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3.22 Mechanical Properties of Materials
Spring 2008

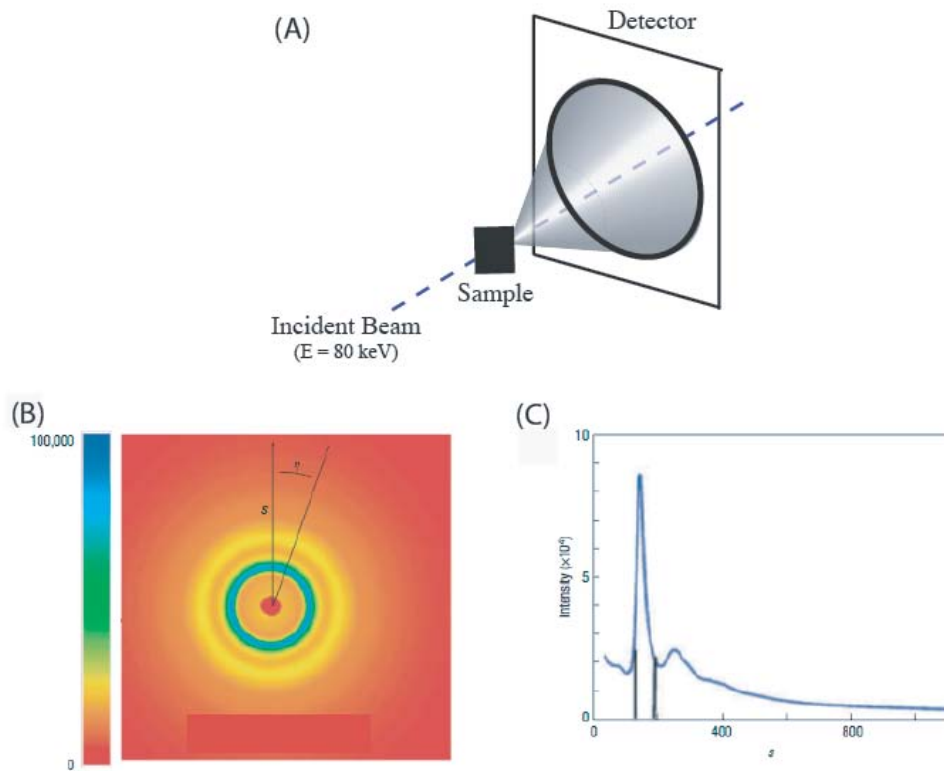
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3.22 Mechanical Behavior of Materials (Spring 2008)

Problem Set #1

Due: Monday, February 25 by 5:00 PM

1. The beauty of a bicycle wheel design is that the spokes (narrow cylindrical wires connecting the wheel center to the perimeter) are loaded in tension, despite the compressive force exerted via the bicyclist's weight. Consider a stainless steel spoke with a diameter of 2 mm, under a 1000 N (224 lbf) load.
 - (a) What is the (tensile) stress on the spoke?
 - (b) What is the elastic strain in the spoke, given that approximation of the stainless steel as a linear elastic continuum is a reasonable approximation? If the spoke is initially 25.50 cm long, what is the spoke length under the maximum applied load?
 - (c) Why would it be a bad idea to place the spoke under a compressive load of 1000 N? Explain this quantitatively, using the elastica.
 - (d) The company that employs you, *Unique Unicycles*, has designed a new bicycle that requires the spokes be loaded in compression. Design an alternative circular spoke made of stainless steel that is able to carry the same load and yet is a reasonable size and weight for the bicycle (e.g., within 25% of the initial spoke weight).
2. Spokes are typically made by drawing a billet of stainless steel through a die to reduce its diameter. During this type of processing it is common for a preferred crystallographic orientation to develop (e.g. a texture), which is a specific microstructural term meaning alignment of the unit cells' orientation. Assume that the two most common orientations to form are $\langle 111 \rangle$ and $\langle 100 \rangle$, aligned parallel to the long axis of the spoke (i.e., either the (111) or the (100) is aligned normal to the spoke long axis).
 - (a) Suppose for a bicycle wheel, it is desirable to maximize the stiffness of the spokes. Which crystallographic orientation is preferable? (For simplicity, use material data for Fe, the major element in stainless steel).
 - (b) Is your answer to part (a) the same if you were to manufacture the spokes from Mo? Why or why not?
 - (c) In a unit cell of Fe, if the stress calculated in question 1(a) is applied in a direction normal to the (111) plane, what are the components of the stress on the $\{100\}$ family of planes?
 - (d) To calculate the strain along these $\langle 100 \rangle$ directions, what other information would you need?
3. X-ray diffraction is a technique commonly used to measure microscopic elastic strains in crystalline metals. These elastic strains alter the atomic plane spacing which can be measured by Bragg's Law. Recently Poulsen and coworkers [Poulsen *et al.*, *Nature Materials* **4**, 33 (2005)] have shown that elastic strains in amorphous materials can also be measured using similar methods. However, these materials do not have atomic planes, so instead strains are measured by the pair correlation function, which is a measure of nearest-neighbor atomic spacings.



Courtesy of Henning Poulsen. Used with permission.

Figure 1: (A) Schematic of the transmission geometry used in your experiments. (B) An example of the two-dimensional diffraction pattern collected [Poulsen *et al.*, *Nature Materials* **4**, 33 (2005)]. The broad halo is indicative of scattering from an amorphous material. Taking a one-dimensional slice from the center of the pattern out to the edge yields a plot of intensity as a function of radial distance as shown in (C) [Poulsen *et al.*, *Nature Materials* **4**, 33 (2005)]. The position of the first peak is related to the nearest neighbor distances.

- (a) Derive a strain equation for this technique, analogous to the macroscopic definition of engineering strain.
- (b) Suppose you load a cylindrical piece of Zr-based metallic glass 3 mm in diameter in compression up to a load $P = 7500$ N, stopping at specific fixed loads to record diffraction data. The incident beam is monochromatic ($E = 80$ keV) and the scattering is conducted in a transmission geometry with a two-dimensional detector placed 400 mm downstream from your sample (Fig. 1A). Figure 1B is an example of the diffraction patterns you collect. The broad halo is indicative of scattering from an amorphous material. Because you used high-energy x-rays and a two-dimensional detector, you are able to differentiate the strain along (\parallel) and normal (\perp) to the direction of loading by measuring the radius of the halo along these two directions (Fig. 1C). Table 1 lists the peak radii along these directions as a function of load. Graph the strain as a function of applied stress along these two directions. How could you extend this test to confirm that the deformation was purely elastic?

Table 1: Position of the first peak along the longitudinal and transverse directions.

P (kN)	Radius \parallel to P (mm)	Radius \perp to P (mm)
0	26.03	26.03
-1.5	26.09	26.01
-3.0	26.17	25.98
-4.5	26.22	25.96
-6.0	26.30	25.94
-7.5	26.36	25.92

- (c) Compare the true vs. engineering strain and the true vs. engineering stress at the state of maximum applied load.
- (d) Calculate the Poisson's ratio ν and the Young's elastic modulus E of the metallic glass continuum.
- (e) What are the strains along the principle stress axes σ_i during the peak compressive load? How would you construct a complete strain tensor using x-ray diffraction? Write out the full small-strain tensor ϵ_{ij} under uniaxial loading using as few variables as possible.
- (f) What would the resulting diffraction pattern look like if you applied a shear stress τ ? a hydrostatic compressive stress σ_{ii} ? (Please show this by drawing a schematic XRD pattern, and comparing it to the given, unstressed state.)
4. A plate of metallic glass is subjected to the following plane strain state: $\epsilon_{11} = \epsilon_{22} \neq 0$; $\epsilon_{12} = \epsilon_{23} = \epsilon_{13} = \epsilon_{33} = 0$. Determine the ratio of the stress components σ_{33}/σ_{11} in terms of the shear elastic modulus G and Poisson's ratio ν .
5. We know that elastic constants are expressed as 4th rank tensors, but can be reduced to a matrix of only 36 components. A cubic crystal (simple cubic, body centered cubic, or face centered cubic) has three independent elastic constants, for example given as the stiffness matrix elements C_{11} , C_{12} , and C_{44} . An isotropic continuum, which is what we've approximated the stainless steel and amorphous metal to be in the above problems, has only two independent elastic constants, for example given as E and ν . By exerting stresses and symmetry operations on such a cubic crystal, prove how 3 independent elastic constants can be reduced to only 2 if the material is elastically isotropic.