24.222 Decisions, Games, and Rational Choice Spring 2008

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## **Final Exam**

Give concise answers to each of the following questions.

- 1. A coin is to be flipped. You call it (heads or tails) and if the coin lands on the face you call, you win \$100. There are two coins: one is an ordinary fair coin; the other is a biased coin, but you are not told which way it is biased, or by how much. Do you have any preference as to which coin is the one to be flipped? If you could choose one or the other coin, for a charge of \$1, would you do it? If so, explain why. If not, are there beliefs and values that a rational person might have that would justify preferring one of the two coins?
- 2. There are, as in Newcomb's problem, two boxes which may or may not have money in them. In this case, you must choose just one of the two boxes. A predictor has predicted whether you would choose box A or B, and if she predicted that you would choose A, she put \$300 in that box, while if she predicted that you would choose box B, she put \$200 in that box. Most people choose box A, but a significant number choose B, and the predictor has proved very reliable in both cases. (She is about 90% accurate when predicting A, and the same when predicting B.). What would you choose in a situation like this? Do you think there would be conditions under which it would be rational to choose B?

3. Consider this game. What are the Nash equilibrium strategies (pure and mixed), and what are their expected payoffs? If you were playing the game (say as the row player) what strategy would you choose, assuming that it is common knowledge that both players are rational? How would you play if the game were to be played 10 times, with the outcome of each round revealed before the next round? (Your payoff in the iterated game is the sum of the utilities for the outcomes of the different rounds.)



4. Consider the game represented by the tree below. (The dotted line indicates thatA's second move is made independently of B's move). Draw the matrix for the normal or strategic form of the game. If you are player B, how should you play (assuming A gives you the chance to move)? Assume it is common belief, at the start of the game, that you are both rational, and so will act to maximize expected utility.



5. In a simple sender-receive game, there are twoplayers (plus nature, or chance), Sender and Receiver. Sender knows the result of a chance move, but receiver does not. Sender can send a message to receivere, who then can make a choice that will determine the outcome for both players, as a function of the result of the chance move. The problem is to determine, in a given such game, what message S should send, if any, and what R should do on receiving one or another message.

Consider the following sender-receiver game, where there are three possible outcomes of the chance move, each with a a prior probability of one third. The numbers in the upper right corrners represent the utility of R, and the numbers in the lower left corners are the utilities for sender.



If you were S, what would you say, in the three possible circumstances? If you were R, how would you respond to any of the messages?

Suppose the game were played ten times, with an independent chance move each time, and the result revealed after each round. Would this make a difference in what messages you would send (if you were S) or to how you would respond (if you were R)? 6. There are three candidates, A B and C, and 100 voters. The voting system is as follows: each voter submits a preferential list. If there is one candidate listed first by a majority of the voters, that is the winner. If not, the candidate with the fewest first place votes is eliminated, and the second place choices of the voters who voted first for the eliminated candidate are added to the votes of the other two. (That is, suppose a voter voted C first and B second, and that C was eliminated on the first round. Then that voter's vote goes to B.).

Give an example of a configuration of preferences (e.g. "31 prefer C to B, and B to A, 17 prefer A to B and B to C, etc.) which is such that, if the configuration were known, some voters would have a motive to vote differently from their actual preferences. Who do you think would win, in your example, assuming that all voters know the configuration of preferences, and vote rationally, given their preferences?

- 7. Consider an election with just two candidates and about 100,000 voters. The pre-election polls are too close to call, but the probability that the election will be decided by just one vote are extremely small. Assume that you strongly prefer one of the two candidates, but that there is a small but non-negligible cost to voting. Would it be rational for you to vote in such circumstances? If so, explain the values and beliefs that you have that make it rational.
- 8. Some economists tend to assume that one is irrational if one acts in the interest of others when those interests conflict with one's own. Some moral theorists argue that it is irrational to act immorally, and that morality sometimes requires one to act in the interest of others when those interests conflict with one's own. Is there a plausible conception of rationality according to which either of these views is right? Is there a plausible conception of rationality that can permit both altruistic and purely self-interested actions to be rational? Explain your answers to these questions.