24.118 – Paradox and Infinity
Problem Set 4: Zeno's Paradoxes

How this problem set will be graded:

- Assessment will be based on the *reasons* you give in support of your answers, rather than the answers themselves. (Keep in mind that even if it is unclear whether your answer is correct, it can be clear whether or not the reasons you have given in support of your answer are good ones. It is only the latter that will be taken into account.)
- *No answer may consist of more than 150 words.* Longer answers will not be given credit.

Problems:

1. Tortoises of the genus *Gopherus* have been clocked walking at speeds of about 0.1 m/s. This means that a tortoise should be able to complete a 100m race in about 1000s (∼16.67 minutes). In order to get to the 100m mark, however, the tortoise must complete infinitely many tasks: she must reach the 50m mark, then reach the 75m mark, then reach the 87.5m mark, and so forth. (For each $i \geq 1$, she must reach the $100(1 - \frac{1}{2^i})$ mark.) How is this possible? How can a tortoise complete infinitely many tasks (one for each $i \geq 1$) in a finite amount of time (1000s)?

2. Lazy wants to run from A to B. He first stops halfway between A and B and takes a break of one second. He then stops halfway between *that* point and B and stops one second, and so on. Will Lazy ever reach point B?

3. One minute before midnight, you give Fool a dollar. Half a minute later, he gives you two dollars. Fifteen seconds later, you give him a dollar. Seven and a half seconds later, Fool gives you four dollars. And so forth. (For each $i \geq 0$, you give Fool one dollar 2^{-2i} minutes before midnight and he gives you 2^{i+1} dollars $2^{-(2i+1)}$ minutes before midnight.) How much money will you have at midnight?

4. Fool has infinitely many dollar bills, and has labelled each one of them with a different natural number (its 'serial number'). One minute before midnight, he gives you a dollar bill. Half a minute later, he gives you two dollars. Fifteen seconds later, he gives you four dollars. And so forth. (For each $i \geq 0$, Fool gives you 2^i dollars 2^{-i} minutes before midnight.) There is, however, a catch. Each time you receive money from Fool, you are required to put together all your dollar bills, and burn the one with the lowest serial number. How much money will you have at midnight?

5. *Extra Credit: Thomson's Lamp.*
   There is a lamp with a toggle button: press the button once and the lamp will go on, press it again and the lamp will go off. One minute before midnight the lamp is off. Half a minute later, the button is pressed. Fifteen seconds later, the button is pressed again. Seven and a half minutes later, it is pressed again. And so forth. (For each $i \geq 1$, the button is pressed 2^{-i} minutes before midnight.) At midnight, will the lamp be on or off? (Neither? Both?)

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