3.00 Thermodynamics Fall 2002
Problem Set 3 Solutions

Solution 3.1

Since the system is adiabatic, there is no exchange of heat across the boundaries. There is also no exchange of mass across the boundaries as mass can carry heat. The first law takes the following form

$$dU = dW \tag{1}$$

By definition, the heat capacity for a material is $C_v = \left(\frac{dq}{dT}\right)_V = \left(\frac{dU}{dT}\right)_V$. This leads to:

$$dU = C_v dT \tag{2}$$
$$\text{where} \tag{3}$$
$$C_v = n\overline{C_v} \tag{4}$$

The minimum work needed to increase the temperature is given by:

$$\int dW = \int_{T_0}^{T_1} n_0 \overline{C_v} dT \tag{5}$$
$$\Delta W = n_0 \overline{C_v}(T_1 - T_0) \tag{6}$$

For an ideal gas: $\overline{C_p} - \overline{C_v} = R$

$$\Delta W = \frac{P_0 V_0}{R T_0} \frac{5R}{2} (T_1 - T_0) \tag{7}$$
$$\Delta W = \frac{1.012 \cdot 10^5 Pa \cdot 72 m^3 \cdot 5}{2} (\frac{298K}{285K} - 1) \tag{8}$$
$$\Delta W = 960kJ \tag{9}$$

For a well insulated room no heat passes through the boundaries, the volume is fixed, and the pressure is fixed. Therefore, as the temperature changes the number of moles of gas in the room adjusts accordingly. This means that the system does some work on the surroundings as molecules leave the system. Imagine that the system has a small orifice attached to a balloon. When the gas molecules leave the system via the orifice they enter the balloon and so do work on the surroundings. This must be taken into account in the integration by using C_p instead of the work-less heat capacity, C_v. In addition, the heat capacity of the gas changes as a function of temperature, $C_p = n(T; P_0, V_0)\overline{C_p}$.

$$\int dU = \int dW = \int_{T_0}^{T_1} \frac{P_0 V_0}{RT} \overline{C_p} dT \tag{10}$$
$$\Delta W = \frac{P_0 V_0}{R} \overline{C_p} \log \frac{T_1}{T_0} \tag{11}$$
$$\Delta W = \frac{1.012 \cdot 10^5 Pa \cdot 72 m^3 \cdot 7}{2} \log \frac{298K}{285K} \tag{12}$$
$$\Delta W = 1137kJ \tag{13}$$

In comparison, it is apparent that the adiabatic temperature increase requires less energy than the constant volume and pressure temperature increase. This problem is not straightforward. Sommerfeld criticizes Emden for dropping a reference energy in the integration while he himself neglects to take into account the temperature dependence of the heat capacity of the system.[1]

Solution 3.2

One mole of an ideal gas has 2 state variables. The four degrees of freedom (n, P, V, and T) for an arbitrary gas are decreased by the constraints, $n = 1$ and $PV = nRT$.

We can write $\overline{U}$ in terms of each combination of two independent variables: $\overline{U}(T,\overline{V}), \overline{U}(T,P), \overline{U}(P,\overline{V})$. The differential form for the molar internal energy of an ideal gas is given by

$$d\overline{U} = \overline{C_v}dT \tag{14}$$

Integration gives:

$$\overline{U}(T) = C_v(T - T_0) = U(T,\overline{V}) = U(T,P) \tag{15}$$

Substitution using the ideal gas law gives:

$$\overline{U(P,\overline{V})} = \frac{\overline{C_v}}{R}(\overline{V}P - \overline{V}_0P_0) \tag{16}$$

Solution 3.3

Because the room is adiabatic and work is being transfered to it at constant rate, then (in the absence of phase transitions in the room) the room temperature must increase as work is transfered to it, regardless of whether the door is open or closed.

Depending on the precise location of the thermometer, the temperature may initially decrease. However, at long times the two curves (open and closed) should look roughly the same.

One could argue that the temperature for the open fridge may be slightly higher at long times because the heat pump would become increasingly less efficient—and also because the heat capacity of the room is slightly smaller without the contribution of the material that is within the fridge. I think the two curves probably converge to the same monotonically increasing value demonstrates sufficient understanding.

Solution 3.4

Change in internal energy of system =Heat flow into Pb and water

$$|mgh| = C_{\text{Pb}}(T_{final} - 20) + C_{\text{H}_2\text{O}}(T_{final} - 20)$$

$$T_{final} = 20 + \frac{|mgh|}{C_{\text{Pb}} + C_{\text{H}_2\text{O}}}$$

[1] A. Sommerfeld, *Thermodynamical and Statistical Mechanics*, Academic Press Inc., New York, 1965. and R. Emden, "Why do we have Winter Heating", Nature, **141**, 908 (1938).

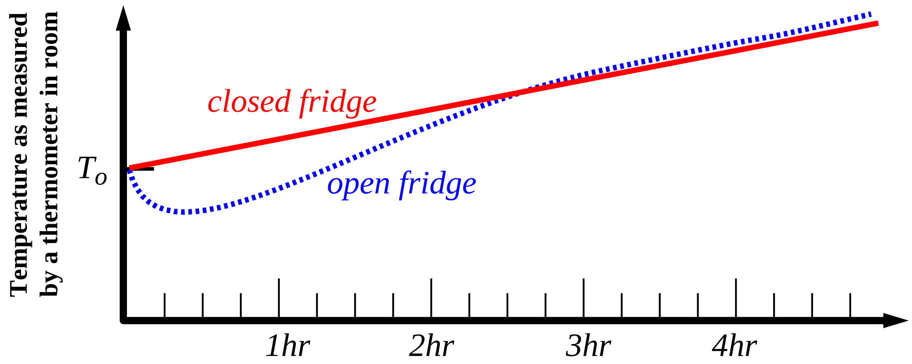

Figure 1: Schematic Solution for Question 2

The second part is tricky. The answer is that the temperature will be lower. The center of mass of the water will raise and so part of the potential energy of the lead weight will be converted to potential energy of the water. $|mgh|$ would be decreased by an amount corresponding to the raise of center of mass of the water.