MIT 24.118, Spring 2013

Extra-credit Problem Set 2.5

Grading
There are two parts. Part I is worth 45% and Part II is worth 55% of the overall grade. You are not required to submit answers to all the questions.

Definition
The concept of Nash Equilibrium is defined at http://en.wikipedia.org/wiki/Nash_equilibrium#Definitions.
The "informal definition" will suffice for this problem set. Note that a pair of actions will fail to be a Nash Equilibrium only if one of the players can get a *strictly* better outcome by changing their action. So if the current outcome for players 1 and 2 is (v_1, v_2) then to show this isn't a Nash Equilibrium, you need to find an action for player 1 or 2 that results in an outcome that is strictly better (e.g. an outcome $v_1^* > v_1$ or $v_2^* > v_2$).

PART I (easier, less philosophical)

1. Two friends are deciding where to eat dinner. They have two options: an Italian restaurant or a Mexican restaurant. They would each like to dine with the other person, but they have different preferences about the restaurants. The pay-off matrix for the game is the following:

Player 2 **Player 1**	**Italian**	**Mexican**
Italian	(2,1)	(0,0)
Mexican	(0,0)	(1,2)

The numbers indicate the value of each outcome to each player. For example, the outcome where both players get Italian has a value of "2" for Player 1 and "1" for Player 2.

(Note: no randomized strategies are allowed).

Are there any Nash Equilibria for this game? If there are, list all of them.

2. Two generals (called "Attacker" and "Defender") are planning their war strategy. The Attacker has the choice of attacking City A or City B. The Defender only has enough troops to defend one of the two cities. If an undefended city is attacked, it is captured easily. If a defended city is attacked, the attackers lose. The pay-off matrix is below:

Defender **Attacker**	**Defend City A**	**Defend City B**
Attack City A	(0,1)	(1,0)
Attack City B	(1,0)	(0,1)

(Note: no randomized strategies are allowed).

Are there any Nash Equilibria for this game? If there are, list all of them.

3. Two students are running in opposite directions along the same narrow corridor. Each student starts out running in the middle of the corridor. They notice each other at the same time. Having seen each other, they now face a choice of whether to run on the left or right side. Unless they run on different sides of the corridor, they will collide. This game can be formalized as follows:

Student 2 **Student 1**	**Left**	**Right**
Left	(0,0)	(1,1)
Right	(1,1)	(0,0)

3.(a). Consider the formalization of this game in the pay-off matrix. Are there any Nash Equilibria for this game? If there are, list all of them.

3.(b). How do people solve this problem (or closely related problems) in everyday life? Feel free to make additional assumptions about the problem that were not stated above. Word limit is 150 words.

PART II (Harder, more philosophical)

3. *The Hitchhiker*
A man is stranded in the desert after his plane crashed on the way to a friend's wedding. He has lost all of his belongings, including cellphone and wallet. The man has enough water to make it to the nearest town. However, it will take two days to walk to the town and so he will miss the wedding.

The man walks alongside the road. He knows that cars rarely ever pass through this desert. Fortunately, a car comes by and the driver pulls over. The man tells the driver that he will pay him $100 for a ride to the town. The man has no cash with him, but tells the driver that he will get the money from an ATM in the town.

The driver accepts the offer and drives the man to the town. Privately, the driver mistrusts the man and believes he will run away once he is dropped off at an ATM. But the driver is happy to help the man anyway, even though it means he has to drive a longer route than normal.

The driver drops off the man at an ATM in the town. The man considers whether to pay the driver or to run away. If he runs away, there is no chance that he gets caught. The man does not care at all about helping the driver by paying him $100 and would like the $100 for himself.

3.(a). Consider the decision facing the man. He can either pay or run. Suppose the man follows Causal Decision Theory (CDT), what would he do? Suppose the man follows Evidential Decision Theory (EDT), what would he do?

If you make any assumptions not stated in the problem, you must state them as part of your answer. (Word limit for whole question is 100 words).

The Hitchhiker, Part II
We now change the original "Hitchhiker" story. In the new story, the driver is *selfish*. This means that he will only pick up the man if he is paid enough money to cover the expense of driving a longer route (which is $50). Moreover, he will only pick up the man if he believes that the man will pay him at the ATM.

In the new story, the man still offers $100 to the driver. The man really does have $100 in his account and he values attending the wedding at more than $100. As in the old story, the man does not care about helping the driver and he is able to run away with no chance of being caught.

We consider the choices of the driver and the man. The driver has the choice of whether to *pick up* or *not*. If the man is picked, he then has the choice of whether to *pay* or *run*.

3.(b)

i. Assume that both the man and the driver use EDT to make their decisions. Describe what the outcome will be.

ii. Assume that both the man and the driver use CDT to make their decisions. Describe what the outcome will be.

For both (i) and (ii), explain any assumptions you make that were not part of the original problem. If you think that either decision theory does not give a clear answer, then explain your reasoning. Word limit is 150 words for the whole of (b).

5. *The Majority Coalition Problem*

In this game there are 9 players and the players are deciding on a way to divide up $9,000 between themselves. The rules of the game are as follows. Each player will submit a proposed allocation of the money between all the players (including himself). If a majority (5 or more) of the players submit the same proposal, then that proposal wins and the money is allocated that way. If no proposal has a majority, then the $9,000 disappears and none of the players get any.

The players can communicate with each other for 12 hours before the deadline for submitting their proposals.

After the 12 hours is up, each player goes into a booth and submits his or her own proposal individually. Nobody gets to see anyone else's proposal.

The proposals consist of amounts in dollars (no cents) given to each player. Here are two possible proposals. The first is the "equal" allocation where all players get the same. The second is an "unequal" allocation where players 1-5 get all the money.

Example 1:

Player	1	2	3	4	5	6	7	8	9
Allocation	$1000	$1000	$1000	$1000	$1000	$1000	$1000	$1000	$1000

Example 2:

Player	1	2	3	4	5	6	7	8	9
Allocation	$1800	$1800	$1800	$1800	$1800	$0	$0	$0	$0

As noted above, if a majority of players submitted one of these two example proposals, then the money would be allocated that way.

Assume that the players are *selfish*. That is, they only care about getting the most money possible for themselves and don't care how much anyone else gets.

5.(a). Are there any Nash Equilibria for this game? If there are, give one example. (Note that players must submit one proposal and cannot choose randomly).

5.(b). Consider the scenario where a group of five players all submit a proposal that allocates themselves all the money (see "Example 2" above). Would this be likely to occur in a real-life version of the game? State any assumptions about the players that would lead to this kind of scenario occurring. (Word limit of 150).

5.(c). We will now change the game so that there are only two players. They still have $9,000 to allocate. As before, the money is only allocated if there is a majority. This means that the players will only get the money if they both submit the same proposal.

Consider all of the games in this problem set. That is, the games in questions 1, 2, 3, and the games in both versions of the Hitchhiker story. Which of the games is this new game most similar to? Justify your answer. (Word limit of 100).