CHRISTINE BREINER: Welcome back to recitation.

Today we're going to talk about some rules of logarithms that you're going to need to remember. We're going to prove why one of them is true, and then I'm going to ask you to use these rules to take a derivative of a function.

So let's just look at these rules first. So I want to point out, as I'm talking about these rules, the first three are written with natural log. But one can also write them in any base as long as the base is the same all the way across. So in any legitimate base that one is allowed to use, so with a positive base, one can use it all the way across instead of the natural log.

So the first one says that the natural log of a product is equal to the sum of the natural logs. So natural log of M times N is equal to the natural log of M plus natural log of N.

The second one says the natural log of a quotient is equal to the difference of the natural logs. So natural log of M divided by N is equal to natural log of M minus natural log of N.

This third one says that the natural log of something raised to a power is that power, as a coefficient, times the natural log of the something. So natural log of M to the k is equal to k times the natural log of M.

And what I want to point out is that there's a distinct difference where the power is. So if the power is inside the argument then this rule holds, but if the power is outside the argument-- so if it's natural log of M, the whole thing raised to a power-- this does not work. This is not equal to what's written above.

And then the third-- the fourth one, sorry. The fourth one is a change of base formula. So if I have, if I have log base something, b-- maybe I want to change the base-- of M, I can rewrite that in the base e. I can write that as natural log of M divided by natural log of b.

And I want to point out, a common mistake people make is sometimes they confuse the second and the fourth because they both have quotients. But notice that the second one is the natural log of a quotient, and the fourth one is about the quotient of natural logs. So that's a distinct difference, and hopefully then you see that they are not-- these two statements are not, in fact, the same statement.

So now what I'd like to do is using what we know about exponential and log functions, I want to prove number one. So let's set out to do that.

Well, in order to make this top line make sense we know that M and N have to be positive. And so I can find-- actually, let me write first what we're doing. We're going to prove one. So with M and N both positive I can find values a and b such that e to the a equals M and e to the b is equal to N. And let me just write out also what that means. Because exponential and log functions are inverses of one another, this means that a is equal to natural log of M and b is equal to natural log of N. So these are equivalent statements. This statement and this statement are equivalent. This statement and this statement are equivalent. So now let's use that information to try and solve the problem. To try and prove number one.

So the natural log of M times N, well, what is that? M is e to the a, N is e to the b. So I can write this as natural log of e to the a times e to the b. What's e to the a times e to the b? This is where we use our rules of exponents. e to the a times e to the b is e to the a plus b. So this is natural log of e to the a plus b.

And now, what's the point? The point is that natural log in exponential functions are inverses of one another, or natural log of e to the x is x. So natural log of e to the a plus b is just a plus b. And I've already recorded for you what those are-- it's natural log of M plus natural log of N. So notice we've done what we set out to do. Natural log of the quantity M times N is equal to natural log of M plus natural log of N.

And in a similar flavor one could immediately do number two, and number three follows quite similarly, as well. It uses-- obviously, these are going to use different rules for exponents besides the product of two exponential functions is equal to the sum of the powers. It's going to use some of those other rules. And I believe that some of these other things might actually also be proven in a later lecture in the actual course. So you'll see these. But I would say, you might want to try and prove two and three, at least, on your own-- might be helpful to look at how those work using the same kind of rules here.

So now what I'd like us to do is using these rules, I'd like us to take a derivative. So what I want us to look at is y equals the square root of x times x plus 4. And we'll just assume that x is bigger than 0. And I want you to find y prime.

Now you could do this just brute force, cranking it out. But I'd like you to try and use the log differentiation technique in order to find this derivative. I'll give you a moment to do it and then I'll come back and I'll show you how I do it.

OK. Welcome back.

So I'm going to use the log differentiation and the rules I have on the side of the board there to take a derivative to find y prime.

So first what we do is we take the log of both sides and then we use some of the rules of logarithms to simplify the expression on the right-hand side. So I will take natural log y is equal to natural log of the square root of x times x plus 4. Now square root-- wow, sorry-- square root is the power of something raised to the 1/2. Right? That's what it means to take a square root. You can take this whole product and raise it to the 1/2. So I'm going to use rule number three and I'm going to bring that 1/2 that is a power out in front of the log. So I can rewrite this expression as 1/2 log of this product. That's one too many parentheses, but that's OK.

OK. So I have 1/2 the natural log of the product of x and x plus 4. So now I'm going to use rule number one which changes the product-- the natural log of a product into the sum of the natural logs. And I can rewrite this as 1/2 natural log x plus 1/2 natural log the quantity x plus 4. Essentially what I'm doing here is I have to distribute this 1/2 because I had one term, and then I'm going to have two terms that are added together, but the 1/2 applies to both of them.

So now I have this nice setup. I have natural log of y is equal to something in terms of x. And now I can take the derivative of a both sides. Now remember, I want to find y prime, so there's some implicit differentiation going on. So let's just be careful when we do that. If I take the derivative of this side I don't just get y prime, I get y prime over y. Where does that come from? Well, d/dx of this expression is the derivative of the natural log evaluated at y, then times the derivative of y.

You've seen this, I think, a lot by now, but just to make sure you understand where both of those come from. So when I take the derivative here I get y prime over y. When I take the derivative here with respect to x, well, derivative of natural log of x is just 1 over x. So I get 1 over 2x. And then the derivative of natural log of x plus 4, if I use the chain rule I get 1 over x plus 4 times the derivative of x plus 4, which is still just 1, so I get 1 over 2 times x plus 4.

So now I wanted us to find y prime. So to find y prime I'm going to move over a little bit more. And just notice that y prime is going to equal y times all of that. Well, I know y. So I'm going to write what y is. y is the square root of x times x plus 4 times this quantity. 1 over 2x plus 1 over 2 times x plus 4.

So that's actually one way to write the derivative of y prime now-- or sorry, the derivative of y. Now I could combine these two fractions into a single fraction and try and make it look a little bit nicer, or I could just leave it this way. This is technically a derivative. So if I started trying to combine things I might find out that I could have just taken the derivative the long way. So this is a nice short way to just get to a place where I can start to find out something about the derivative of y.

So I guess I'll stop there.