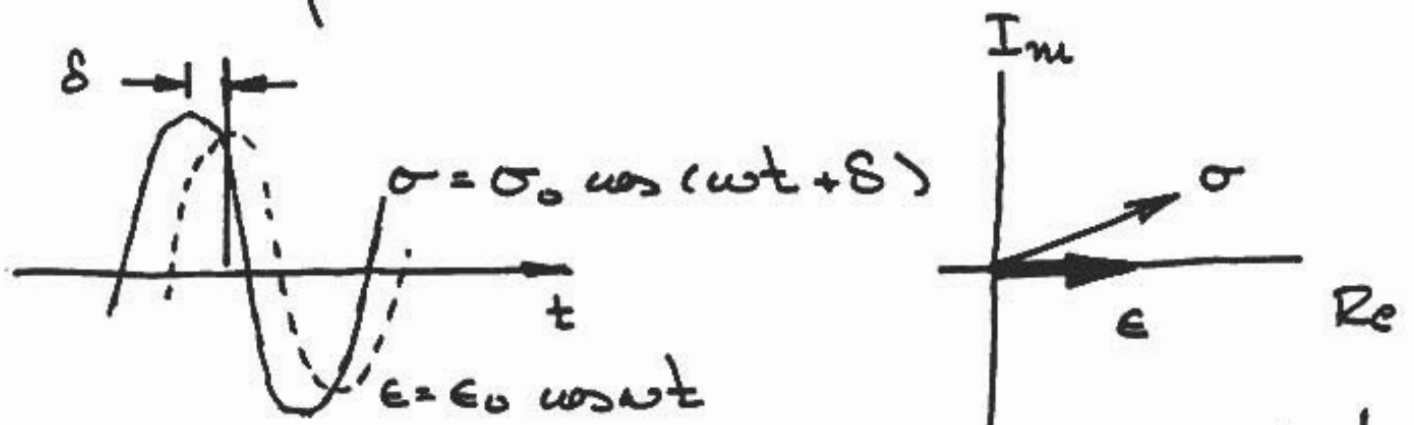


Dynamic Loading

- Strain lags stress



$$\sigma = \sigma_0' \cos \omega t + \sigma_0'' \sin \omega t = (\sigma_0' + i\sigma_0'') e^{i\omega t}$$

- Energy storage and loss

$$\begin{aligned} W &= \oint \sigma d\epsilon = \oint \sigma \frac{d\epsilon}{dt} dt \\ &= \int_0^{2\pi/\omega} (\sigma_0' \cos \omega t) (-\epsilon_0 \omega \sin \omega t) dt \\ &\quad + \int_0^{2\pi/\omega} (\sigma_0'' \sin \omega t) (-\epsilon_0 \omega \sin \omega t) dt \\ &= 0 + \pi \sigma_0'' \epsilon_0 \end{aligned}$$

$$W_{st} = \int_0^{\pi/2\omega} (\sigma_0' \cos \omega t) (-\epsilon_0 \omega \sin \omega t) dt = -\frac{1}{2} \sigma_0' \epsilon_0$$

$$\frac{W_{dis}}{W_{st}} = \frac{\pi \sigma_0'' \epsilon_0}{\frac{1}{2} \sigma_0' \epsilon_0} = 2\pi \frac{\sigma_0''}{\sigma_0'} = 2\pi \tan \delta$$

- Complex modulus

$$\begin{aligned} E^* &= E' + i E'' = \frac{\sigma_0'}{\epsilon_0} + i \frac{\sigma_0''}{\epsilon_0} \\ &= \frac{\sigma_0^*}{\epsilon_0^*} \end{aligned}$$

See Figure 12.1, "Temperature dependence of mechanical loss tangent for nylon," in McCrum, N.G. *Anelastic and Dielectric Effects in Polymeric Solids*. Mineola NY: Dover, 1991.

Sample: latex 1
Size: 9.2400 x 3.0000 x 1.0200 mm
Method: Temperature Ramp

DMA

File: C:\TA\Data\DMA\Mike T\latex1.003
Operator: Mike
Run Date: 20-Jun-03 10:57
Instrument: DMA Q800 V3.13 Build 74

