PROBLEM 1

(a) (i) $$s_{ji} = \frac{\partial \varepsilon_i}{\partial \sigma_j} = \frac{1}{(\partial \sigma_j / \partial \varepsilon_i)} \quad \text{—— ①}$$

$$u = \frac{1}{2}\sum_{j=1}^{6} \sigma_j \varepsilon_j$$

$$\Rightarrow 2\frac{\partial u}{\partial \varepsilon_j} = \sigma_j \quad \text{—— ②}$$

plugging in the expression for σ_j from ② in ①

$$s_{ji} = \frac{1}{2\,\partial\left(\frac{\partial u}{\partial \varepsilon_j}\right)/\partial \varepsilon_i} = \frac{1}{2\cdot\left(\partial^2 u / \partial \varepsilon_j \partial \varepsilon_i\right)}$$

$$= \frac{1}{2\cdot\left(\partial^2 u / \partial \varepsilon_i \partial \varepsilon_j\right)} = \frac{1}{2\,\partial(\partial u / \partial \varepsilon_i)/\partial \varepsilon_j}$$

$$= \frac{1}{\partial \sigma_i / \partial \varepsilon_j} = \frac{\partial \varepsilon_j}{\partial \sigma_i} = s_{ij}$$

(ii) $$s_{ij} = s_{ji} \Rightarrow s_{12} = s_{21} \quad \text{—— ①}$$

If apply only $\sigma_1 \Rightarrow \varepsilon_1 = s_{11}\sigma_1 \Rightarrow s_{11} = 1/E_1$

Similarly only $\sigma_2 \Rightarrow \varepsilon_2 = s_{22}\sigma_2 \Rightarrow s_{22} = 1/E_2$

If only $\sigma_1 \Rightarrow \varepsilon_1 = \sigma_1 / E_1$

$$\varepsilon_2 = s_{21}\sigma_1$$

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{s_{21}\sigma_1}{(\sigma_1/E_1)} = -E_1 \cdot s_{21}$$

$$\Rightarrow s_{21} = -\frac{\nu_{12}}{E_1} \quad \text{—— ②}$$

Similarly if only $\sigma_2 \Rightarrow s_{12} = -\dfrac{\nu_{21}}{E_2}$ —— ③

From ①, $S_{12} = S_{21}$

$$\Rightarrow \quad -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}$$

$$\Rightarrow \quad \boxed{\nu_{12} \cdot E_2 = \nu_{21} \cdot E_1}$$

(b) From previous part, $S_{11} = 1/E_1$, $S_{22} = 1/E_2$

similarly $S_{33} = 1/E_3$

Also $S_{12} = -\frac{\nu_{21}}{E_2}$

similarly $S_{13} = -\frac{\nu_{31}}{E_3}$, $S_{23} = -\frac{\nu_{32}}{E_3}$

(c) Rigid die $\Rightarrow \varepsilon_2 = \varepsilon_3 = 0$

cylindrical die $\Rightarrow \sigma_2 = \sigma_3$

Hydrostatic state $\Rightarrow \sigma_1 = \sigma_2 = \sigma_3 = \sigma$

& $\sigma_4 = \sigma_5 = \sigma_6 = 0$

$$\varepsilon_2 = S_{12}\sigma_1 + S_{22}\sigma_2 + S_{23}\sigma_3$$

$$= -\frac{\nu_{21}}{E_2}\sigma + \frac{1}{E_2}\sigma - \frac{\nu_{32}}{E_3}\sigma$$

$$\Rightarrow \quad \boxed{(1-\nu_{21})E_3 = \nu_{32}E_2} \text{ — (i)}$$

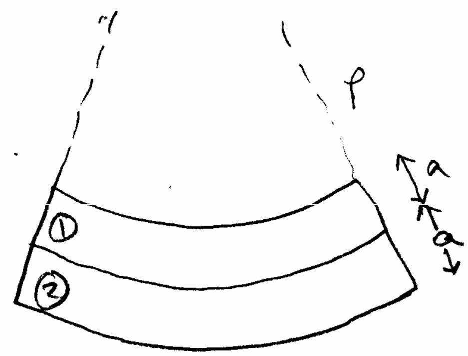
(ii) $\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]}$

$= 0$ (as $\sigma_1 = \sigma_2 = \sigma_3$)

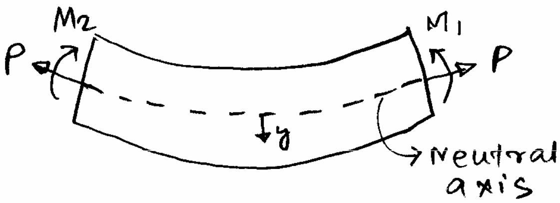
$\Rightarrow$ The material will never yield no matter what stress you apply.

PROBLEM 2

E_1	$E_2 = 0.2\,E_1$
α_1	$\alpha_2 = 10\,\alpha_1$
a	a
b	b



Layer 1



$$P = \frac{1}{a}\left[\frac{E_1 I_1 + E_2 I_2}{\rho}\right]$$

$$I_1 = \frac{ba^3}{12} = I_2$$

σ at any point 'y' away from the neutral axis can be calculated as:

strain in layer ① at 'y' from neutral axis $= \varepsilon^{①}(y)$

$$\varepsilon_y^{①} = \frac{P}{a_1 b E_1} + \frac{y}{\rho} \qquad \left[-\frac{a_1}{2} < y < \frac{a_1}{2}\right]$$

$$\Rightarrow \sigma_y^{①} = E_1 \cdot \varepsilon_y^{①} = \frac{P}{a_1 b} + \frac{yE_1}{\rho}$$

$$= \frac{1}{\rho}\left[\frac{1}{a^2 b}\left(E_1 \frac{ba^3}{12} + E_2 \frac{ba^3}{12}\right) + yE_1\right]$$

$$\sigma_y^{①} = \frac{1}{\rho}\left[\frac{a}{12}(E_1 + E_2) + yE_1\right]$$

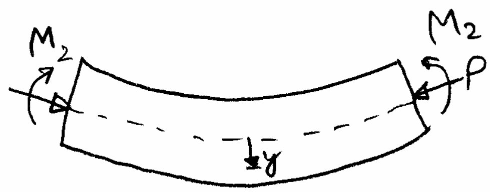
$\sigma_{max}^{①}$ is at $y = a/2$

$$\Rightarrow \sigma_{max}^{①} = \frac{1}{\rho}\left[\frac{a}{12}(E_1 + 0.2E_1) + \frac{aE_1}{2}\right]$$

$$= 0.6\left(\frac{E_1 a}{\rho}\right)$$

$$\sigma^{(1)}_{min}\left(y=-\frac{a}{2}\right) = -0.4\left(\frac{E_1 a}{\rho}\right)$$

Layer 2



similarly

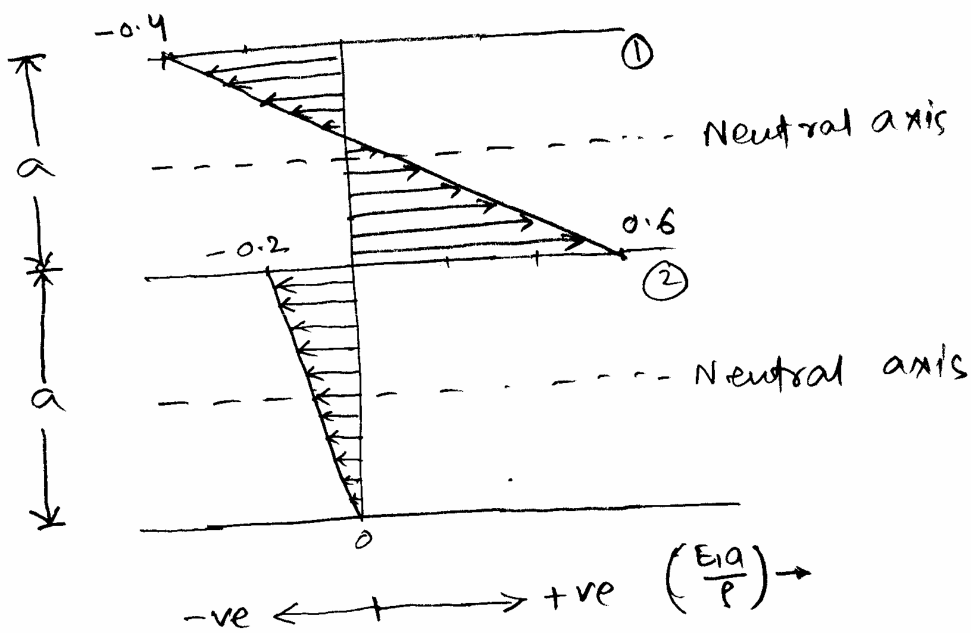
$$\sigma^{(2)}_{y} = -\frac{1}{\rho}\left[\frac{a}{12}(E_1+E_2) - yE_2\right]$$

$$\sigma^{(2)}_{max}\left(y=\frac{a}{2}\right) = -\frac{1}{\rho}\left[\frac{a}{12}\,1.2E_1 + 0.2E_1\left(-\frac{a}{2}\right)\right]$$

$$= 0$$

$$\sigma^{(2)}_{min}\left(y=-\frac{a}{2}\right) = -\frac{1}{\rho}\left[\frac{a}{10}E_1 + \frac{a}{10}E_1\right] = -0.2\left(\frac{aE_1}{\rho}\right)$$

(b)



$\boxed{\text{Stress profile, } \sigma(y)}$

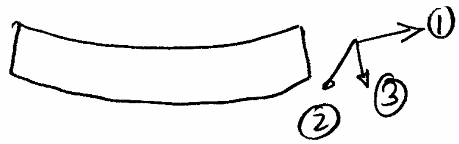
(c) In bilayer beam, stress in the depth direction is negligible & can be neglected.

Let say the longitudinal dir^n = 1

in the depth dir^n (into the page) = 2

through the thickness dir^n = 3

i.e.,



$\sigma_1 \neq 0, \quad \sigma_2 = \sigma_3 = 0$

von Mises criterion

$$\Rightarrow \sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}$$

$$= \sqrt{\frac{1}{2}(\sigma_1^2+\sigma_1^2)} = |\sigma_1| = \sigma_{yield}$$

absolute value of σ

for layer ①, $|\sigma^{①}_{max}| > |\sigma^{①}_{min}|$

$$\Rightarrow \text{yielding when } \underbrace{0.6\left(\frac{E_1 a}{\rho}\right)}_{=|\sigma^{①}_{max}|} = \sigma^{①}_{yield}$$

for layer ②,

$$|\sigma^{②}_{max}| < |\sigma^{②}_{min}|$$

$$\Rightarrow \text{yielding when } |\sigma^{②}_{min}| = \sigma^{②}_{yield}$$

$$\Rightarrow 0.2\left(\frac{E_1 a}{\rho}\right) = \sigma^{②}_{yield}$$

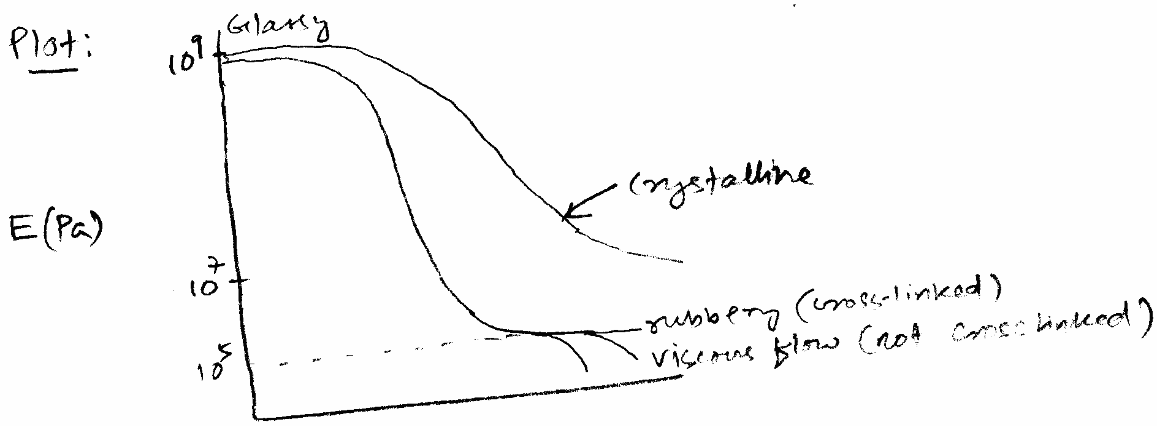
same increase in temp $\Rightarrow \rho$ is same

$$\Rightarrow \frac{E_1 a}{\rho} = \frac{\sigma^{①}_{yield}}{0.6} = \frac{\sigma^{②}_{yield}}{0.2} \Rightarrow \boxed{\frac{\sigma^{②}_{yield}}{\sigma^{①}_{yield}} = 0.33}$$

PROBLEM 3

(a) → in class notes for description.

→ Plot:



Problem 3(b)

$$Given:$$
$$\tau_1 = \tau, \tau_2 = 3\tau, t_1 = 2\tau, t_2 = 7\tau,$$
$$\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma, \varepsilon(t_1) = \varepsilon, \varepsilon(t_2) = 2.5\varepsilon$$

$$J(t) = a\sqrt{t} + b$$
$$\varepsilon(t_1) = \Delta\sigma_1 \times J(t_1 - \tau_1) \Rightarrow \frac{\varepsilon}{\Delta\sigma} = a\sqrt{\tau} + b - [1]$$
$$\varepsilon(t_2) = \Delta\sigma_1 \times J(t_2 - \tau_1) + \Delta\sigma_2 \times J(t_2 - \tau_2)$$
$$\Rightarrow \frac{2.5\varepsilon}{\Delta\sigma} = \left(\sqrt{6}a\sqrt{\tau} + b\right) + \left(2a\sqrt{\tau} + b\right)$$
$$\Rightarrow \frac{2.5\varepsilon}{\Delta\sigma} = 4.45a\sqrt{\tau} + 2b - [2]$$

Solve [1] and [2] to get values of 'a' and 'b'

$$a \approx 0.2\frac{\varepsilon}{\Delta\sigma \cdot \sqrt{\tau}}, b \approx 0.8\frac{\varepsilon}{\Delta\sigma}$$

(c) Maxwell model

Relaxation response: $\epsilon = \text{const} \Rightarrow \sigma = E\varepsilon \exp\left(-\frac{Et}{\eta}\right)$

- Typically relaxation can't be represented by single exponential term
- σ doesn't typically decay to zero.

creep response: $\frac{d\sigma}{dt} = 0 \Rightarrow \varepsilon = \frac{\sigma}{E} + \frac{\sigma}{\eta} t$ (for $t < t_1$)

$\Rightarrow$ not generally true for viscous materials

Voigt Model

Relaxation response: $\epsilon = \text{const} \Rightarrow \sigma = E\varepsilon$

inadequate representation of relaxation

creep response: $\sigma = \text{const} \Rightarrow \sigma = \sigma_0$

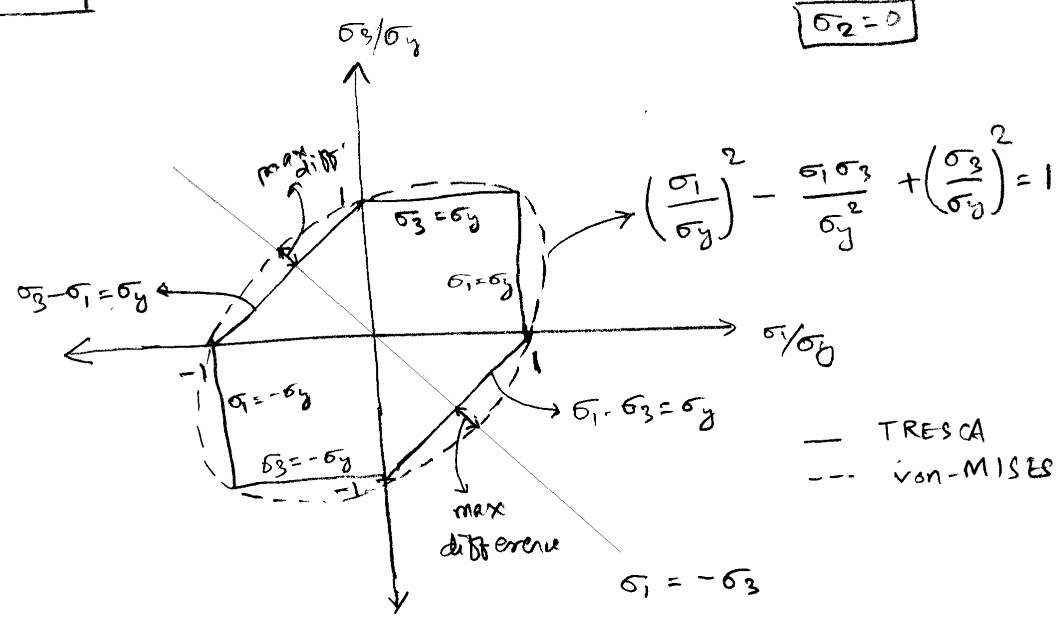
$$\Rightarrow \varepsilon = \frac{\sigma_0}{E}\left[1 - \exp\left(-\frac{Et}{\eta}\right)\right] \quad (t < t_1)$$

$$\varepsilon = \frac{\sigma_0}{E} \exp\left(-\frac{Et}{\eta}\right) \quad (t > t_1)$$

OK

PROBLEM 4

(a)



(b) $\sigma_y = \sigma_y^0 \left[1 - \frac{kT}{Q_b} \ln\left(\frac{\dot{\gamma}_0}{\dot{\gamma}}\right)\right]$

$\Rightarrow \frac{5}{6} \cancel{\sigma_y^0} = \cancel{\sigma_y^0} \left[1 - \frac{\cancel{kT}}{60\cancel{kT}} \ln\left(\frac{\dot{\gamma}_0}{\dot{\gamma}}\right)\right]$

$\Rightarrow \frac{1}{60} \ln\left(\frac{\dot{\gamma}_0}{\dot{\gamma}}\right) = 1/6 \Rightarrow \ln\left(\frac{\dot{\gamma}_0}{\dot{\gamma}}\right) = 10 \quad — (1)$

$r_p = \frac{K_I^2}{2\pi \sigma_y^2} \Rightarrow \frac{r_p(T)}{r_p(1.2T)} = \left[\frac{\sigma_y(1.2T, \dot{\gamma})}{\sigma_y(T, \dot{\gamma})}\right]^2$

$\sigma_y(1.2T, \dot{\gamma}) = \sigma_y^0 \left[1 - \frac{1.2\cancel{kT}}{60\cancel{kT}} \times 10\right] = \sigma_y^0\left[1 - \frac{12}{60}\right]$

$= \frac{48}{60} \sigma_y^0$

$\sigma_y(T, \dot{\gamma}) = \sigma_y^0 \left[1 - \frac{kT}{60kT} \times 10\right] = \frac{5\sigma_y^0}{6} = \frac{50\sigma_y^0}{60}$

$\frac{r_p(T)}{r_p(1.2T)} = \left(\frac{48}{50}\right)^2 \cong 0.922$

$\Rightarrow$ As $T \uparrow \Rightarrow r_p \uparrow$ or as $T \downarrow \Rightarrow r_p \downarrow$

$\Rightarrow$ material becomes more brittle $\Rightarrow$ gets ductile-brittle transition

(as $T \downarrow$ or as $\dot{\gamma} \uparrow$)