18.01 Single Variable Calculus
Fall 2006

18.01 Practice Exam 2 Fall 2006

Problem 1. $f(x) = 2x^3+3x^2-12x+1$

$f'(x) = 6x^2+6x-12 \qquad f'(x)=0 \quad x=1,-2$

$f''(x) = 12x+6 \qquad f''(x)=0 \quad x=-1/2$

$f''(1)=18>0$ so $(1,-6)$ is a loc. min

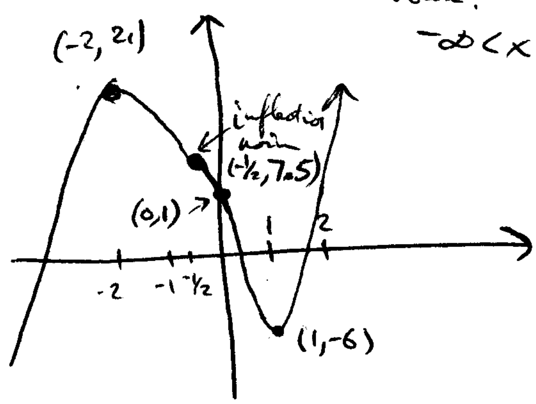
$f''(-2)=-18<0$ so $(-2,21)$ is a loc. max

$x\to\infty \quad f(x)\to\infty$

$x\to-\infty \quad f(x)\to-\infty$

concave up: $-1/2<x<\infty$

concave down: $-\infty<x<-1/2$



Problem 2. $V=\pi r^2 h = 64\pi$

$r^2h=64 \quad h=64/r^2$

$A = 2\pi rh + \pi r^2$

$\quad = \frac{128\pi}{r} + \pi r^2$

$0<r<\infty$

$\frac{dA}{dr} = -\frac{128\pi}{r^2} + 2\pi r \qquad \frac{dA}{dr}=0,\ r=4$ (so $h=4$)

$A(r=4) = 48\pi$

$\frac{d^2A}{dr^2} = \frac{256\pi}{r^3} + 2\pi > 0$ at $r=4$

so $r=4, h=4, A=48\pi$ is the minimum, since as $r\to0$ or as $r\to\infty$, $A\to\infty$.

Problem 3. a) $\int e^{-3x}dx = -\frac{1}{3}e^{-3x}+C$

b) $\int \cos^2 x \sin x\, dx = -\int u^2 du$

$u=\cos x$

$du=-\sin x\, dx$

$= -\frac{1}{3}u^3 + C$

$= -\frac{1}{3}\cos^3 x + C$

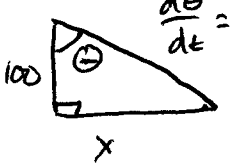
c) $\int \frac{x\,dx}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{-1}{2}\cdot 2u^{1/2} + C$

$u=1-x^2$

$du=-2x\,dx$

$= -\sqrt{1-x^2}+C$

Problem 4.



$\frac{d\theta}{dt} = \frac{\pi}{4}$ rad/min $\qquad \tan\theta = x/100 \qquad 100\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

$\left.\frac{dx}{dt}\right|_{\theta=\pi/3} = 100\cdot4\cdot\frac{\pi}{4} = \boxed{100\pi} \approx 314$ m/min $=18$ km/h

Problem 5. a) $e^{-x}\sqrt{1+cx} \approx (1-x)(1+\frac{1}{2}cx) = 1+(\frac{c}{2}-1)x - \frac{1}{2}cx^2$. so f is const to first order if $c=2$.

b) $\frac{dx}{\sqrt{1-x^2}} = 2t\,dt$

$\arcsin x = t^2+c$

$x=\sin(t^2+c)$

$1=x(0)=\sin c$

$c=\pi/2$

$x=\sin(t^2+\frac{\pi}{2})$

Problem 6. a) $\frac{\ln(1+x)-\ln(1+0)}{x-0} = \left.\frac{d}{dx}(\ln(1+x))\right|_{x=c}$

$\ln(1+x) = \frac{1}{1+c}x < x$, if $x>0$. $\quad 0<c<x$

b) Suppose $f(a)=f(b)=0, a\neq b$. So there is $a<d<b$ s.t. $f'(d)=0$. No. $f'(d)=3d^2+1>0$ for any d. f can't have 2 zeros.