EXAM 1

(1) (10 points) Find $\int_{-2}^{3} 2x^2 \left[\, |x| \,\right] dx$. (Here, as usual, $[x]$ denotes the largest integer $\leq x$.)

(2) (10 points) Let f be an integrable function on $[a, b]$ and $a < d < b$. Further suppose that
$$\int_{a+d}^{b+d} f(x-d)dx = 4, \qquad \int_{-a}^{-d} f(-x)dx = 7.$$
Find
$$\int_{d}^{b} 2f(x)dx.$$

(3) (10 points) Suppose A, B are inductive sets. Prove $A \cap B$ is an inductive set. Give an example of inductive sets A, B such that $A - B$ is not an inductive set.

(4) (15 points) Let f be a bounded, integrable function on $[0, 1]$. Suppose there exists $C \in \mathbb{R}$ such that $f(x) \geq C > 0$ for all $x \in [0, 1]$. Prove that $g(x) = 1/f(x)$ is integrable on $[0, 1]$.

(5) (15 points) Suppose f is defined for all $x \in (-1, 1)$ and that $\lim_{x \to 0} f(x) = A$. Show there exists a constant $c < 1$ such that $f(x)$ is bounded for all $x \in (-c, c)$.

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