## Summation

Compute the following sums:

a) $\displaystyle\sum_{k=1}^{5} k^2$

b) $\displaystyle\sum_{k=1}^{3} (2k)^2$

c) $\displaystyle\sum_{n=1}^{4} (-1)^n n$

d) $\displaystyle\sum_{k=0}^{5} 2^k$

### Solution

The most difficult part of these problems is interpreting the summation notation. That will grow easier with practice.

a) $\displaystyle\sum_{k=1}^{5} k^2$

$$
\begin{aligned}
\sum_{k=1}^{5} k^2 &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 \\
&= 1 + 4 + 9 + 16 + 25 \\
&= 55
\end{aligned}
$$

This answer is more than 5 times the smallest number in the sum and less than 5 times the largest, so we know it's at least plausible.

b) $\displaystyle\sum_{k=1}^{3} (2k)^2$

$$
\begin{aligned}
\sum_{k=1}^{3} (2k)^2 &= (2 \cdot 1)^2 + (2 \cdot 2)^2 + (2 \cdot 3)^2 \\
&= 4 + 16 + 36 \\
&= 56
\end{aligned}
$$

By using values such as $2k$ and $2k + 1$ in our sums, we can restrict our attention to exclusively even or odd numbers.

c) $\displaystyle\sum_{n=1}^{4}(-1)^n n$

$$\sum_{n=1}^{4}(-1)^n n \;=\; (-1)^1 1 + (-1)^2 2 + (-1)^3 3 + (-1)^4 4$$
$$=\; -1 + 2 + -3 + 4$$
$$=\; 2$$

Multiplying the summand by $(-1)^n$ gives us a sum with alternating signs. One way to check your results when computing sums like this one is to estimate the extent to which the negative terms cancel out the positive ones.

d) $\displaystyle\sum_{k=0}^{5} 2^k$

$$\sum_{k=0}^{5} 2^k \;=\; 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$
$$=\; 1 + 2 + 4 + 8 + 16 + 32$$
$$=\; 63$$

Not all summations start with the index value 1.

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