Example: Change of Variables

Example: $\int_1^2 (x^3+2)^5 x^2\,dx$

Before, we would have tried to handle this integral by substitution, using $u = x^3 + 2$. We're going to do the same thing here, taking into account the limits.

First we compute $du = 3x^2$. We'll be integrating u^5, and $\frac{1}{3}\,du$ will replace $x^2\,dx$. All that's left to set up the integral is to figure out the new limits; this is one of the reasons we use dx and du — to remind ourselves which variable is involved in the integration.

Initially, x is varying between 1 and 2. So $u_1 = 1^3+2 = 3$ and $u_2 = 2^3+2 = 10$. Now we can finish the problem:

$$\begin{aligned}
\int_{x=1}^{x=2} (x^3+2)^5 x^2\,dx &= \int_{u=3}^{u=10} u^5 \frac{1}{3} u\,du \\
&= \frac{u^6}{18}\Big|_{u=3}^{u=10} \qquad \left(\textbf{not } \tfrac{u^6}{18}\Big|_1^2\right) \\
&= \frac{1}{18}(10^6 - 3^6)
\end{aligned}$$

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