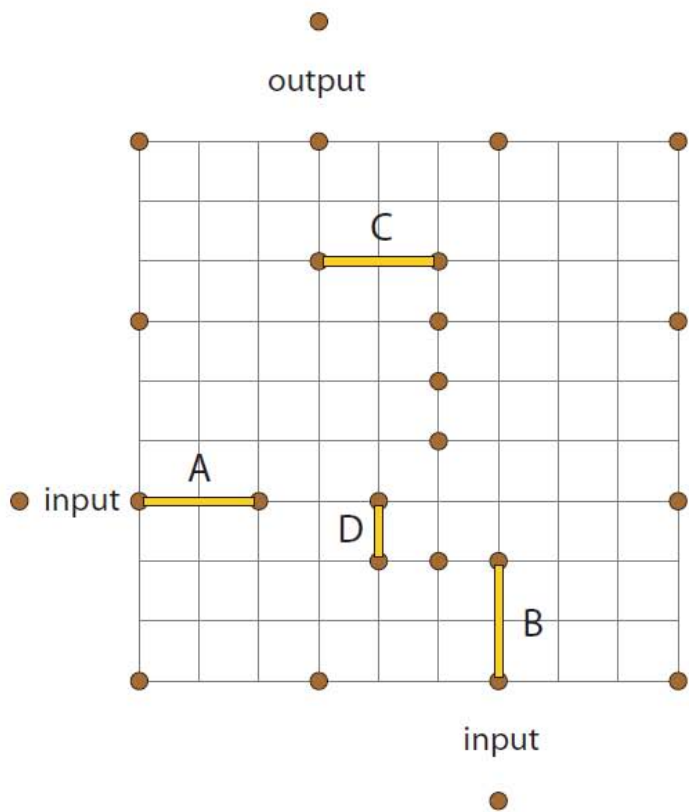


(c) OR vertex

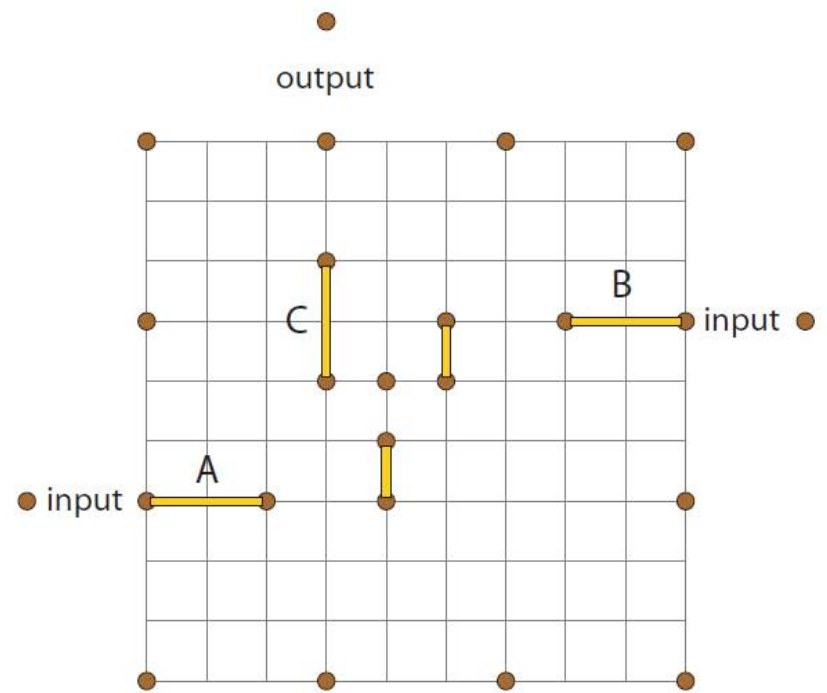
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# Plank Puzzles [Hearn 2004]



(a) AND

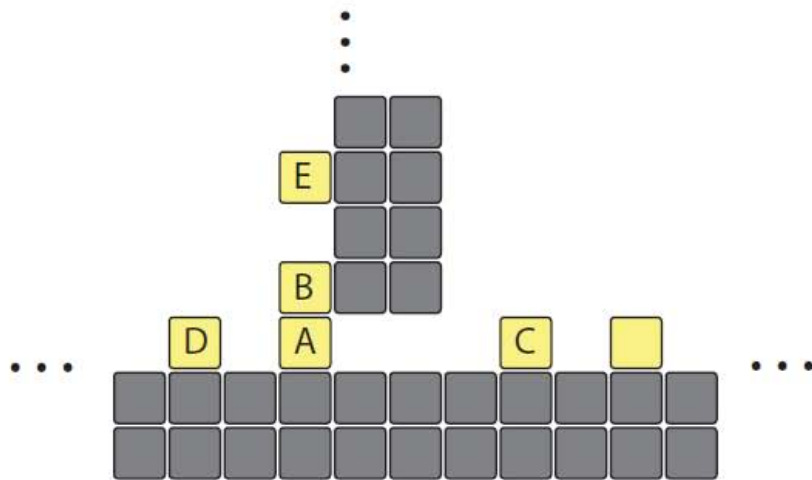


(b) OR

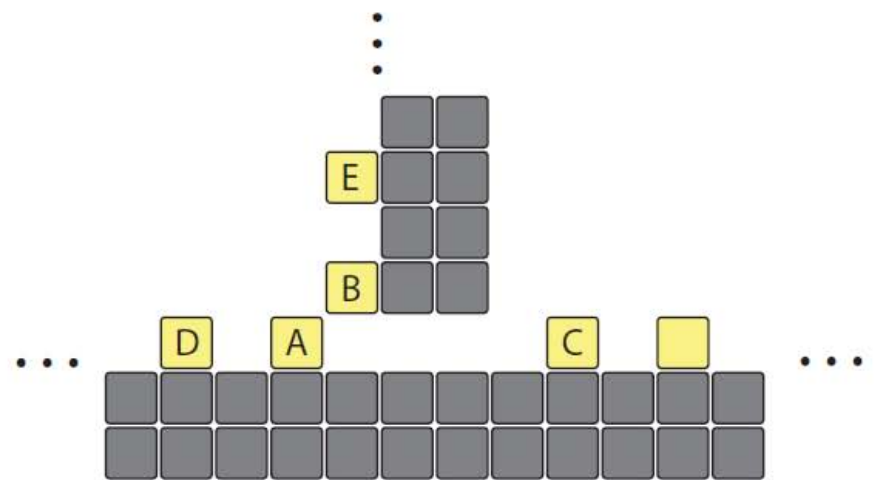
# Sokoban

[Hearn & Demaine 2002]

---



(a) AND



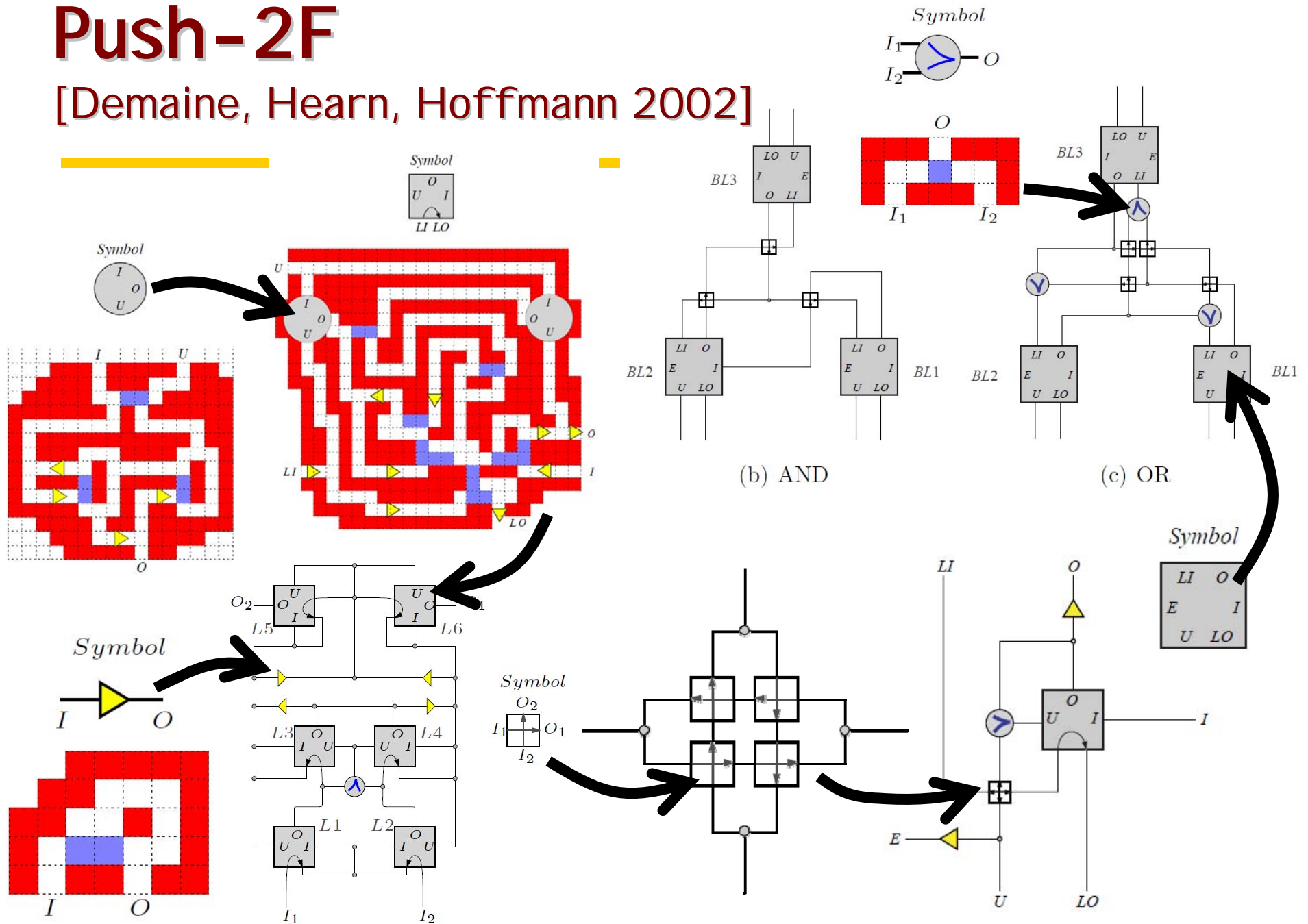
(b) OR

PSPACE-completeness known [Culberson 1998]

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# Push-2F

[Demaine, Hearn, Hoffmann 2002]

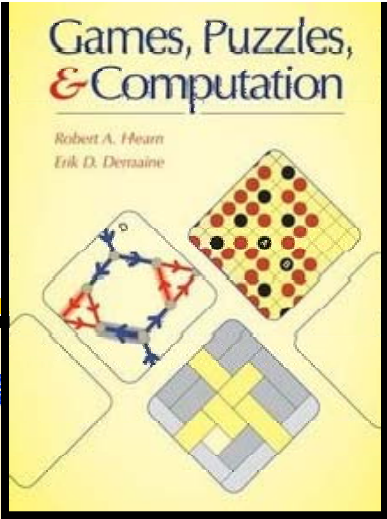


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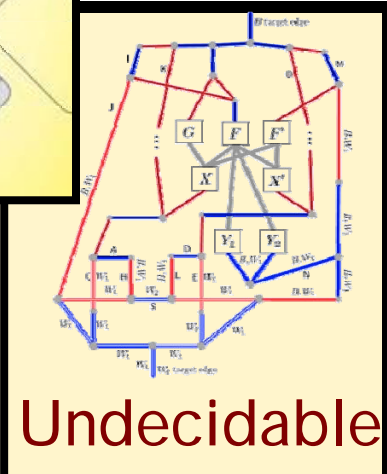
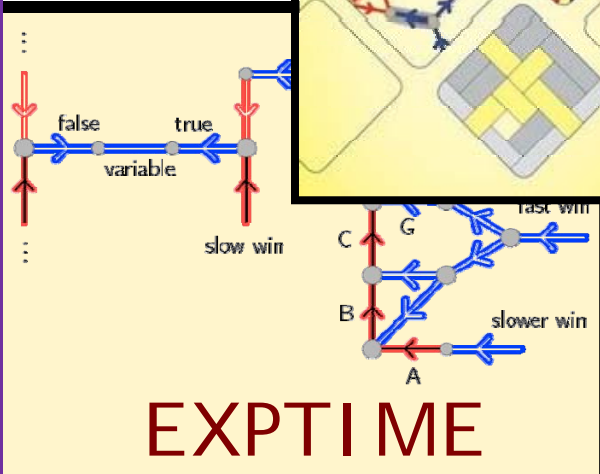
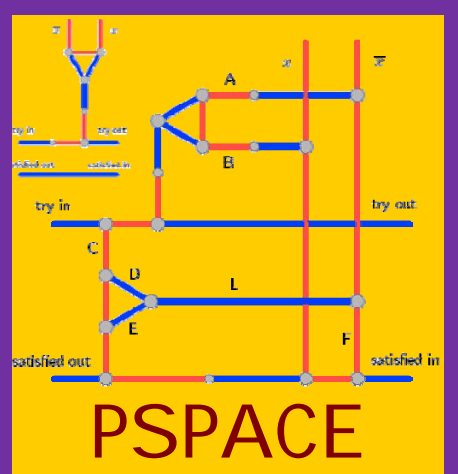
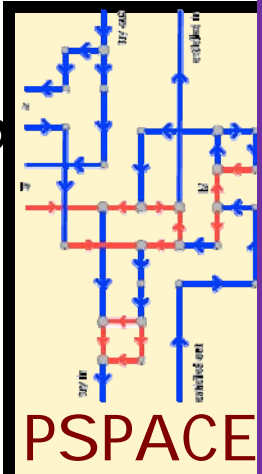


# Constraint Logic

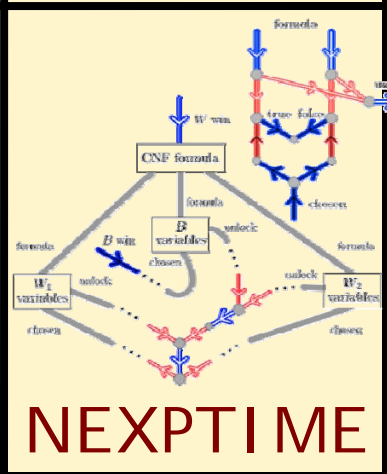
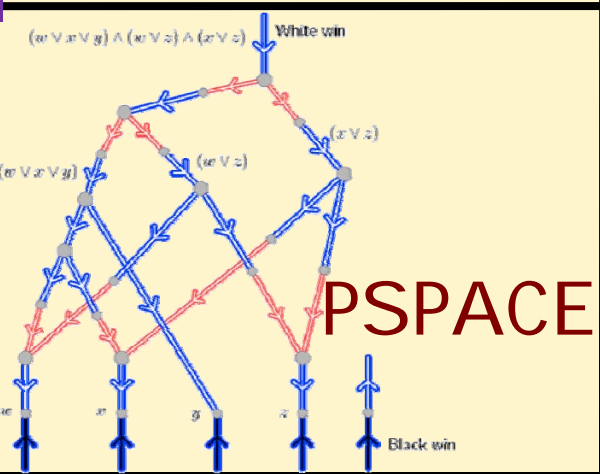
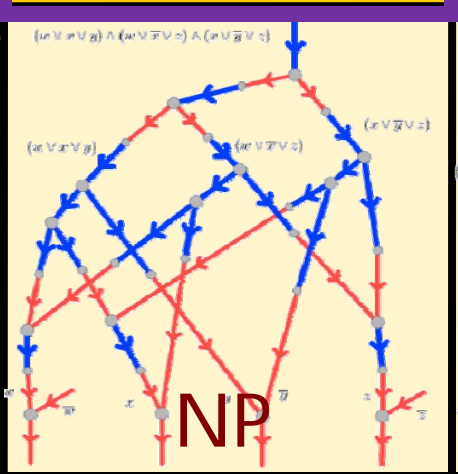
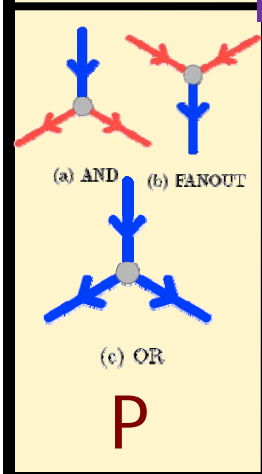
[Hearn & Demaine 2009]



unbounded



bounded



0 players  
(simulation)

1 player  
(puzzle)

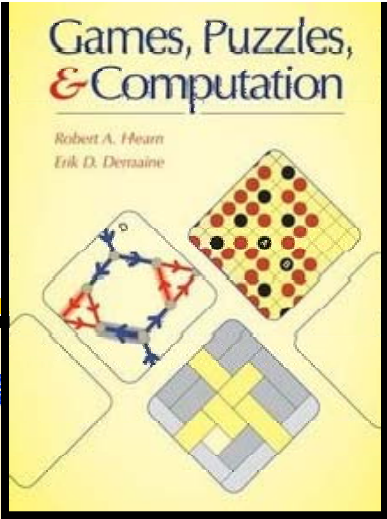
2 players  
(game)

team,  
imperfect info

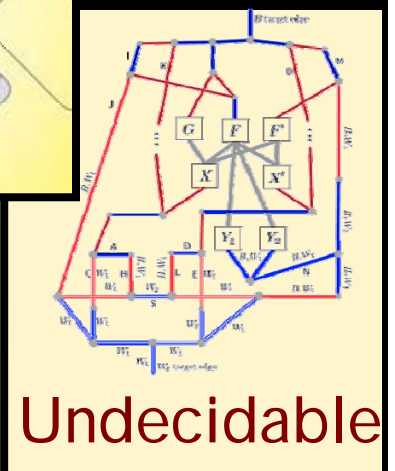
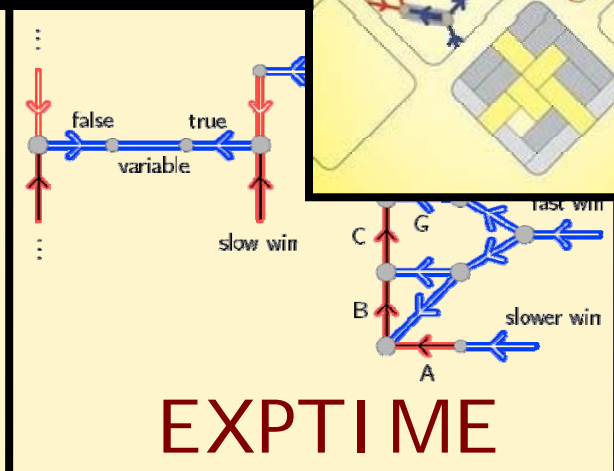
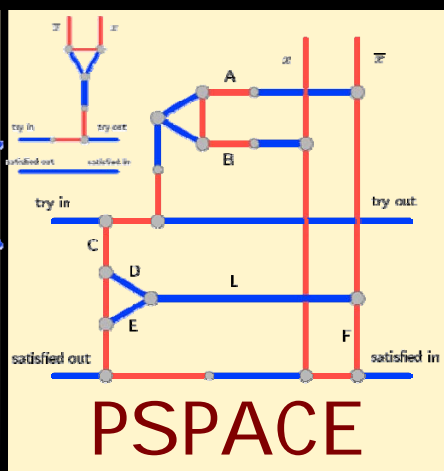
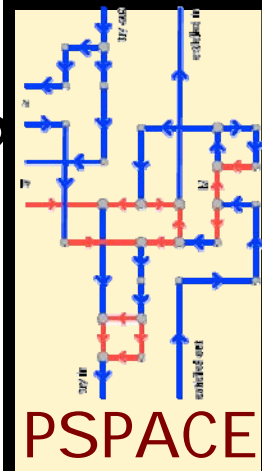


# Constraint Logic

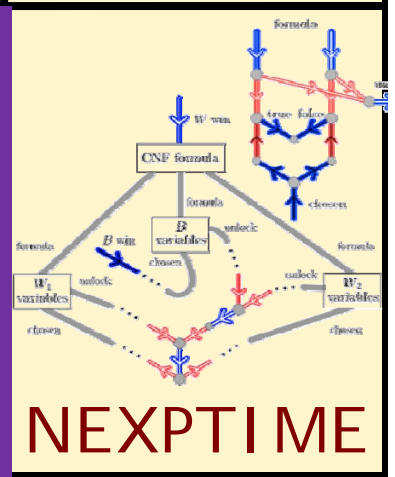
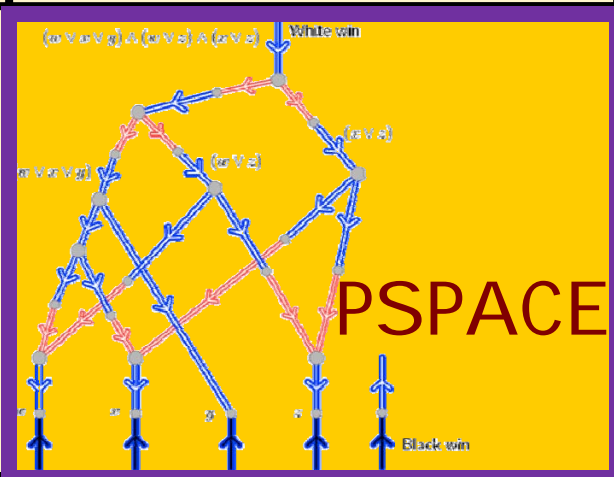
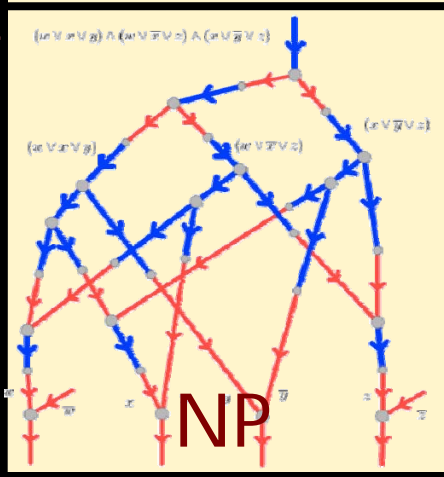
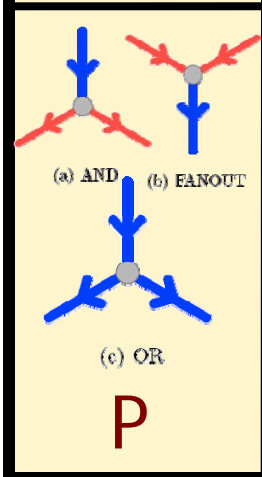
[Hearn & Demaine 2009]



unbounded



bounded



0 players  
(simulation)

1 player  
(puzzle)

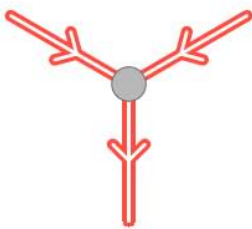
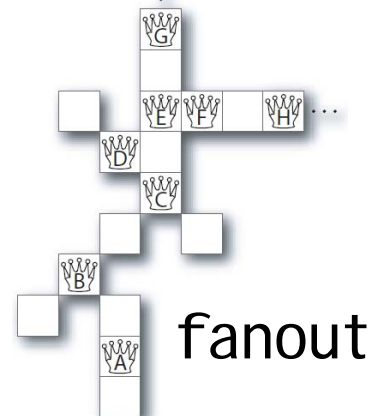
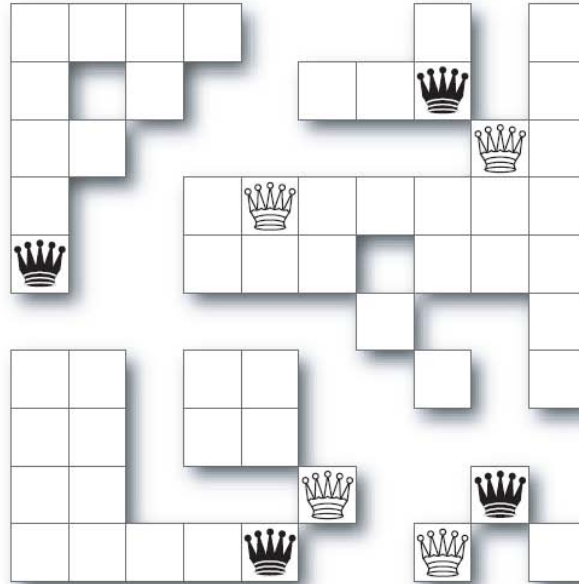
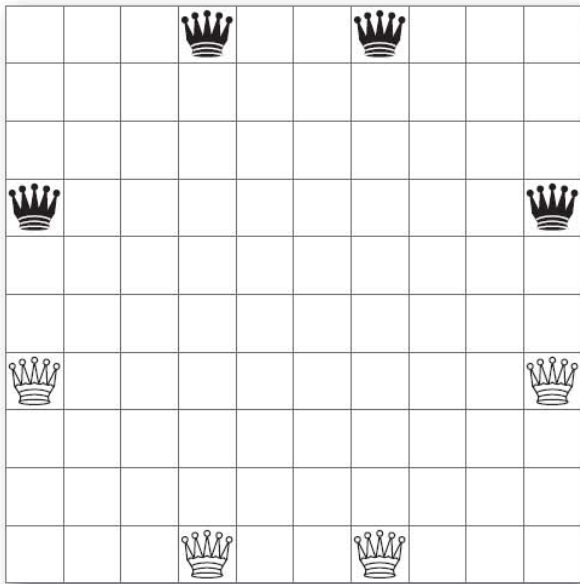
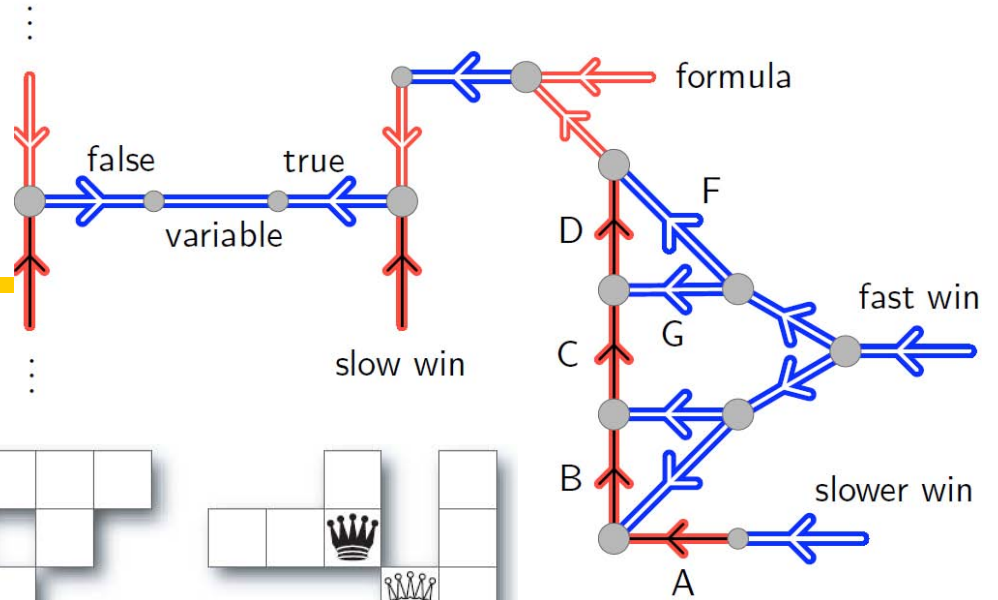
2 players  
(game)

team,  
imperfect info

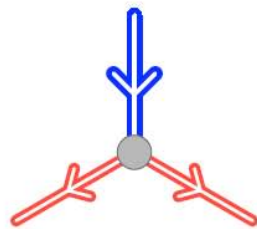
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# Amazons

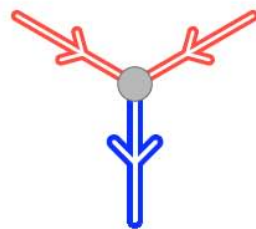
[Hearn 2005]



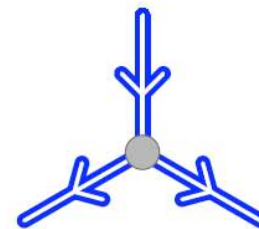
(a) CHOICE



(b) AND



(c) FANOUT



(d) OR



(e) VARIABLE

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<http://ocw.mit.edu>

ES.268 The Mathematics in Toys and Games  
Spring 2010

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