Example: $\sin(10t)$

The chain rule (composition rule) says that $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$. In other words, the derivative of the composition of functions $f(g(t))$ is the derivative of the outside function $f(x)$ times the derivative of the inside function $g(t)$.

For the example $\frac{d}{dt}\sin(10t)$, the inside function is $x = 10t$ and the outside function is $y = \sin x$. Using the rules we know, we can compute that $\frac{dy}{dx} = \cos x$ and $\frac{dx}{dt} = 10$, so:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \cos x \cdot 10.$$

Since we're the only ones who know the value of x in this formula, we replace x by $10t$ to get:

$$\frac{dy}{dt} = \cos(10t) \cdot 10 = 10\cos(10t).$$

Once you've had more practice using the chain rule, you won't always need the variable x that represents the inside function. When you look at $\frac{d}{dt}\sin(10t)$ you might say to yourself: "The derivative of the outside function, sine, is cosine. I'm plugging $10t$ into it. And the derivative of $10t$ is just 10. So $\frac{d}{dt}\sin(10t) = \cos(10t) \cdot 10$.

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