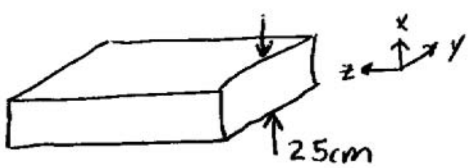
3.044 MATERIALS PROCESSING

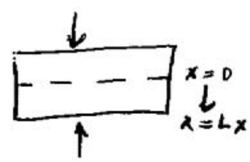
LECTURE 6

Ex. 1: glass fiber (ceramic)

Ex. 2: plasma spray (ceramic and metal)

Ex. 3: hot rolling steel slabs (metal)





look at iron-carbon (steel) phase diagram, red hot is about 900 – 1000°C,
need to heat into gamma field to make it soft and eliminate ceramic carbide phase

Problem Statement: How long in furnace to heat the block to 900°C

Geometry: rectangular, long on z, long on y, $L_x = 12.5\text{cm}$
$\Rightarrow$ 1-D slab, half-thickness $L_x = 12.5\text{cm}$

Boundary Conditions:

1. $T_0 = 25°\text{C}$
2. @ x = 0 symmetry: $\frac{\partial T}{\partial x} = 0$
3. @ x = L_x convection: $q_{\text{conv}} = h(T - T_f)$, $h = 100\frac{\text{W}}{\text{m}^2\text{K}}$ and $T_f = 1000°\text{C}$

Governing Equation: $B_i = \dfrac{hL}{k}$

$h = 100,\ L = 0.125\text{m},\ k = 35\frac{\text{W}}{\text{mK}}$

$B_i = \dfrac{100 \cdot 0.125}{35} = 0.36 \Rightarrow$ must deal with both **conduction and convection**

Date: February 27th, 2012.

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}} \qquad \text{With B.C @ x} = L_x: \quad -k\frac{\partial T}{\partial x} = h(T - T_f)$$

Graph for sheet @ x = 0: $\dfrac{T - T_f}{T_i - T_f} = f(F_0)$

$T = 900,\ T_f = 1000,\ T_i = 25 \approx 0 \Rightarrow \boxed{\Theta = 0.1 \text{ target}}$

From Graph: $\quad F_0 \approx 8 = \dfrac{\alpha t}{L^2},\ L = 0.125\text{m},\ \alpha = \dfrac{k}{\rho c_p} = \dfrac{35}{7700\left[\frac{\text{kg}}{\text{m}^3}\right]\ 0.8\left[\frac{\text{kJ}}{\text{kg K}}\right]}$

Solution: $\boxed{t = 22,000s \approx 6 \text{ hours}}$

How to decrease time?: $\dfrac{T - T_f}{T_i - T_f} = f(k,\ c_p,\ \rho,\ t,\ L_x,\ h)$

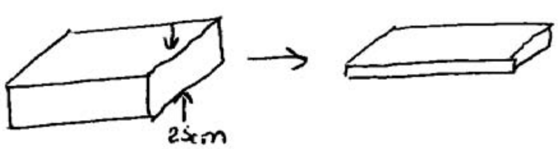
1. thinner L $\rightarrow$ constrained by casting
2. ~~higher h (fluid) $\rightarrow$ molten metal, salt~~
3. hotter T_f $\rightarrow$ high energy, doesn't drastically change time
4. $\boxed{\text{preheat } T_i} \Rightarrow$ vertical integration, combine casting and rolling temperatures such that steel is still hot from casting as it goes through hot rolling process

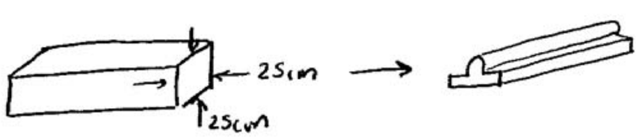
Solution: increase T_i to 500°C

$\Theta = 0.2,\ F_0 \approx 5 - 6,$

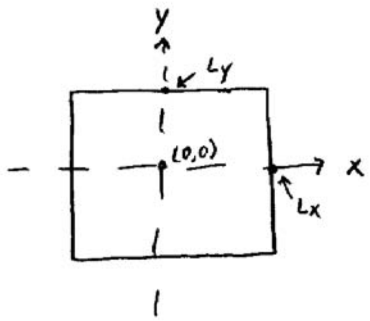
$\boxed{t \approx 4 \text{ hrs}}$

<u>**Ex. 3b:**</u> Multidimensional





Geometry: z long

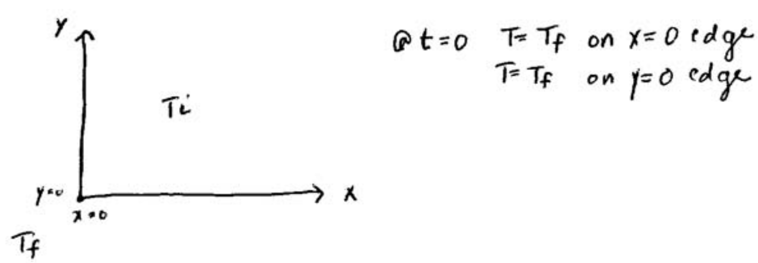


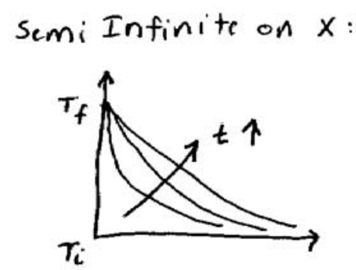
Governing Equation: $\frac{\partial T}{\partial x} = \alpha \nabla^2 T = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$

<u>Superposition Principle:</u>

If there is only 1 dimensionless temperature for all dimensions (all dimensions share the same boundary conditions), then

$$\boxed{\Theta(x, y, t) = \Theta(x, t)\Theta(y, t)}$$





$$\Theta(x, t) = erf \frac{x}{2\sqrt{\alpha t}}$$
$$\Theta(y, t) = erf \frac{y}{2\sqrt{\alpha t}}$$

Full Solution:

$$\boxed{\Theta(x, y, t) = \text{erf}\frac{x}{2\sqrt{\alpha t}}\text{erf}\frac{y}{2\sqrt{\alpha t}}}$$

In 3 dimensions:

$$\Theta(x, y, z, t) = \text{erf X erf Y erf Z}$$

Problem Statement: How long does it take to heat to 900°C in 2 dimensions?

Solution: t when $\Theta = 0.1$ @ x = 0 and y = 0

$$\Theta(x,t) = f(F_{0,x})$$
$$\Theta(y,t) = f(F_{0,y})$$
$$\Theta(x,y,t) = \Theta(x,t)\Theta(y,t)$$

By Symmetry:

$$F_{0,x} = F_{0,y}$$
$$\Theta(x,t) = \Theta(y,t) = \sqrt{0.1} = 0.32$$
$$F_0 = 4$$
$$\boxed{t \approx 3\text{hrs}}$$