Infinite Discontinuities

In an *infinite* discontinuity, the left- and right-hand limits are infinite; they may be both positive, both negative, or one positive and one negative.

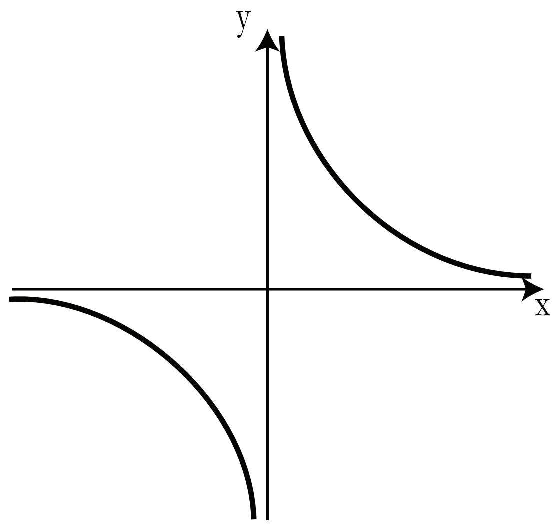

Figure 1: An example of an infinite discontinuity: $\dfrac{1}{x}$

From Figure 1, we see that $\lim_{x\to 0^+} \dfrac{1}{x} = \infty$ and $\lim_{x\to 0^-} \dfrac{1}{x} = -\infty$. Saying that a limit is ∞ is different from saying that the limit doesn't exist – the values of $\frac{1}{x}$ are changing in a very definite way as $x \to 0$ from either side. (Note that it's not true that $\lim_{x\to 0} \dfrac{1}{x} = \infty$ because ∞ and $-\infty$ are different.)

There are two more things we can learn from this example. First, sketch the graph of $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$; it also has an infinite discontinuity at $x = 0$. Notice that the derivative of the function $\frac{1}{x}$ is always negative. It may seem strange to you that the derivative is decreasing as x approaches 0 from the positive side while $\frac{1}{x}$ is increasing, but very often the graph of the derivative will look nothing like the graph of the original function.

What the graph of the derivative $-\frac{1}{x^2}$ is showing you is the slope of the graph of $\frac{1}{x}$. Where the graph of $\frac{1}{x}$ is not very steep, the graph of $-\frac{1}{x^2}$ lies close to the x-axis. Where the graph of $\frac{1}{x}$ is steep, the graph of $-\frac{1}{x^2}$ is far away from the x-axis. The value of $-\frac{1}{x^2}$ is always negative, and the graph of $\frac{1}{x}$ always slopes downward.

Finally, $\frac{1}{x}$ is an odd function and $-\frac{1}{x^2}$ is an even function. When you take the derivative of an odd function you always get an even function and vice-versa. If you can easily identify odd and even functions, this is a good way to check

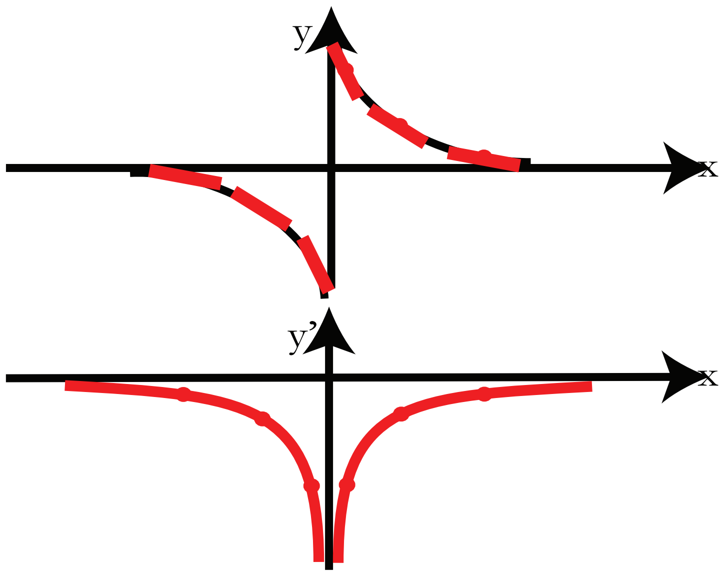

Figure 2: Top: graph of $f(x) = \dfrac{1}{x}$ and Bottom: graph of $f'(x) = -\dfrac{1}{x^2}$

your work.