## Session \#9: Homework Solutions

## Problem \# 1

Estimate the ionic radius of $\mathrm{Cs}^{+}$. The lattice energy of CsCl is $633 \mathrm{~kJ} / \mathrm{mol}$. For CsCl the Madelung constant, M , is 1.763 , and the Born exponent, n , is 10.7. The ionic radius of $\mathrm{Cl}^{-}$is known to be $1.81 \AA$.

## Solution

$$
\mathrm{E}_{\text {lattice }}=\frac{\mathrm{Mq}_{1} \mathrm{q}_{2}}{4 \pi \varepsilon_{0} \mathrm{r}_{\mathrm{o}}}\left(1-\frac{1}{\mathrm{n}}\right) \text { and } \mathrm{r}_{0}=\mathrm{r}_{\mathrm{Cs}^{+}}+\mathrm{r}_{\mathrm{Cl}^{-}}
$$

Solve first for $r_{0}$

$$
\begin{aligned}
& r_{0}=\frac{M q_{1} q_{2} N_{A_{v}}}{4 \pi \varepsilon_{0} \mathrm{E}_{\text {lattice }}}\left(1-\frac{1}{\mathrm{n}}\right)=\frac{1.763\left(1.6 \times 10^{-19}\right)^{2} 6.02 \times 10^{23}}{4 \pi 8.85 \times 10^{-12} 6.33 \times 10^{5}}\left(1-\frac{1}{10.7}\right) \\
&=3.50 \times 10^{-10} \mathrm{~m}=3.50 \AA=r_{\mathrm{Cs}^{+}}+r_{\mathrm{Cr}} \\
& \therefore r_{\mathrm{Cs}^{+}}=3.50-1.81=1.69 \AA
\end{aligned}
$$

## Problem \#2

(a) CFCs have been implicated in ozone depletion. Show that when Freon 12 $\left(\mathrm{CCl}_{2} \mathrm{~F}_{2}\right)$ is exposed to ultraviolet radiation, the compound decomposes to produce chlorine.
(b) Draw the Lewis structure of Freon 12 and indicate the polarities of each bond within this compound.
(c) Determine the percent ionic character of the $\mathrm{C}--\mathrm{Cl}$ and $\mathrm{C}-\mathrm{F}$ bonds.

## DATA:

Average Bond Energies (kJ/mol)

## Single Bonds

H-H 435
F-F 155
Cl-Cl 242
C-C 347

## Multiple Bonds

$C=C \quad 610$
$\mathrm{C} \equiv \mathrm{C} \quad 836$

## Solution

(a) Chlorine will be liberated if the $\mathrm{C}-\mathrm{Cl}$ bond breaks, therefore, compute its strength and show that ultraviolet photons have enough energy to break the bond.

$$
\begin{aligned}
\mathrm{E}_{\mathrm{C}-\mathrm{Cl}} & =\sqrt{\mathrm{E}_{\mathrm{C}-\mathrm{C}} \times \mathrm{E}_{\mathrm{Cl}-\mathrm{Cl}}}+96.3\left(\chi_{\mathrm{C}}-\chi_{\mathrm{Cl}}\right)^{2} \\
& =\sqrt{347 \times 242}+96.3(2.55-3.16)^{2} \\
& =326 \times 10^{3} / 6.02 \times 10^{23}=5.41 \times 10^{-19} \mathrm{~J} / \text { bond }
\end{aligned}
$$

photon will break $\therefore$ this bond if $\mathrm{E}_{\mathrm{ph}}>\mathrm{E}_{\text {bond }} \Rightarrow$ critical $\lambda$ is

$$
\lambda=\frac{\mathrm{hc}}{\mathrm{E}_{\text {bond }}}=\frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{5.41 \times 10^{-19}}=3.66 \times 10^{-7} \mathrm{~m}
$$

which lies in the u.v. part of the electromagnetic spectrum.
(b) Draw the Lewis structure of Freon 12 and indicate the polarities of each bond within this compound.


## Problem \#3

The compound trichloroethylene has the composition $\mathrm{C}_{2} \mathrm{Cl}_{3} \mathrm{H}$.
(a) Formulate this compound in "Lewis" notation. Example:

(b) List the types of atomic orbitals which, upon overlap, lead to the formation of the bonds indicated in (a).
(c) List all bonds (number and type) involved in the formation of this compound.

## Solution

(a) $\mathrm{C}_{2} \mathrm{Cl}_{3} \mathrm{H}$ in Lewis notation:


Note that the H may be located on any one of the $4\left(\mathrm{sp}^{2}\right)$ hybrid orbitals without changing the properties of the compound.
(b)


On each carbon there are: three equivalent $\mathrm{sp}^{2}$ bonding orbitals in a plane and one $p$ orbital normal to the $\mathrm{sp}^{2}$ plane.


On each chlorine there is: one singly occupied p orbital.


On the hydrogen we have: one singly occupied 1s orbital.
(c) $2 \mathrm{sp}^{2}-\mathrm{sp}^{2}$ orbitals $\rightarrow 1 \sigma \mathrm{C}^{\sigma}-\mathrm{C}$

2 p -p orbitals $\rightarrow 1 \pi \mathrm{C}^{\pi}-\mathrm{C}$
6 bonds
$3 \mathrm{sp}^{2}-\mathrm{p}$ orbitals $\rightarrow 3 \sigma \mathrm{C}-\mathrm{Cl}$
$1 \mathrm{sp}^{2}-\mathrm{s}$ orbitals $\rightarrow 1 \sigma \mathrm{C}-\mathrm{H}$

## Problem \#4

Frequently the statement is made that the crystal energy ( $\Delta \mathrm{E}_{\text {cryst }}$ ) is predominantly given by the attractive energy term. How can you account for this approximation?

## Solution

$\mathrm{E}_{\text {cryst }}=\mathrm{K}(1-1 / n)=\mathrm{K}-1 / \mathrm{nK}$ where $-1 / \mathrm{nK}$ is the repulsive contribution to $\mathrm{E}_{\text {cryst }} \cdot 1 / \mathrm{n} \cong 1 / 8$. This means only $\sim 12 \%$ of the total crystal energy.

## Problem \#5

For benzene $\left(\mathrm{C}_{6} \mathrm{H}_{6}\right)$, which has a schematic carbon arrangement:

(a) Determine the total number and types of covalent bonds.
(b) For each bonding type, determine the bonding orbitals which, by overlap, lead to their formation.

Solution
(a)


From the schematic we recognize three single and three double carbon-to-carbon bonds which are respectively of the $\sigma$ (sigma) and $\sigma+\pi$ (sigma +pi$)$ variety. Left out in the schematic are the six $\mathrm{C}-\mathrm{H}$ bonds which are of the $\sigma$ variety:

(b) In the ring structure of benzene we have three double bonds, which means that all six carbon atoms are $s p^{2}$ hybridized. Accordingly:
$3 \pi$ bonds involve lateral ( $p-p$ ) obrital overlap.
6 (C-C) $\sigma$ bonds involve ( $\mathrm{sp}^{2}-\mathrm{sp}^{2}$ ) orbital overlap.
$6(\mathrm{C}-\mathrm{H}) \sigma$ bonds involve ( $\mathrm{sp}^{2}-\mathrm{s}$ ) orbital overlap.
(It should be noted that the experimentally determined bond energy for the carbon ring is larger than the sum of the bond energies of the double plus single bonds. This increased bond energy is attributed to the existence of alternating double bonds which permit the establishment of "equivalent resonance structures"

in which the double bonds are said to be delocalized:


## Problem \# 6

Why is double bonding encountered in some carbon compounds, but not in germanium compounds - although both exhibit $s p^{3}$ hybridization?

## Solution

Double bond formation, the result of lateral p orbital overlap, is contingent on close approach of the atoms. Such approach is possible for carbon with its K shell, but not for the larger germanium with its fully occupied M shell.

## Problem \#7

List the individual steps with the corresponding chemical equations used in constructing a Born-Haber cycle for the formation of $\mathrm{CaBr}_{2}$ from the element and identify those which you expect to be exothermic.

## Solution

1) $\mathrm{Ca}(\mathrm{s}) \rightarrow \mathrm{Ca}(\mathrm{v})$
2) $\mathrm{Br}_{2}(\mathrm{~g}) \rightarrow 2 \mathrm{Br} \bullet$
3) $\mathrm{Ca} \rightarrow \mathrm{Ca}^{++}+2 \mathrm{e}^{-}$

3a) $\mathrm{Ca} \rightarrow \mathrm{Ca}^{+}+1 \mathrm{e}^{-}$
3b) $\mathrm{Ca}^{+} \rightarrow \mathrm{Ca}^{++}+1 \mathrm{e}^{-}$
4) $\mathrm{Br}+1 \mathrm{e}^{-} \rightarrow \mathrm{Br}^{1-}$
5) $\mathrm{Ca}^{2+}+2 \mathrm{Br}^{1-} \rightarrow \mathrm{CaBr}_{2}$
$\Delta \mathrm{H}_{\text {vap }}{ }^{(+)}$
$\Delta \mathrm{H}_{\text {dissoc }}(+)$
$\Delta \mathrm{H}_{\text {ioniz }}(+)$
$\Delta \mathrm{H}_{\text {ioniz }}(+)$
EA $(-) \rightarrow$ exothermic
$\mathrm{DE}_{\text {cryst }}(-) \rightarrow$ exothermic
$\Delta \mathrm{H}_{\text {reaction }}=\Delta \mathrm{H}_{\text {vap }}+\Delta \mathrm{H}_{\text {dissoc }}+1 \Delta \mathrm{H}_{\text {ioniz }}+2 \Delta \mathrm{H}_{\text {ioniz }}+2 \mathrm{EA}+\Delta \mathrm{E}_{\text {cryst }}$ (it is acceptable to note $\mathrm{Ca} \rightarrow \mathrm{Ca}^{++}+2 \mathrm{e}^{-}$if you realize that 1 and 2 ionization is involved)

## Problem \#8

(a) Given the ionic radii, $\mathrm{Cs}^{+}=1.67 \AA, \mathrm{Cl}^{-}=1.81 \AA$, and the Madelung constant $\mathrm{M}(\mathrm{CsCl})=1.763$, determine to the best of your ability the molar Crystal energy ( $\Delta \mathrm{E}_{\text {cryst }}$ ) for CsCl .
(b) Not given additional data, do you expect the value obtained to be larger or smaller than theoretical, and by how many percent do you anticipate to be off?

## Solution

(a) Given the radii $\mathrm{Cs}^{+}=1.67 \AA$ and $\mathrm{Cl}^{-}=1.81 \AA$, we can assume that $\mathrm{r}_{0}$ is the sum of the two. However, we need to know the exponential constant of the repulsive term which is not provided. Considering only the attractive force:
$\Delta \mathrm{E}_{\text {cryst }}=\frac{-\mathrm{e}^{2} \mathrm{~N}_{A} \mathrm{MQ}_{1} \mathrm{Q}_{2}}{4 \pi \varepsilon_{0} \mathrm{r}_{\mathrm{o}}}$
where: $\mathrm{Q}_{1}=\mathrm{Q}_{2}=1$
$\mathrm{M}=1.763$
$\mathrm{N}_{\mathrm{A}}=6.02 \times 10^{23}$ particle/mole
$\Delta \mathrm{E}_{\text {cryst }}=\frac{-\left(1.6 \times 10^{-19} \mathrm{coul}\right)^{2} \times 6.02 \times 10^{23} \times 1.763 \times 1 \times 1}{4 \pi 8.85 \times 10^{-12} \times(1.81+1.67) \times 10^{-10} \mathrm{~m}}$
$=7.02 \times 10^{5} \mathrm{~J} / \mathrm{mole}$
(b) Because we didn't include the repulsive part, we expect the theoretical energy to be smaller by $1 / \mathrm{n}$ times then calculated above. Because n is in the range of 10 we are off by around $\mathbf{+ 1 0 \%}$

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